VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2015

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER Letter

FURTHER MATHEMATICS

Written examination 2

Monday 2 November 2015

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
5	5	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 43 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

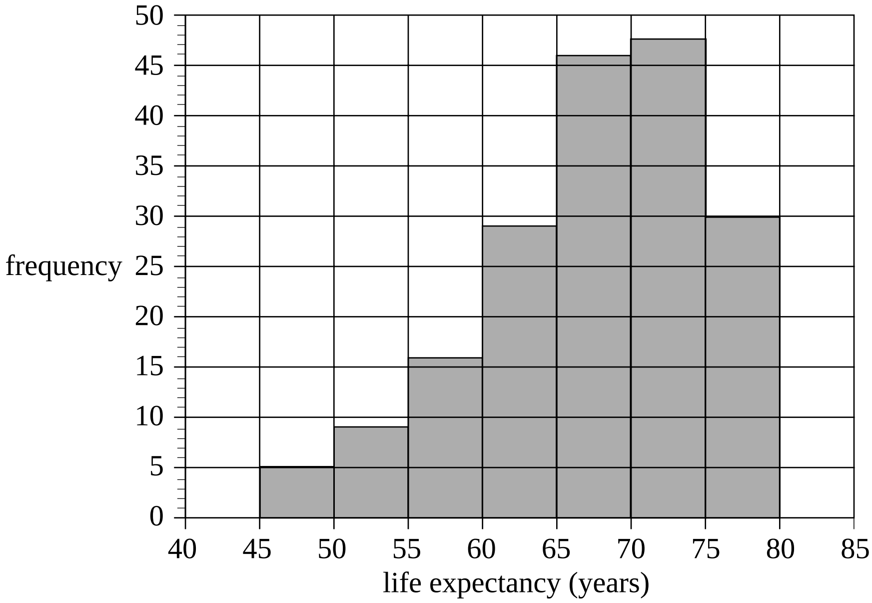
Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Core

Question 1 (3 marks)

The histogram below shows the distribution of life expectancy of people for 183 countries.



a. For this distribution, the modal interval is ______. 1 mark

b. In how many of these countries is life expectancy less than 55 years? 1 mark

c. In what percentage of these 183 countries is life expectancy between 75 and 80 years?
Write your answer correct to one decimal place. 1 mark

Question 2 (3 marks)

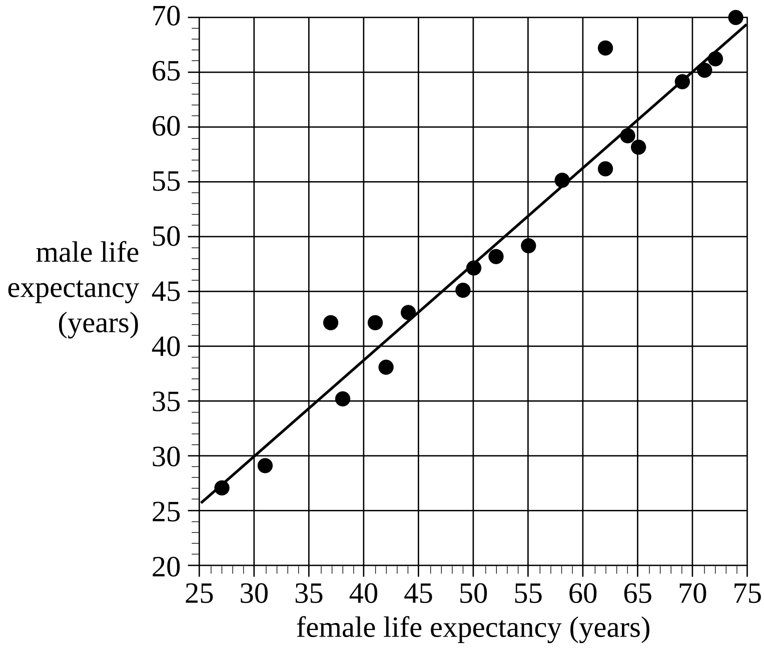
The parallel boxplots below compare the distribution of life expectancy for 183 countries for the years 1953, 1973 and 1993.

a. Describe the shape of the distribution of life expectancy for 1973. 1 mark

b. Explain why life expectancy for these countries is associated with the year. Refer to specific statistical values in your answer. 2 marks

Question 3 (3 marks)

The scatterplot below plots male life expectancy (*male*) against female life expectancy (*female*) in 1950 for a number of countries. A least squares regression line has been fitted to the scatterplot as shown.



The slope of this least squares regression line is 0.88

a. Interpret the slope in terms of the variables *male* life expectancy and *female* life expectancy. 1 mark

The equation of this least squares regression line is

$$male = 3.6 + 0.88 \times female$$

b. In a particular country in 1950, *female* life expectancy was 35 years.

Use the equation to predict *male* life expectancy for that country. 1 mark

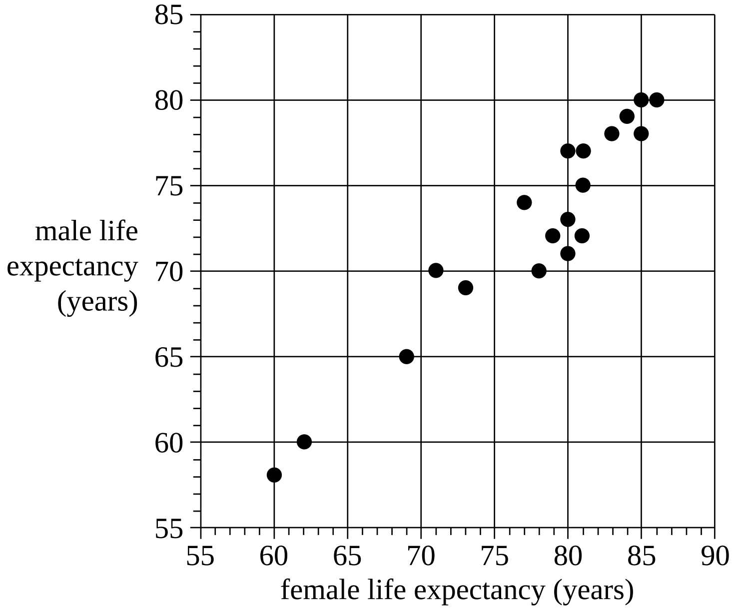
c. The coefficient of determination is 0.95

Interpret the coefficient of determination in terms of male life expectancy and female life expectancy. 1 mark

Question 4 (2 marks)

The table below shows male life expectancy (*male*) and female life expectancy (*female*) for a number of countries in 2013. The scatterplot has been constructed from this data.

Life expectancy (in years) in 2013	
male	***female***
80	85
60	62
73	80
70	71
70	78
78	83
77	80
65	69
74	77
70	78
75	81
58	60
80	86
69	73
79	84
72	81
78	85
72	79
77	81
71	80



a. Use the scatterplot to describe the association between *male* life expectancy and *female* life expectancy in terms of strength, direction and form. 1 mark

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b. Determine the equation of a least squares regression line that can be used to predict *male* life expectancy from *female* life expectancy for the year 2013.

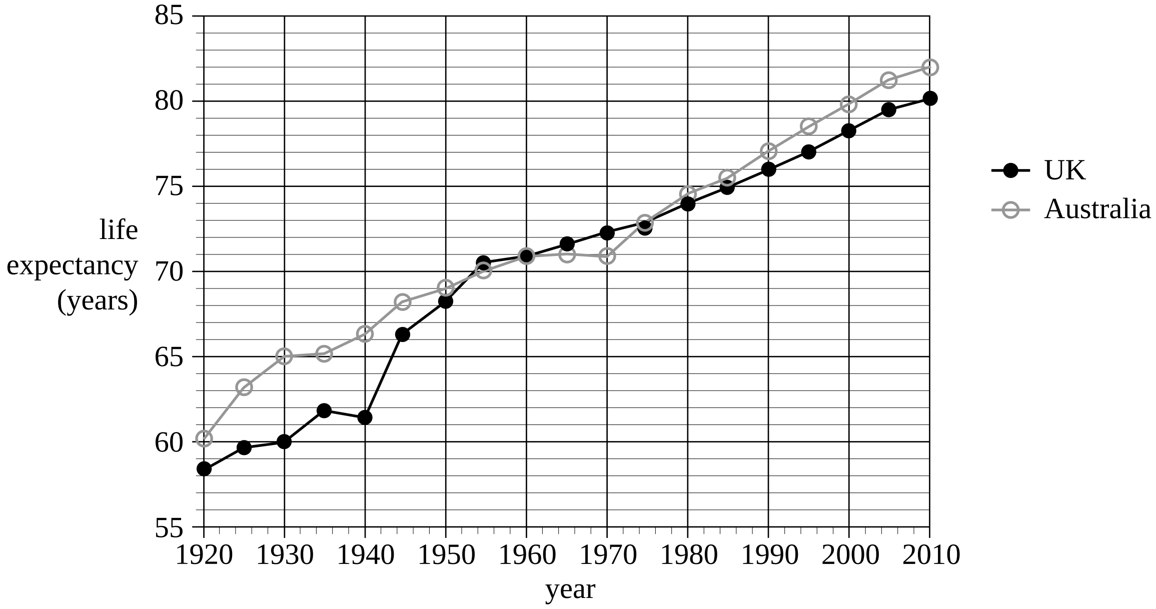
Complete the equation for the least squares regression line below by writing the intercept and slope in the boxes provided.

Write these values correct to two decimal places. 1 mark

male = [ ] + [ ] × *female*

Question 5 (4 marks)

The time series plot below displays the *life expectancy*, in years, of people living in Australia and the United Kingdom (UK) for each *year* from 1920 to 2010.



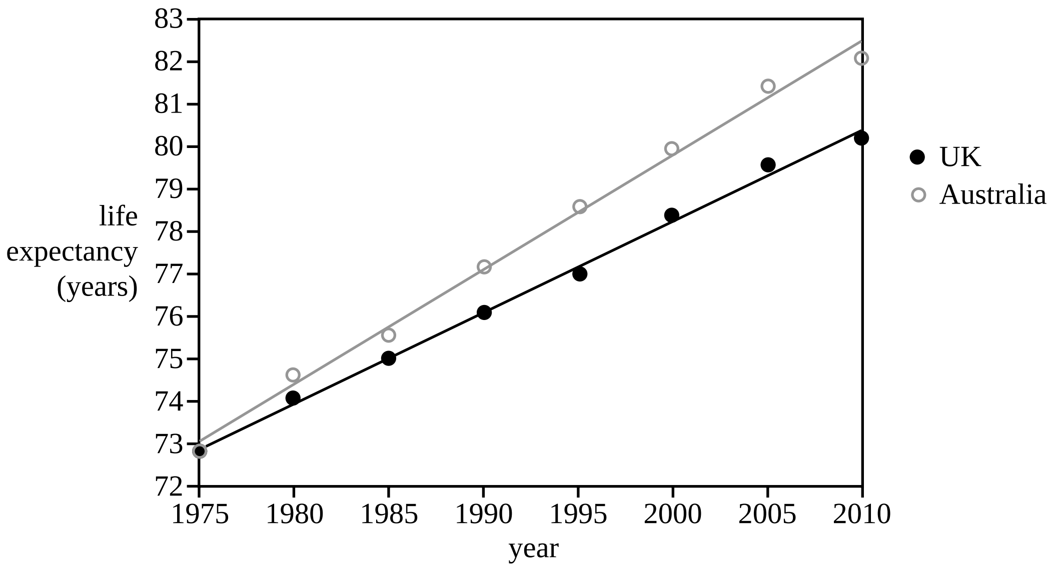
a. By how much did *life expectancy* in Australia increase during the period 1920 to 2010?
Write your answer correct to the nearest year. 1 mark

b. In 1975, the life expectancies in Australia and the UK were very similar.

From 1975, the gap between the life expectancies in the two countries increased, with people in Australia having a longer life expectancy than people in the UK.

To investigate the difference in life expectancies, least squares regression lines were fitted to the data for both Australia and the UK for the period 1975 to 2010.

The results are shown below.



The equations of the least squares regression lines are as follows.

Australia: *life expectancy* = –451.7 + 0.2657 × *year*

UK: *life expectancy* = –350.4 + 0.2143 × *year*

i. Use these equations to predict the difference between the life expectancies of Australia and the UK in 2030.

Give your answer correct to the nearest year. 2 marks

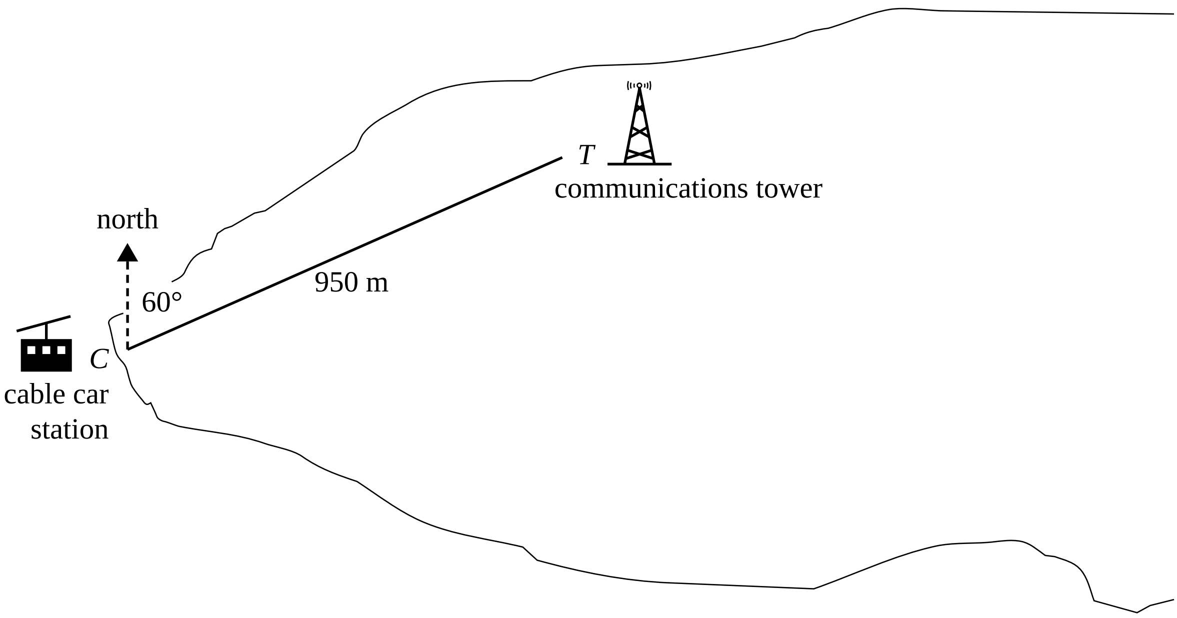
ii. Explain why this prediction may be of limited reliability. 1 mark

END OF CORE
TURN OVER

Question 2 (4 marks)

There are plans to construct a series of straight paths on the flat top of the mountain.

A straight path will connect the cable car station at *C* to a communications tower at *T*, as shown in the diagram below.

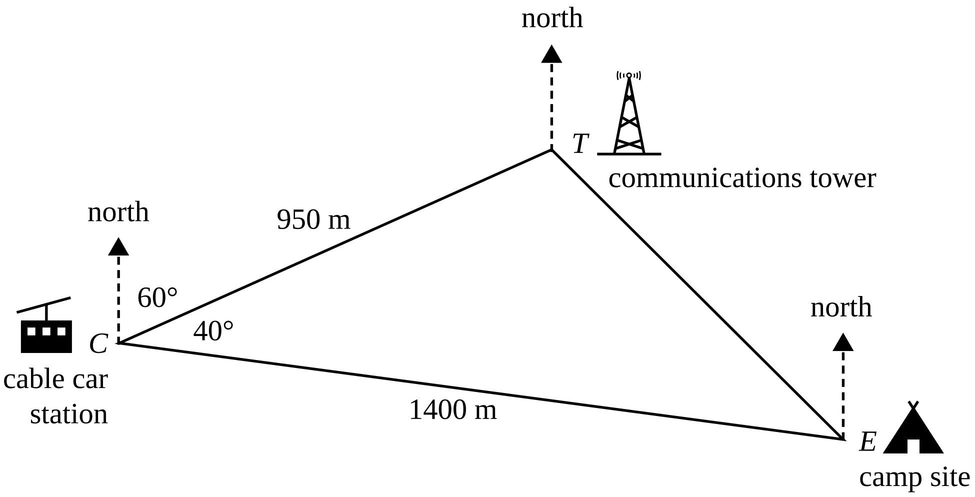


The bearing of the communications tower from the cable car station is 060°.

The length of the straight path between the communications tower and the cable car station is 950 m.

a. How far north of the cable car station is the communications tower? 1 mark

Paths will also connect the cable car station and the communications tower to a camp site at E, as shown below.



The length of the straight path between the cable car station and the camp site is 1400 m.

The angle TCE is 40°.

b. **i.** What will be the length of the straight path between the communications tower and the camp site?

Write your answer correct to the nearest metre. 1 mark

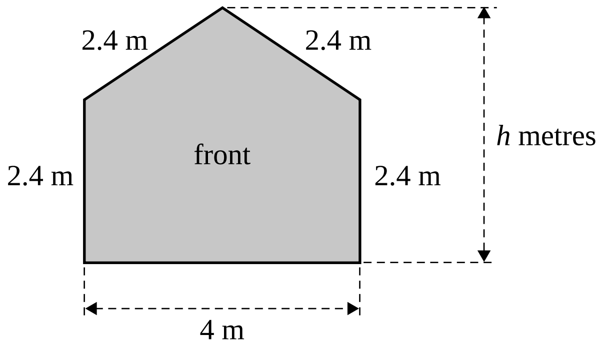
ii. Use the cosine rule to find the bearing of the camp site from the communications tower.

Write your answer correct to the nearest degree. 2 marks

Question 3 (3 marks)

Cabins are being built at the camp site.

The dimensions of the front of each cabin are shown in the diagram below.



The walls of each cabin are 2.4 m high.

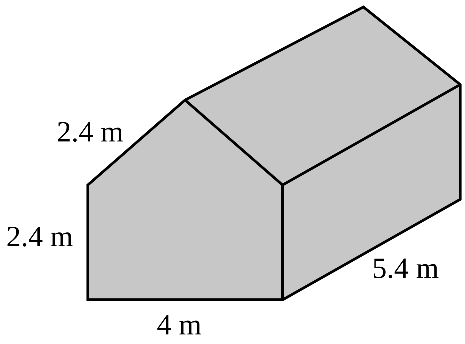
The sloping edges of the roof of each cabin are 2.4 m long.

The front of each cabin is 4 m wide.

The overall height of each cabin is h metres.

a. Show that the value of h is 3.73, correct to two decimal places. 1 mark

Each cabin is in the shape of a prism, as shown in the diagram below.

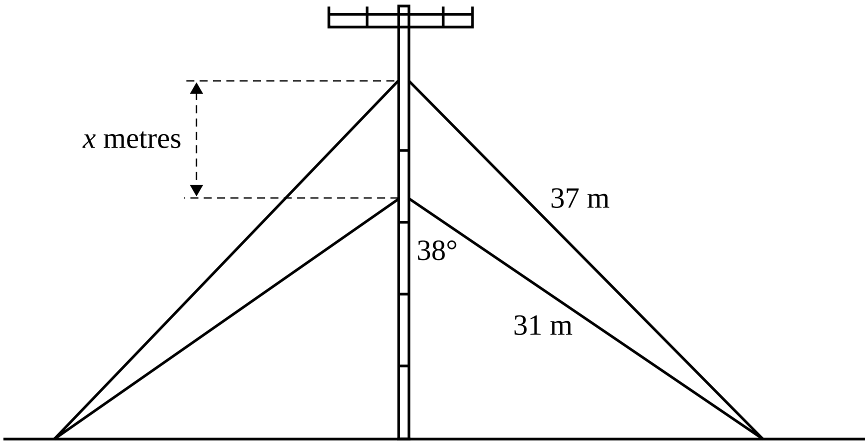


b. All external surfaces of one cabin are to be painted, excluding the base.

What is the total area of the surface to be painted?
Write your answer correct to the nearest square metre. 2 marks

Question 4 (2 marks)

Wires support the communications tower, as shown in the diagram below.



The shortest wire is 31 m long.
The shortest wire makes an angle of 38° with the communications tower.
The longest wire is 37 m long.
The longest wire is attached to the communications tower x metres above the shortest wire.

What is the value of x?
Write your answer in metres, correct to one decimal place.

END OF MODULE 2
TURN OVER

Module 6: Matrices

Question 1 (5 marks)

Students in a music school are classified according to three ability levels: beginner (B), intermediate (I) or advanced (A).

Matrix S_0, shown below, lists the number of students at each level in the school for a particular week.

$$S_0 = \begin{bmatrix} 20 \\ 60 \\ 40 \end{bmatrix} \begin{matrix} B \\ I \\ A \end{matrix}$$

a. How many students in total are in the music school that week? 1 mark

The music school has four teachers, David (D), Edith (E), Flavio (F) and Geoff (G).

Each teacher will teach a proportion of the students from each level, as shown in matrix P below.

$$\begin{matrix} & D & E & F & G \\ P = & [0.25 & 0.5 & 0.15 & 0.1] \end{matrix}$$

The matrix product, $Q = S_0P$, can be used to find the number of students from each level taught by each teacher.

b. **i.** Complete matrix Q, shown below, by writing the missing elements in the shaded boxes. 1 mark

$$Q = \begin{bmatrix} 5 & \square & 3 & 2 \\ 15 & 30 & \square & 6 \\ 10 & 20 & 6 & 4 \end{bmatrix}$$

ii. How many intermediate students does Edith teach? 1 mark

The music school pays the teachers \$15 per week for each beginner student, \$25 per week for each intermediate student and \$40 per week for each advanced student.

These amounts are shown in matrix C below.

$$C = \begin{matrix} B & I & A \\ [15 & 25 & 40] \end{matrix}$$

The amount paid to each teacher each week can be found using a matrix calculation.

c. **i.** Write down a matrix calculation in terms of Q and C that results in a matrix that lists the amount paid to each teacher each week. 1 mark

ii. How much is paid to Geoff each week? 1 mark

Question 2 (3 marks)

The ability level of the students is assessed regularly and classified as beginner (B), intermediate (I) or advanced (A).

After each assessment, students either stay at their current level or progress to a higher level.

Students cannot be assessed at a level that is lower than their current level.

The expected number of students at each level after each assessment can be determined using the transition matrix, T_1, shown below.

$$\begin{array}{c} \text{before assessment} \\ \begin{array}{ccc} B & I & A \end{array} \end{array}$$

$$T_1 = \begin{bmatrix} 0.50 & 0 & 0 \\ 0.48 & 0.80 & 0 \\ 0.02 & 0.20 & 1 \end{bmatrix} \begin{matrix} B \\ I \\ A \end{matrix} \quad \text{after assessment}$$

a. The element in the third row and third column of matrix T_1 is the number 1.

Explain what this tells you about the advanced-level students. 1 mark

Let matrix S_n be a state matrix that lists the number of students at beginner, intermediate and advanced levels after n assessments.

The number of students in the school, immediately before the first assessment of the year, is shown in matrix S_0 below.

$$S_0 = \begin{bmatrix} 20 \\ 60 \\ 40 \end{bmatrix} \begin{matrix} B \\ I \\ A \end{matrix}$$

b. **i.** Write down the matrix S_1 that contains the expected number of students at each level after one assessment.

Write the elements of this matrix correct to the nearest whole number. 1 mark

ii. How many intermediate-level students have become advanced-level students after one assessment? 1 mark

Question 3 (7 marks)

A new model for the number of students in the school after each assessment takes into account the number of students who are expected to leave the school after each assessment.

After each assessment, students are classified as beginner (*B*), intermediate (*I*), advanced (*A*) or left the school (*L*).

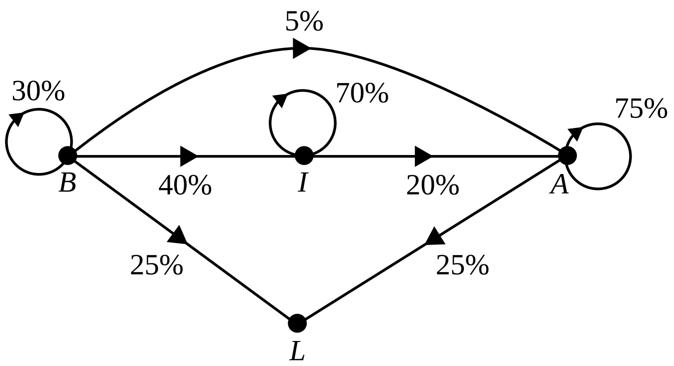
Let matrix T_2 be the transition matrix for this new model.

Matrix T_2, shown below, contains the percentages of students who are expected to change their ability level or leave the school after each assessment.

$$T_2 = \begin{array}{c} \text{before assessment} \\ \begin{array}{cccc} B & I & A & L \end{array} \\ \left[\begin{array}{cccc} 0.30 & 0 & 0 & 0 \\ 0.40 & 0.70 & 0 & 0 \\ 0.05 & 0.20 & 0.75 & 0 \\ 0.25 & 0.10 & 0.25 & 1 \end{array}\right] \begin{array}{c} B \\ I \\ A \\ L \end{array} \text{ after assessment} \end{array}$$

a. An incomplete transition diagram for matrix T_2 is shown below.

Complete the transition diagram by adding the missing information. 2 marks



The number of students at each level, immediately before the first assessment of the year, is shown in matrix R_0 below.

$$R_0 = \begin{bmatrix} 20 \\ 60 \\ 40 \\ 0 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix}$$

Matrix T_2, repeated below, contains the percentages of students who are expected to change their ability level or leave the school after each assessment.

$$\begin{matrix} & \text{before assessment} \\ & \begin{matrix} B & \quad I & \quad A & \quad L \end{matrix} \end{matrix}$$

$$T_2 = \begin{bmatrix} 0.30 & 0 & 0 & 0 \\ 0.40 & 0.70 & 0 & 0 \\ 0.05 & 0.20 & 0.75 & 0 \\ 0.25 & 0.10 & 0.25 & 1 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix} \quad \text{after assessment}$$

b. What percentage of students is expected to leave the school after the first assessment? 1 mark

c. How many advanced-level students are expected to be in the school after two assessments? Write your answer correct to the nearest whole number. 1 mark

d. After how many assessments is the number of students in the school, correct to the nearest whole number, first expected to drop below 50? 1 mark

Another model for the number of students in the school after each assessment takes into account the number of students who are expected to join the school after each assessment.

Let R_n be the state matrix that contains the number of students in the school immediately after n assessments.

Let V be the matrix that contains the number of students who join the school after each assessment.

Matrix V is shown below.

$$V = \begin{bmatrix} 4 \\ 2 \\ 3 \\ 0 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix}$$

The expected number of students in the school after n assessments can be determined using the matrix equation

$$R_{n+1} = T_2 \times R_n + V$$

where

$$R_0 = \begin{bmatrix} 20 \\ 60 \\ 40 \\ 0 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix}$$

e. Consider the intermediate-level students expected to be in the school after three assessments.

How many are expected to become advanced-level students after the next assessment?

Write your answer correct to the nearest whole number. 2 marks

END OF QUESTION AND ANSWER BOOK

Adjusted Finance Questions to Reflect the new Study Design

Question 1

A sound system used by a musical production business was initially purchased at a cost of $3,800

After 2 years, the value of the sound system had depreciated to $3150.

a. Assuming the flat rate of depreciation was used, show that the value of the sound system was depreciated by $325 each year.

1 mark

b. The value of the sound system will continue to depreciate by $325 each year.

i. Write down a recurrence relation that models the value V_n of the sound system in terms of V_{n+1} and the initial value V_0.

ii. How many years after the initial purchase will it take for the sound system to have a value of $550?

2 marks

c. The business also purchased recording equipment at a cost of $2,100. After 5 years, the value of the equipment had depreciated to $1040 using the reducing balance method of depreciation.

Calculate the annual percentage rate by which the recording equipment depreciated. Write your answer correct to two decimal places.

Question 2

Jane and Michael, the owners of the business, decide to set up a music scholarship.

To fund the scholarship, they invest in a perpetuity that pays interest at the rate of 3.68% per annum. The interest from this perpetuity is used to provide an annual scholarship of $460.

a. Calculate the minimum amount they must invest in the perpetuity to fund the scholarship.

b. For how many years will they be able to provide the scholarship?

2 marks

Question 3

As their business grows, Jane and Michael decide to invest some of their earnings.

They each choose a different investment strategy.

Jane opens an account with Red Bank, with an initial deposit of $4000. Interest is calculated at the rate of 3.6% per annum, compounding monthly.

a. i. The value of Jane's investment after n months can be evaluated using a rule of the form:

$$V_n = V_0 \times R^n$$

State the values of V_0 and R.

2 marks

ii. Determine the value of Jane's investment after 6 months. Give your answer to the nearest cent.

1 mark

b. Michael decides to open an account with Blue Bank, with an initial deposit of $2,000. At the end of each quarter, he adds an additional $200 to his account. Interest is compounded at the end of each quarter.

The equation below can be used to determine the balance of Michael's account at the end of the first quarter:

Account balance = $2000 \times (1 + 0.008) + 200$

Show that the annual compounding rate of interest is 3.2%

1 mark

c. Determine the balance in Michael's account, after the $200 has been added, at the end of 5 years.
Write your answer to the nearest cent.

1 mark

Question 4

Jane and Michael borrow $50 000 to expand their business.
Interest on the unpaid balance is charged to the loan account monthly.
The $50 000 is to be fully paid off in equal monthly repayments of $485.60 for 12 years.

a. Determine the annual compounding rate of interest. Write your answer correct to two decimal places.

1 mark

b. Calculate the total amount that will be paid off the principal at the end of the first year. Write your answer to the nearest dollar.

1 mark

c. Halfway through the term of the loan, at the end of the sixth year, Jane and Michael make an additional one off payment of $3 500.
Assume no other changes are made to the loan conditions.

Determine how much time Jane and Michael will save in repaying their loan. Give your answer to the nearest number of months.

2 marks