VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2014

SOLUTIONS

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter |

FURTHER MATHEMATICS

Written examination 2

Monday 3 November 2014

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 38 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

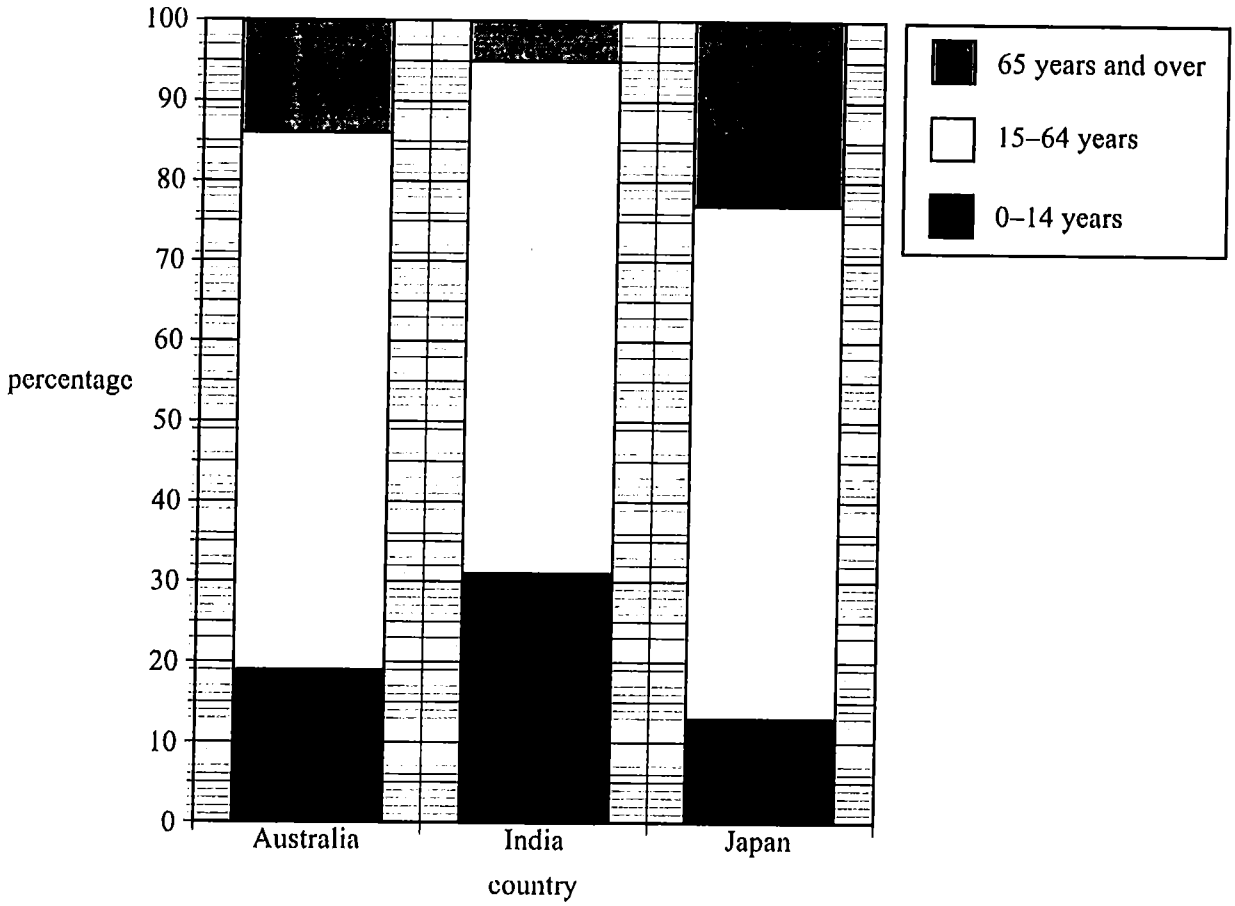
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Core

Question 1 (3 marks)

The segmented bar chart below shows the age distribution of people in three countries, Australia, India and Japan, for the year 2010.



Source: Australian Bureau of Statistics, *3201.0 – Population by Age and Sex, Australian States and Territories*, June 2010

a. Write down the percentage of people in Australia who were aged 0–14 years in 2010.

Write your answer, correct to the nearest percentage. 1 mark

19%

b. In 2010, the population of Japan was 128 000 000.

How many people in Japan were aged 65 years and over in 2010? 1 mark

23% of 128000000 = 0.23 × 128000000 = 29,440,000

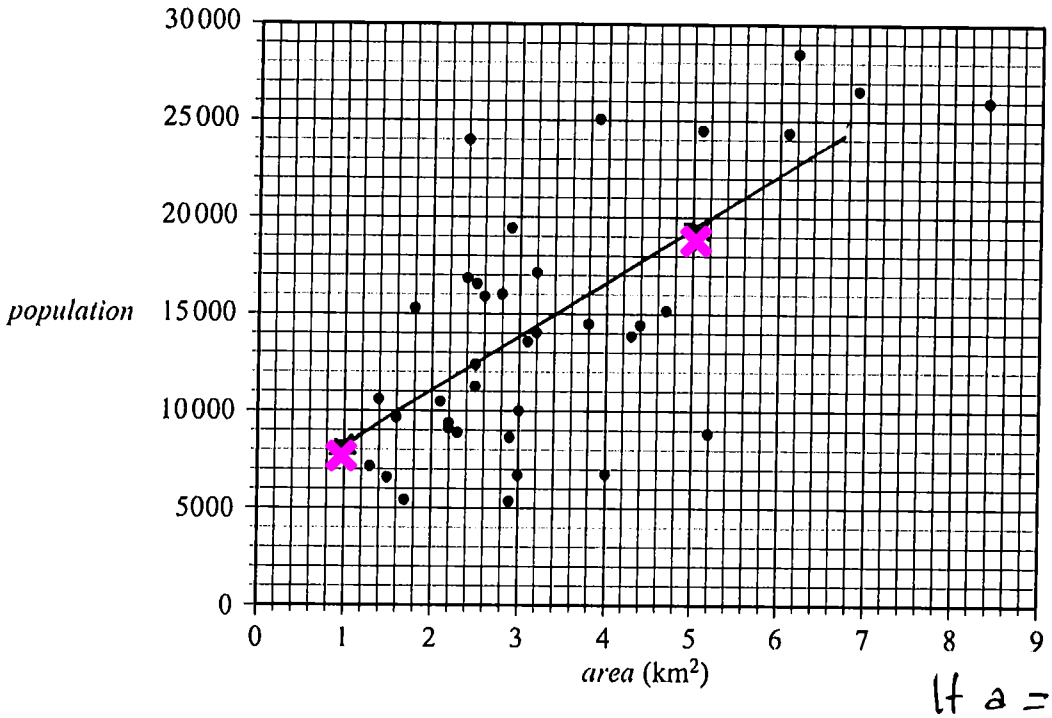
c. From the graph above, it appears that there is no association between the percentage of people in the 15–64 age group and the country in which they live.

Explain why, quoting appropriate percentages to support your explanation. 1 mark

The percentage of 15-64 year old age group is relatively unchanged across all countries (67% in Australia, 64% in India, 64% in Japan)

Question 2 (6 marks)

The scatterplot below shows the *population* and *area* (in square kilometres) of a sample of inner suburbs of a large city.



Find two points on least squares line and join them up

If $a = 1$,

$P = 5330 + 2680$

$= 8010$

$\therefore (1, 8010)$ ✖

If $a = 5$,

$P = 5330 + 5 \times 2680$

$= 18730$

$(5, 18730)$ ✖

The equation of the least squares regression line for the data in the scatterplot is

$$population = 5330 + 2680 \times area$$

a. Write down the dependent variable.

1 mark

Population

b. Draw the least squares regression line on the **scatterplot above**.

1 mark

(*Answer on the scatterplot above.*)

c. Interpret the slope of this least squares regression line in terms of the variables *area* and *population*.

2 marks

On average, the population increases by 2680 per 1 km^2 increase in area.

This is a template response

d. Wiston is an inner suburb. It has an area of 4 km^2 and a population of 6690.

The correlation coefficient, r, is equal to 0.668

i. Calculate the residual when the least squares regression line is used to predict the population of Wiston from its area. 1 mark

Population = 5330 + 2680 × 4

= 16050

Residual = Actual − Predicted

= 6690 − 16050 = −9360

Negative sign MUST be included

ii. What percentage of the variation in the population of the suburbs is explained by the variation in area?

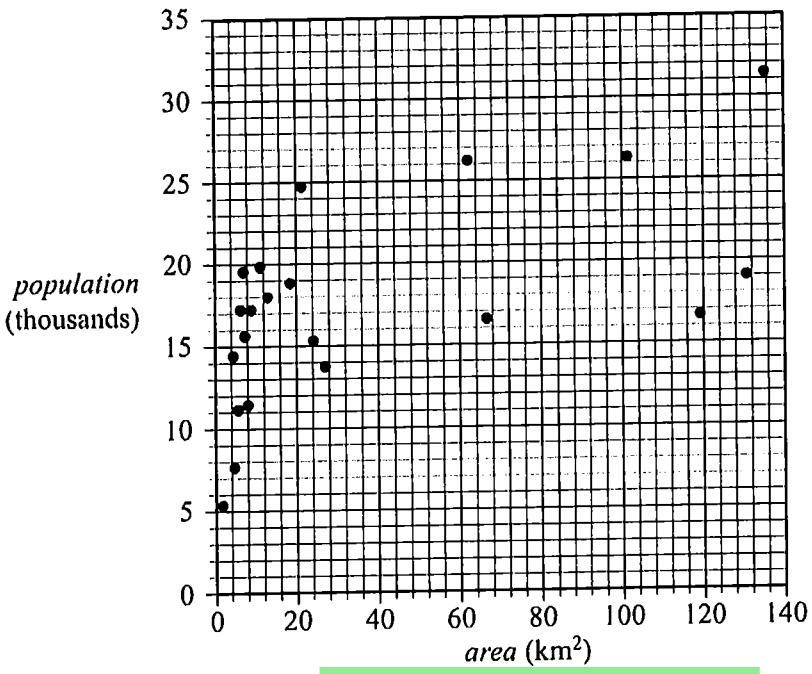
Write your answer, correct to one decimal place. 1 mark

$r^2 = (0.668)^2 = 0.4462$

$\therefore$ 44.6%

Question 3 (2 marks)

The scatterplot and table below show the *population*, in thousands, and the *area*, in square kilometres, for a sample of 21 outer suburbs of the same city.



Area (km²)	*Population* (thousands)
1.6	5.2
4.4	14.3
4.6	7.5
5.6	11.0
6.3	17.1
7.0	19.4
7.3	15.5
8.0	11.3
8.8	17.1
11.1	19.7
13.0	17.9
18.5	18.7
21.3	24.6
24.2	15.2
27.0	13.6
62.1	26.1
66.5	16.4
101.4	26.2
119.2	16.5
130.7	18.9
135.4	31.3

log(area)

Use CAS with new variable being log(area)

In the outer suburbs, the relationship between *population* and *area* is non-linear.

A **log** transformation can be applied to the variable *area* to linearise the scatterplot.

a. Apply the **log** transformation to the data and determine the equation of the least squares regression line that allows the population of an outer suburb to be predicted from the logarithm of its area.

Write the slope and intercept of this regression line in the boxes provided below.

Write your answers, correct to one decimal place. 1 mark

$$\textit{population} = \boxed{7.7} + \boxed{7.7} \times \log_{10}(\textit{area})$$

b. Use this regression equation to predict the population of an outer suburb with an area of 90 km².

Write your answer, correct to the nearest one thousand people. 1 mark

Population = 7.7 + 7.7 × log₁₀(area)

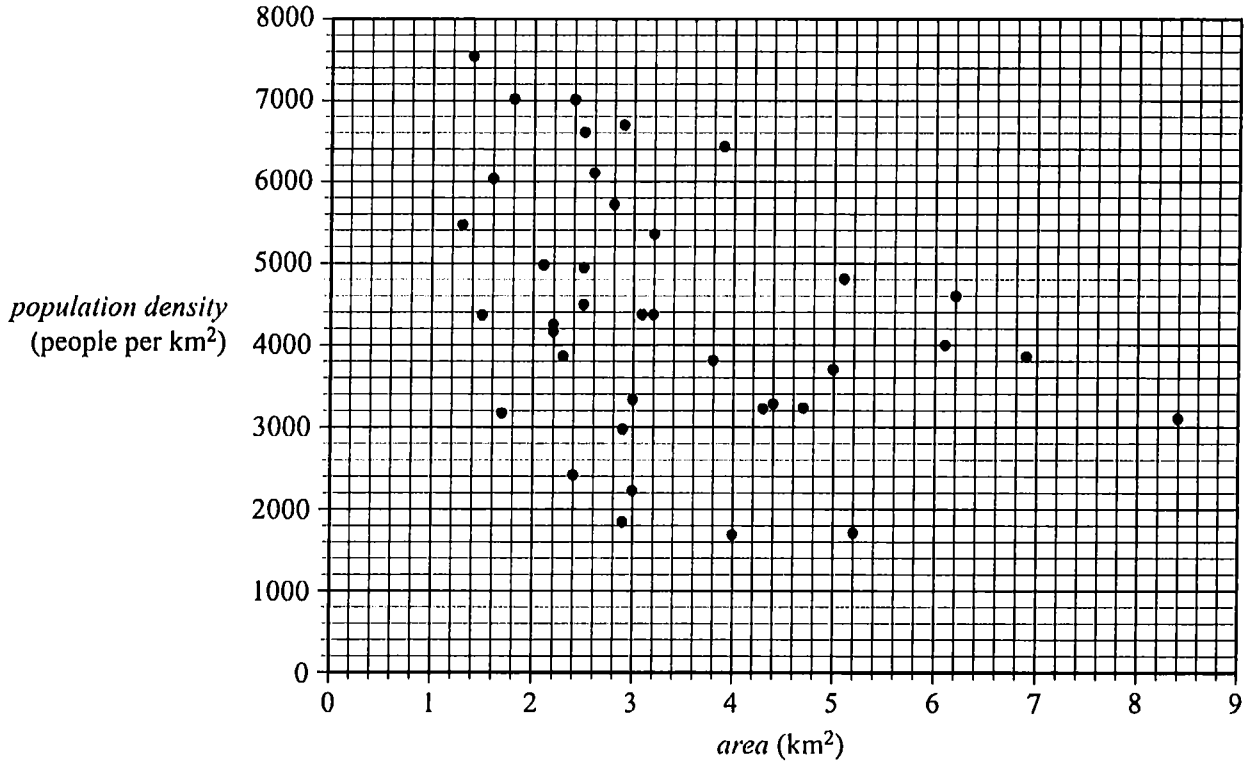
Population = 7.7 + 7.7 × log₁₀(90)

= 22.75 thousand

∴ ≃ 23,000

Question 4 (4 marks)

The scatterplot below shows the *population density*, in people per square kilometre, and the *area*, in square kilometres, of 38 inner suburbs of the same city.



For this scatterplot, $r^2 = 0.141$

a. Describe the association between the variables *population density* and *area* for these suburbs in terms of strength, direction and form. 1 mark

weak, negative, linear

(if you find the square root of 0.141, then r = -0.38. This indicates a weak association, and from the scatterplot it is clearly a negative association)

b. The mean and standard deviation of the variables *population density* and *area* for these 38 inner suburbs are shown in the table below.

	Population density (people per km^2)	*Area* (km^2)
Mean	4370	3.4
Standard deviation	1560	1.6

i. One of these suburbs has a population density of 3082 people per square kilometre.

Determine the standard z-score of this suburb's population density.

Write your answer, correct to one decimal place. 1 mark

$$z = \frac{3082 - 4370}{1560} \simeq -0.8$$

Assume the areas of these inner suburbs are approximately normally distributed.

ii. How many of these 38 suburbs are **expected** to have an area that is two standard deviations or more above the mean?

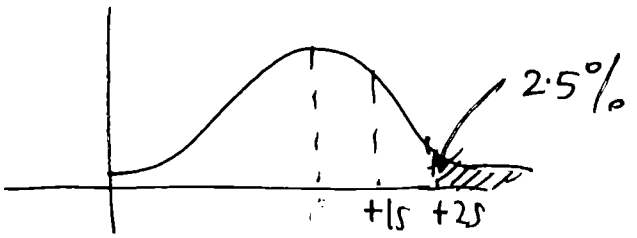
Write your answer, correct to the nearest whole number. 1 mark

$$\frac{2.5}{100} \times 38 \simeq 1$$

iii. How many of these 38 inner suburbs **actually** have an area that is two standard deviations or more above the mean? 1 mark

$$3.4 + 2 \times 1.6 = 6.6 \text{ km}^2$$

From scatterplot, there are 2 suburbs with area greater than 6.6 km^2



END OF CORE

Finance Module Rewritten to Reflect the New Study Design

Question 1

A sponsor of a cricket club has invested in a $20 000 perpetuity.

The annual interest from this perpetuity is $750.

The interest from the perpetuity is given to the best player in the club every year, for a period of 10 years.

a. What is the annual interest rate for this perpetuity investment?

$Q = \frac{rV_0}{100}$

$\therefore 750 = \frac{r \times 20000}{100} \therefore r = 3.75\%$ p.a 1 mark

b. After 10 years, how much money is still invested in the perpetuity?

$ 20,000

In a perpetuity, the balance remains constant forever! 1 mark

Question 2

The cricket club has invested $45 550 in an account for 4 years. After 4 years of compound interest, the value of the investment is $60 000.

a. How much interest was earned over the 4 years?

60000 − 45550

= $14,450 1 mark

Interest on the account had been calculated and paid quarterly.

b. What was the annual rate of interest for this investment? Write your answer, correct to one decimal place.

$V_{16} = 45550\left(1+\frac{r/4}{100}\right)^{16}$

$\therefore 60000 = 45550\left(1+\frac{r/4}{100}\right)^{16}$

Solving: $r = 6.9\%$ p.a

The $60 000 was re-invested in another account for 12 months. The new account paid interest at the rate of 7.2% per annum, compounding monthly. At the end of each month, the cricket club added an additional $885 to the investment.

c. This investment strategy can be modelled by a linear recursion relation of the form:

$$V_{n+1} = RV_n + D, \qquad V_0 = K$$

where V_n is the value of the investment at the end of n months

i. Write down the values of R, D and K in the spaces provided:

$R = 1 + \frac{r/12}{100}$

$\therefore R = 1 + \frac{7.2/12}{100}$

$= 1.006$

$R =$ 1.006

$D =$ 885

$K =$ 60000

3 marks

ii. By using this recurrence relation, and showing two separate calculations, determine the value of the investment at the end of the second month. Give your answer to the nearest cent.

$V_{n+1} = 1.006 \cdot V_n + 885$

$\therefore V_1 = 1.006 \times 60000 + 885 = 60,921$

$V_2 = 1.006 \times 60921 + 885 = \$62497.47$

Must show the two calculations as above for 2 marks

2 marks

iii. Calculate the account balance at the end of 12 months. Give your answer to the nearest dollar.

N = 12

I = 7.2

PV = −60000

FV = ?

PMT = −885

PY = 12

CY = 12

gives FV = 75443.01

∴ Balance = $75443.

1 mark

Question 3

The cricket club borrowed $400 000 to build a clubhouse. Interest is calculated at the rate of 4.5% per annum, compounding monthly. The cricket club will make monthly repayments of $2500. After a number of monthly repayments, the balance of the loan will be reduced to $143 585.33 What percentage of the next monthly repayment will reduce the balance of the loan? Write your answer, correct to the nearest percentage.

N = ?
I = 4.5
PV = 400000
PMT = −2500
FV = −143585.33
P/Y = 12
C/Y = 12

gives N = 180

2 marks

N = 181
I = 4.5
PV = 400 000
PMT = −2500
FV = ?
P/Y = 12
C/Y = 12 gives FV = −141623.78

∴ Balance is reduced by:

$143,585.33 − $141,623.78
= $1961.55

Required percentage $= \frac{1961.55}{2500} \times \frac{100}{1}$

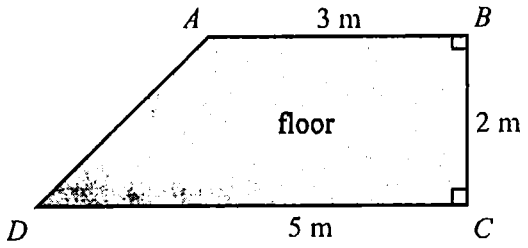
$\approx 78\%$

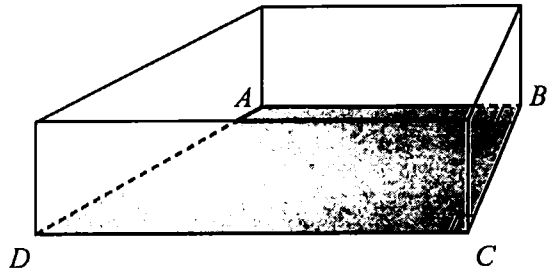
Module 2: Geometry and trigonometry

Question 1 (2 marks)

The floor of a chicken coop is in the shape of a trapezium.

The floor, *ABCD*, and the chicken coop are shown below.

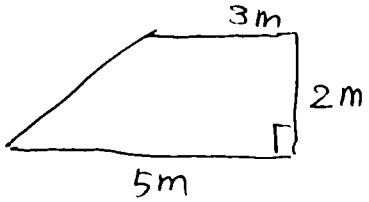




$AB = 3$ m, $BC = 2$ m and $CD = 5$ m.

a. What is the area of the floor of the chicken coop?

Write your answer in square metres. 1 mark

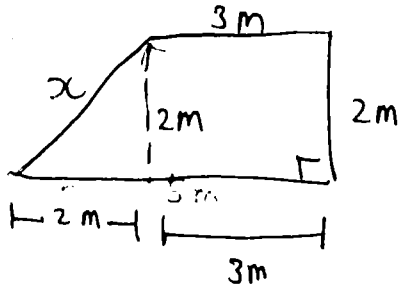


$$A = \frac{h}{2}(a+b) = \frac{2}{2} \times (3+5) = 8\ m^2$$

b. What is the perimeter of the floor of the chicken coop?

Write your answer in metres, correct to one decimal place. 1 mark

$$P = 2.828 + 5 + 2 + 3$$
$$= 12.828$$
$$\simeq 12.8\ m$$

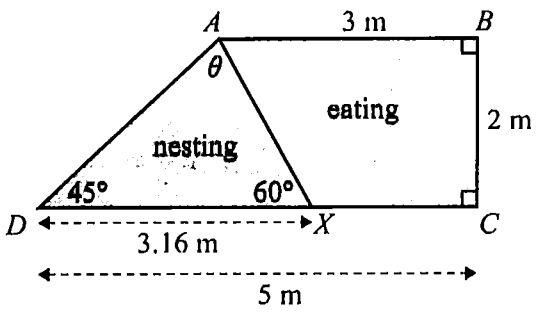


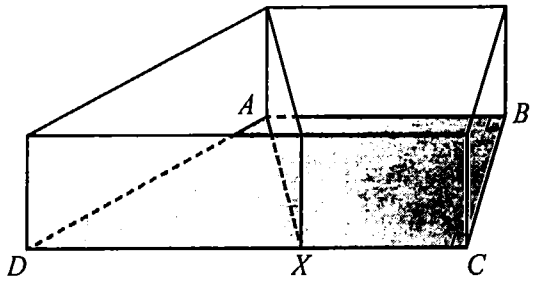
$$x^2 = 2^2 + 2^2$$
$$\therefore x = \sqrt{8}$$
$$\simeq 2.828\ m$$

Question 2 (6 marks)

The chicken coop has two spaces, one for nesting and one for eating.

The nesting and eating spaces are separated by a wall along the line AX, as shown in the diagrams below.





$DX = 3.16$ m, $\angle ADX = 45°$ and $\angle AXD = 60°$.

a. Write down a calculation to show that the value of θ is 75°. 1 mark

$\theta = 180° - 45° - 60°$

$= 75°$

b. The sine rule can be used to calculate the length of the wall AX.

Fill in the missing numbers below. 1 mark

$$\frac{AX}{\sin \boxed{45}^\circ} = \frac{\boxed{3.16}}{\sin \boxed{75}^\circ}$$

c. What is the length of AX?

Write your answer in metres, correct to two decimal places. 1 mark

$$AX = \frac{3.16 \sin(45°)}{\sin(75°)} \approx 2.31 \text{ m}$$

d. Calculate the area of the floor of the nesting space, ADX.

Write your answer in square metres, correct to one decimal place. 1 mark

$A = 0.5 \times 2.31 \times 3.16 \sin(60°)$

$\approx 3.2 \text{ m}^2$

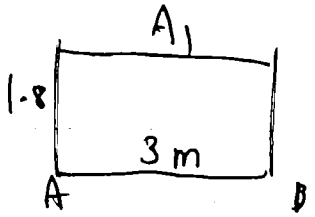
The height of the chicken coop is 1.8 m.

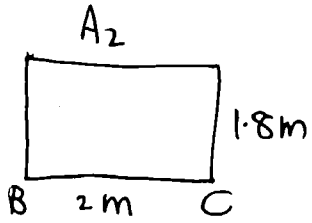
Wire mesh will cover the roof of the eating space.

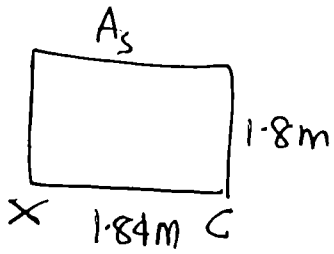
The area of the walls along the lines AB, BC and CX will also be covered with wire mesh.

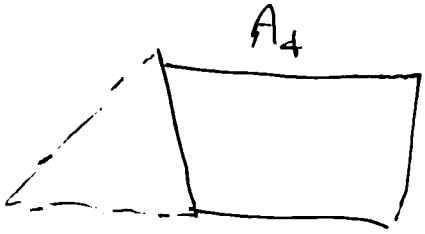
e. What total area, in square metres, will be covered by wire mesh?

Write your answer, correct to the nearest square metre. 2 marks









$A_4 = 8\,m^2 -$ Area ΔADX

$A_4 = 8\,m^2 - 3.1653\,m^2$

$\simeq 4.835\,m^2$

$\therefore$ Total area $= 1.8 \times 3 + 2 \times 1.8 + 1.84 \times 1.8 + 4.835\,m^2$

$\simeq 17.147\,m^2$

$\simeq 17\,m^2$

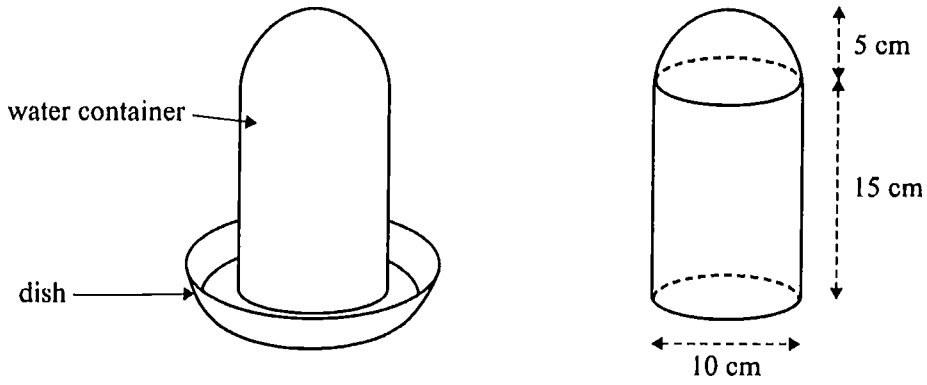
Question 3 (5 marks)

The chicken coop contains a circular water dish.

Water flows into the dish from a water container.

The water container is in the shape of a cylinder with a hemispherical top.

The water container and the dish are shown in the diagrams below.



The cylindrical part of the water container has a diameter of 10 cm and a height of 15 cm.

The hemisphere has a radius of 5 cm.

a. What is the surface area of the hemispherical top of the water container?

Write your answer, correct to the nearest square centimetre. 1 mark

$S = 2\pi r^2 = 2\pi \times 5^2 \simeq 157 cm^2$

b. What is the maximum volume of water that the water container can hold?

Write your answer, correct to the nearest cubic centimetre. 2 marks

$V = \pi r^2 h + \frac{2}{3}\pi r^3$

$\therefore V = \pi \times 5^2 \times 15 + \frac{2}{3} \times \pi \times 5^3$

$V \simeq 1439.86 \simeq 1440 cm^3$

The eating space of the chicken coop also has a feed container.

The feed container is similar in shape to the water container.

The volume of the water container is three-quarters of the volume of the feed container.

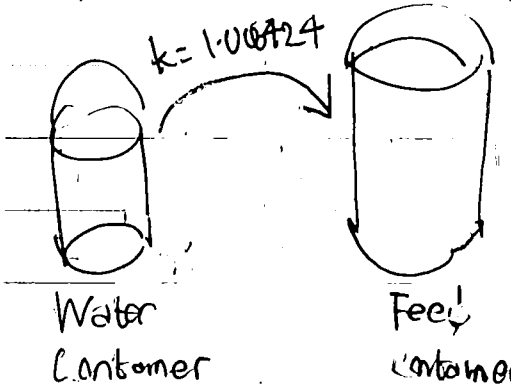
The surface area of the water container is 628 cm^2.

$k^3 = \frac{4}{3}$

$k = \sqrt[3]{\frac{4}{3}} = 1.1006424$

c. What is the surface area of the feed container?

Write your answer, correct to the nearest square centimetre. 2 marks



$k^2 = (1.1006424)^2$

$= 1.2114137$

Now must find area scale factor

$\therefore$ Area of feed container

$= 628 \times 1.2114137$

$= 760.77 = 761 cm^2$

Question 4 (2 marks)

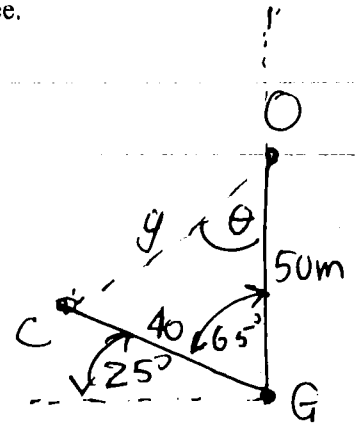
One of the chickens escapes into a neighbouring field through an open gate.

The chicken's owner is 50 m due north of the gate, searching for the chicken.

The chicken is 40 m from the gate on a bearing of 295°.

What is the bearing of the chicken from its owner?

Write your answer, correct to the nearest degree.



$g^2 = 40^2 + 50^2 - 2 \times 40 \times 50 \cos(65°)$

$g = \sqrt{40^2 + 50^2 - 2 \times 40 \times 50 \cos(65°)}$

$g \simeq 49.09 \text{ m}$

$$\frac{\sin(\theta)}{40} = \frac{\sin(65°)}{49.086933}$$

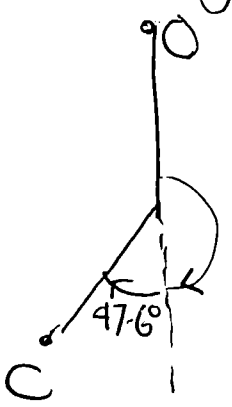
$$\therefore \theta = \sin^{-1}\left(\frac{40 \sin(65°)}{49.086933}\right)$$

$\theta = 47.6°$

∴ Bearing of C from O $= 180° + 47.6°$

$= 227.6°$

$\simeq 228°$



END OF MODULE 2

Module 6: Matrices

Question 1 (6 marks)

A small city is divided into four regions: Northern (N), Eastern (E), Southern (S) and Western (W).

The number of adult males (M) and the number of adult females (F) living in each of the regions in 2013 is shown in matrix V below.

$$V = \begin{matrix} \\ N \\ E \\ S \\ W \end{matrix} \begin{matrix} M & F \\ \end{matrix} \begin{bmatrix} 1360 & 1460 \\ 1680 & 1920 \\ 900 & 1060 \\ 1850 & 1770 \end{bmatrix}$$

a. Write down the order of matrix V. 1 mark

4 X 2

b. How many adult males lived in the Western region in 2013? 1 mark

1850

c. In terms of the population of the city, what does the sum of the elements in the second column of matrix V represent? 1 mark

Total no. of adult females living in the city

An election is to be held in the city.

All of the adults in each of the regions of the city will vote in the election.

One of the election candidates, Ms Aboud, estimates that she will receive 45% of the male votes and 55% of the female votes in the election.

This information is shown in matrix P below.

$$P = \begin{matrix} \\ M \\ F \end{matrix} \begin{bmatrix} 0.45 \\ 0.55 \end{bmatrix}$$

d. Explain, in terms of rows and columns, why the matrix product $V \times P$ is defined. 1 mark

V order: 4×2 P order: 2×1

No. of columns in V = no. of rows in P

Therefore, V.P is defined

The product of matrices V and P is shown below.

$$V \times P = \begin{bmatrix} 1360 & 1460 \\ 1680 & 1920 \\ 900 & 1060 \\ 1850 & 1770 \end{bmatrix} \times \begin{bmatrix} 0.45 \\ 0.55 \end{bmatrix} = \begin{bmatrix} w \\ 1812 \\ 988 \\ 1806 \end{bmatrix}$$

e. Using appropriate elements from the matrix product $V \times P$, write a calculation to show that the value of w is 1415. 1 mark

$1360 \times 0.45 + 1460 \times 0.55 = 1415$

f. How many votes does Ms Aboud expect to receive in the election? 1 mark

$1415 + 1812 + 988 + 1806$

$= 6021$

$\therefore$ 6021 votes.

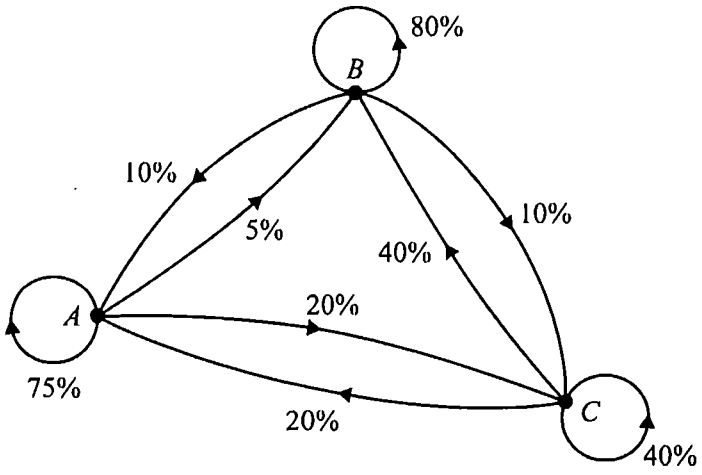
Question 2 (6 marks)

There are three candidates in the election: Ms Aboud (A), Mr Broad (B) and Mr Choi (C).

The election campaign will run for six months, from the start of January until the election at the end of June.

A survey of voters found that voting preference can change from month to month leading up to the election.

The transition diagram below shows the percentage of voters who are expected to change their preferred candidate from month to month.



a. **i.** Of the voters who prefer Mr Choi this month, what percentage are expected to prefer Ms Aboud next month? 1 mark

20%

ii. Of the voters who prefer Ms Aboud this month, what percentage are expected to change their preferred candidate next month? 1 mark

25%

In January, 12 000 voters are expected in the city. The number of voters in the city is expected to remain constant until the election is held in June.

The state matrix that indicates the number of voters who are expected to have a preference for each candidate in January, S_1, is given below.

$$S_1 = \begin{bmatrix} 6000 \\ 3840 \\ 2160 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

b. How many voters are expected to change their preference to Mr Broad in February? 1 mark

$6000 \times \frac{5}{100} + 2160 \times \frac{40}{100}$

$= 1164$

The information in the transition diagram has been used to write the transition matrix, T, shown below.

$$T = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \text{this month} \\ \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.75 & 0.10 & 0.20 \\ 0.05 & 0.80 & 0.40 \\ 0.20 & 0.10 & 0.40 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \end{array} \quad \text{next month}$$

c. **i.** Evaluate the matrix $S_3 = T^2 S_1$ and write it down in the space below.
Write the elements, correct to the nearest whole number. 1 mark

$$S_3 = \begin{bmatrix} 4900.2 \\ 4633.8 \\ 2466 \end{bmatrix}$$

ii. What information does matrix S_3 contain? 1 mark

The number of expected voters for each candidate in March.

d. Using matrix T, how many votes would the winner of the election in June be expected to receive?
Write your answer, correct to the nearest whole number. 1 mark

$$S_6 = T^5 S_1$$

$$= \begin{bmatrix} 4334.06 \\ 5302.95 \\ 2362.996 \end{bmatrix}$$

∴ Winner expected to get 5303 votes

Question 3 (3 marks)

Mr Choi may need to withdraw from the election at the end of May.

Matrix T, shown below, shows the percentage of voters who change their preferred candidate, from month to month, **before** Mr Choi would withdraw from the election.

$$T = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \text{this month} \\ \begin{bmatrix} 0.75 & 0.10 & 0.20 \\ 0.05 & 0.80 & 0.40 \\ 0.20 & 0.10 & 0.40 \end{bmatrix} \end{array} \begin{array}{l} \\ A \\ B \\ C \end{array} \quad \text{next month}$$

Matrix T_1, shown below, shows the percentage of voters who change their preferred candidate, from May to June, **after** Mr Choi would withdraw from the election.

$$T_1 = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \text{May} \\ \begin{bmatrix} 0.75 & 0.15 & 0.6 \\ 0.25 & 0.85 & 0.4 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ A \\ B \\ C \end{array} \quad \text{June}$$

Consider the voters who preferred Mr Broad in May and who were expected to prefer Mr Choi in June.

a. What percentage of these voters are now expected to prefer Mr Broad in June? 1 mark

50%

Matrix T says that 10% of B's votes would go to C
Matrix T_1 now says that 85% of B'svotes stay with B
So of the 10% that were going to C, half or 50% are now going to B

The state matrix that indicates the number of voters who are expected to have a preference for each candidate in January, S_1, is given below.

$$S_1 = \begin{bmatrix} 6000 \\ 3840 \\ 2160 \end{bmatrix} \begin{array}{l} A \\ B \\ C \end{array}$$

b. If Mr Choi withdraws, how many votes is Mr Broad expected to receive in the election in June?

Write your answer, correct to the nearest vote. 2 marks

$S_5 = T^4 S_1 = \begin{bmatrix} 4453.6 \\ 5154.3 \\ 2392.1 \end{bmatrix}$

The matrix for June is now:

$$\begin{bmatrix} 0.75 & 0.15 & 0.6 \\ 0.25 & 0.85 & 0.4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4453.6 \\ 5154.3 \\ 2392.1 \end{bmatrix} = \begin{bmatrix} 5548.6 \\ 6451.4 \\ 0 \end{bmatrix} \begin{array}{l} A \\ B \\ C \end{array}$$

∴ Mr Broad expected to receive 6451 votes.

END OF QUESTION AND ANSWER BOOK