



## PARADE COLLEGE

### Mathematics Methods 3&4-CAS

### Probability Analysis SAC 2

**Name of Student:** \_\_\_\_\_

**Date:** Thursday 11 September 2014

**Reading Time:** 3.30pm to 3.40pm (10 minutes)

**Writing Time:** 3.40pm to 5.15pm (95 minutes)

**Location:** Greening Auditorium

#### Topics:

- Discrete random variables and their probability distributions (chapter 14)
- The binomial distribution (chapter 15)
- Markov chains (chapter 16)
- Continuous random variables and their probability distributions (chapter 17)
- The normal distribution (chapter 18)

#### Directions to students:

- This task consists of 4 extended response question totaling 60 marks.
- Students are permitted to use one bound reference which adheres to the VCAA rules on reference material for exam 2.
- Students are permitted to use approved CAS technology.
- Students are **not** permitted to bring blank sheets of paper and white out liquid/tape.
- Answer **all** questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.

This task assesses all three outcomes.

**Outcome 1**

Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

**Outcome 2**

Identify important information, variables and constraints.  
Apply mathematical ideas and content from the specified areas of study.  
Analyse and interpret results with consideration of limitations and constraints

**Outcome 3**

Select and appropriately use a computer algebra system and other technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

**Question 1** (2+2+2+3+1+3=13 marks)

It is found that when a married couple go and see a movie together there is a 95% chance that they will see a romantic comedy if the previous movie they saw together was a romantic comedy, and a 70% chance that they will see an action movie if the previous movie they saw together was an action movie.

A transition matrix for the probabilities of a married couple seeing either a romantic comedy or an action movie together given the previous movie seen together is

$$\begin{bmatrix} 0.95 & 0.30 \\ 0.05 & 0.70 \end{bmatrix}$$

a) Marshall and Inga are a married couple who decide to go and see a movie together. It is equally likely that they will see either a romantic comedy or an action movie. Find the probability that the second time they go to the movies together, Marshall and Inga will see a romantic comedy.

(2 marks)

b) As it happens, Marshall and Inga saw an action movie the last time they went to the movies. They decide to go to the movies together four more times in a four month period following this last movie.

Find (**correct to four decimal places**):

- i) the probability that the fourth movie they see together during this four month period will be a romantic comedy.

(2 marks)

- ii) the probability that the four movies they see together during this four month period will all be romantic comedies.

(2 marks)

On a particular day it was found that 212 married couples saw a romantic comedy together and 84 married couples saw an action movie together.

c) Find the number of married couples expected to see each type of movie in the long term.

(3 marks)

After a four month period, Marshall says that he doesn't want to see any more action movies and would rather see some thrillers.

If Marshall and Inga see a thriller together, then the probability that the next movie they see together is also a thriller is  $p - 0.45$ .

If they see a romantic comedy together, then the probability that the next movie they see together is a thriller is  $p - 0.2$ .

d) i) Set up a transition matrix to model this situation.

(1 mark)

It is known that if the first movie Marshall and Inga see together is a thriller, then the probability that the third movie they see together is a romantic comedy is 0.68.

iii) Find the value of  $p$  correct to four decimal places.

(3 marks)

**Question 2** (1+2+1+2+2+2+3+5=18 marks)

Marshall and Inga's daughters Hannah and Molly want a pet to play with in the backyard and have their hearts set on a Dalmation dog. Inga is not too fond of spots and will only consider a Dalmation with not too many spots.

Canine Bio Engineering is a bio-tech company that has bred a genetically modified Dalmation dog (*canis familiaris minima maculosa*) for spot-intolerant people who want a Dalmation dog but don't want it to have too many spots.

The number of spots,  $X$ , on the modified Dalmation dog is a discrete random variable with probability distribution as shown in the following table:

$x$	2	3	4	5	6
$\Pr(X = x)$	0.3	0.4	$k$	0.05	0.02

a) Show that  $k = 0.23$

(1 mark)

b) Find the expected number of spots on a modified Dalmation dog.

(2 marks)

The genetically modified Dalmation dogs are kept in a secure open range enclosure.

c) If two dogs are randomly chosen from the enclosure, find the probability that:

i) both dogs have three spots

(1 mark)

ii) the total number of spots on both dogs is equal to seven

(2 marks)

The breeders have also found that the probability that a dog has spots on its ears is 0.3. In the enclosure, let  $Y$  represent the number of dogs with spots on their ears.

d) Eight modified Dalmation dogs are randomly chosen from the enclosure.

Find **correct to four decimal places**, the probability that:

i) three of the dogs have spots on their ears.

(2 marks)

ii) at least two of the dogs have spots on their ears.

(2 marks)

iii) exactly five of the dogs have spots on their ears given that at least two of the dogs have spots on their ears.

(3 marks)

A random group of genetically modified Dalmation dogs manage to escape from the enclosure.

e) Find the smallest number of dogs in the group if the probability that at least one of the dogs in the group has spots on its ears is greater than 0.80.

(5 marks)



**Question 3** (1+2+3+3+3=12 marks)

Marshall and Inga do some research to estimate the food requirements of a dog. They found that the amount of pet food required per month by an adult dog is a normally distributed random variable with a mean of 50 kg and a standard deviation of 10 kg.

a) Find, **correct to four decimal places**, the probability that an adult dog will require an amount of pet food between 33 kg and 54 kg

(1 mark)

b) In a random sample of 14 dogs, find the probability, **correct to four decimal places** that exactly six of them will require between 33 kg and 54 kg of food.

(2 marks)

An adult dog is considered to have a small appetite if the amount of pet food it requires is less than 30 kg, a large appetite if the amount is greater than  $a$  kg and a normal appetite otherwise. The probability of a dog having a large appetite is equal to 0.12.

c) i) Find, **to the nearest kg**, the value of  $a$ .

(3 marks)

ii) Find, **correct to four decimal places**, the probability that an adult dog requires more than 26 kg per month given that they have a small appetite.

(3 marks)

The amount of pet food required by a pup is a normally distributed random variable with a standard deviation of 8 kg. The probability of requiring less than 26 kg is 0.34.

d) Find, **to the nearest kg**, the mean amount of pet food required by a pup.

(3 marks)

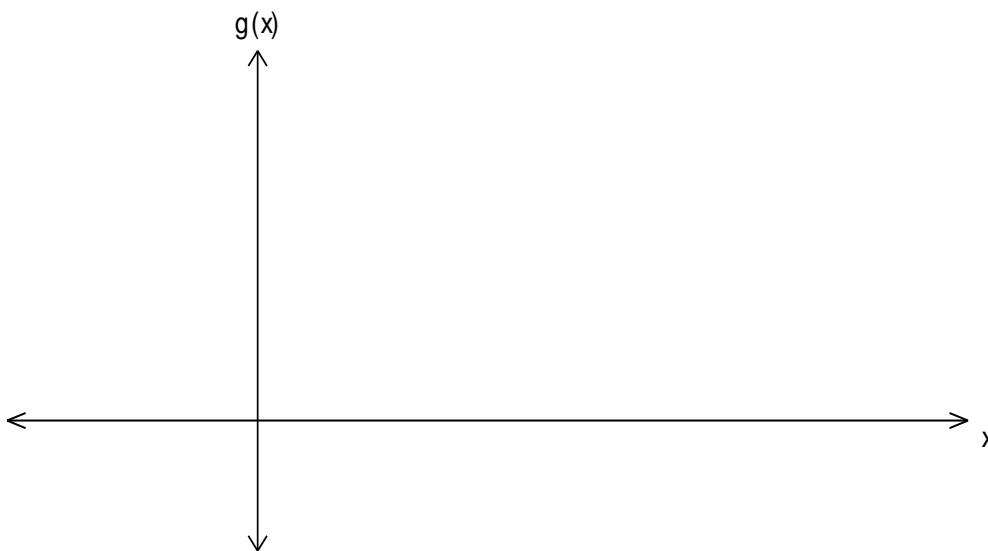
**Question 4** (3+1+1+2+1+2+3+2+2=17 marks)

Since purchasing the dog, Marshall and Inga have found that the girls, Hannah and Molly spend a minimum of 2 hours in the backyard on the weekend. In fact, Marshall found that the time Hannah and Molly spend in the backyard on the weekend, in hours is a continuous random variable,  $X$ , and has a probability distribution function given by

$$g(x) = \begin{cases} 0.1x - 0.2 & \text{if } 2 \leq x \leq 6 \\ 0.4e^{-2(x-6)} & \text{if } x > 6 \\ 0 & \text{elsewhere} \end{cases}$$

- a) Sketch a graph of this probability distribution function on the axes below, labelling all important information.

(3 marks)



- b) i) What is the probability that they will spend more than 6 hours in the backyard?

(1 mark)

- ii) Hence, verify that the function is a probability distribution.

(1 mark)

c) Find the mean time to the nearest hour, that Hannah and Molly spend in the backyard on a weekend.

(2 marks)

d) Find the time that Hannah and Molly spend in the backyard most often.

(1 mark)

e) What is the probability **correct to four decimal places** that they will spend between 5 and 6.5 hours in the backyard?

(2 marks)

f) If it is known that Hannah and Molly spend less than 6.5 hours in the backyard, what is the probability **correct to four decimal places** that they spend between 5 and 6.5 hours in the backyard?

(3 marks)

g) Find their median playing time in the backyard, **correct to two decimal places**.  
(2 marks)

h) Find the time interval, from 2 hours onwards which Hannah and Molly would spend in the backyard on 90% of occasions (**Correct to two decimal places**).  
(2 marks)

**End of Task**