



Victorian Certificate of Education 2011

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures

Words

Letter

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MATHEMATICAL METHODS (CAS)

Written examination 1

Tuesday 8 November 2011

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 13 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

- a. Differentiate $\sqrt{4-x}$ with respect to x .

1 mark

- b. If $g(x) = x^2 \sin(2x)$, find $g'\left(\frac{\pi}{6}\right)$.

2 marks

TURN OVER

Question 2

- a. Find an antiderivative of $\frac{1}{3x-4}$ with respect to x .

1 mark

- b. Solve the equation $4^x - 15 \times 2^x = 16$ for x .

3 marks

Question 3

- a. State the range and period of the function

$$h: R \rightarrow R, h(x) = 4 + 3\cos\left(\frac{\pi x}{2}\right).$$

2 marks

- b. Solve the equation

$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2} \text{ for } x \in [0, \pi].$$

2 marks

TURN OVER

Question 4

If the function f has the rule $f(x) = \sqrt{x^2 - 9}$ and the function g has the rule $g(x) = x + 5$

- a. find integers c and d such that $f(g(x)) = \sqrt{(x+c)(x+d)}$

2 marks

- b. state the maximal domain for which $f(g(x))$ is defined.

2 marks

Question 6

Consider the simultaneous linear equations

$$\begin{aligned} kx - 3y &= k + 3 \\ 4x + (k + 7)y &= 1 \end{aligned}$$

where k is a real constant.

- a.** Find the value of k for which there are infinitely many solutions.

3 marks

- b.** Find the values of k for which there is a unique solution.

1 mark

Question 7

A biased coin is tossed three times. The probability of a head from a toss of this coin is p .

- a. Find, in terms of p , the probability of obtaining
- i. three heads from the three tosses

- ii. two heads and a tail from the three tosses.

1 + 1 = 2 marks

- b. If the probability of obtaining three heads equals the probability of obtaining two heads and a tail, find p .

2 marks

Question 8

Two events, A and B , are such that $\Pr(A) = \frac{3}{5}$ and $\Pr(B) = \frac{1}{4}$.

If A' denotes the complement of A , calculate $\Pr(A' \cap B)$ when

a. $\Pr(A \cup B) = \frac{3}{4}$

2 marks

b. A and B are mutually exclusive.

1 mark

Question 9

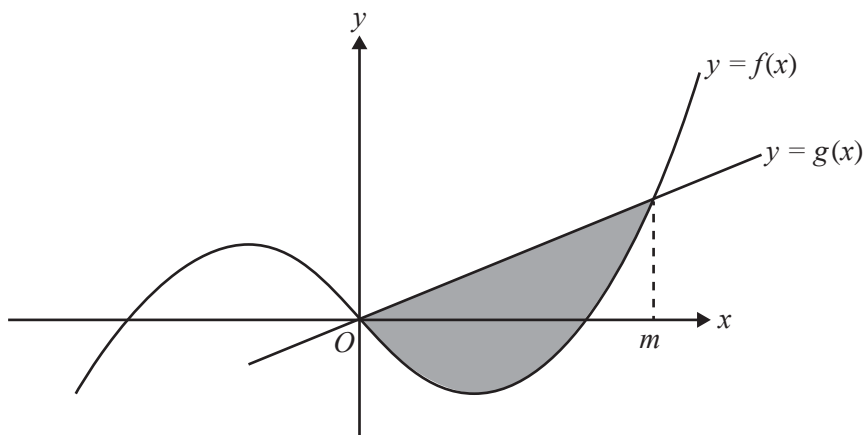
Parts of the graphs of the functions

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3 - ax \quad a > 0$$

$$g: \mathbb{R} \rightarrow \mathbb{R}, g(x) = ax \quad a > 0$$

are shown in the diagram below.

The graphs intersect when $x = 0$ and when $x = m$.



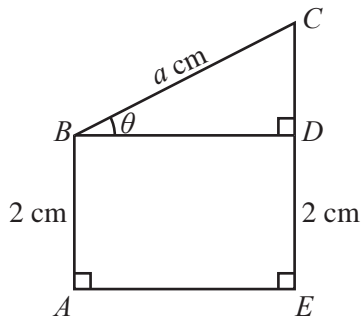
Question 9 – continued

Question 10

The figure shown represents a wire frame where $ABCE$ is a convex quadrilateral. The point D is on line segment EC with $AB = ED = 2$ cm and $BC = a$ cm, where a is a positive constant.

$$\angle BAE = \angle CEA = \frac{\pi}{2}$$

Let $\angle CBD = \theta$ where $0 < \theta < \frac{\pi}{2}$.



- a. Find BD and CD in terms of a and θ .

2 marks

- b. Find the length, L cm, of the wire in the frame, including length BD , in terms of a and θ .

1 mark

- c. Find $\frac{dL}{d\theta}$, and **hence** show that $\frac{dL}{d\theta} = 0$ when $BD = 2CD$.

2 marks

- d. Find the maximum value of L if $a = 3\sqrt{5}$.

1 mark