Ex 3C: Questions

ANSWERS

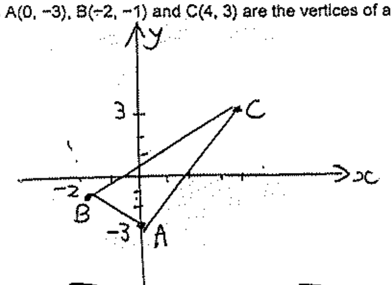
Question 1

Find the distance between the following pairs of points. In each case, give your answer as i) an exact value and ii) a decimal approximation correct to two decimal places (if appropriate)

a. (2, 5) and (6, 8) $(x_1,y_1)=(2,5)$ $(x_2,y_2)=(6,8)$ $d=\sqrt{(6-2)^2+(8-5)^2}$ $=\sqrt{4^2+3^2}$ $=\sqrt{25} = 5$	b. (-1, 2) and (4, 14) (x_1,y_1) - (x_2,y_2) $(-1,2)$ $(4,14)$ $d=\sqrt{(4--1)^2+(14-2)^2}$ $=\sqrt{5^2+12^2}$ $=\sqrt{169}$ $=13$	c. (-1, 2) and (-7, -5) (x_1,y_1) (x_2,y_2) $(-1,2)$ $(-7,-5)$ $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{(-7--1)^2+(-5-2)^2}$ $=\sqrt{(-6)^2+(-7)^2}$ $=\sqrt{36+49}=\sqrt{85}\approx 9.22$
d. (-4, 5) and (1, 1) (x_1,y_1) (x_2,y_2) $(-4,5)$ $(1,1)$ $d=\sqrt{(1--4)^2+(1-5)^2}$ $d=\sqrt{(5)^2+(-4)^2}$ $d=\sqrt{25+16}$ $d=\sqrt{41}\approx 6.40$	e. (-4, 6) and (6, -4) (x_1,y_1) (x_2,y_2) $(-4,6)$ $(6,-4)$ $d=\sqrt{(6--4)^2+(-4-6)^2}$ $d=\sqrt{(10)^2+(-10)^2}$ $d=\sqrt{100+100}$ $d=\sqrt{200}\approx 14.14$	

Question 2

Prove that the points A(0, −3), B(−2, −1) and C(4, 3) are the vertices of an isosceles triangle.



Calculate length of $\overline{BC}$

$(x_1,y_1)=(-2,-1)$

$(x_2,y_2)=(4,3)$

$d=\sqrt{(4--2)^2+(3--1)^2}$

$d=\sqrt{6^2+4^2}$

$d=\sqrt{36+16}=\sqrt{52}$

Calculate $\overline{AC}$

$(x_1,y_1)=(0,-3)$

$(x_2,y_2)=(4,3)$

$d=\sqrt{(4-0)^2+(3--3)^2}$

$d=\sqrt{4^2+6^2}=\sqrt{52}$

Calculate $\overline{BA}$

$(x_1,y_1)=(-2,-1)$

$(x_2,y_2)=(0,-3)$

$d=\sqrt{(-2-0)^2+(-1--3)^2}$

$=\sqrt{(-2)^2+(2)^2}$

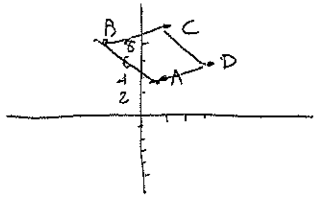
$=\sqrt{8}$

Since $\overline{BC}=\overline{AC}=\sqrt{52}$, it is isosceles

Question ~~3~~ 3

The vertices of a quadrilateral are: A(1, 4), B (-1, 8), C(1, 9) and D (3, 5).

a. Sketch the quadrialateral.



b. Find the lengths of the sides:

i. AB

$(x_1,y_1)=(1,4)$ $(x_2,y_2)=(-1,8)$

$d=\sqrt{(-1-1)^2+(8-4)^2} = \sqrt{(-2)^2+4^2}=\sqrt{20}$

ii. BC

$(x_1,y_1)=(-1,8)$, $(x_2,y_2)=(1,9)$

$d=\sqrt{(1--1)^2+(9-8)^2} = \sqrt{(2)^2+(1)^2}=\sqrt{5}$

iii. CD $(x_1,y_1)=(1,9)$, $(x_2,y_2)=(3,5)$

$d=\sqrt{(3-1)^2+(5-9)^2}$

$=\sqrt{(2)^2+(-4)^2}=\sqrt{20}$

iv. AD

$(x_1,y_1)=(1,4)$, $(x_2,y_2)=(3,5)$

$d=\sqrt{(5-4)^2+(3-1)^2}=\sqrt{1^2+2^2}$

$=\sqrt{5}$

c. Find the lengths of the diagonals AD and AC.

$\overline{BD}$: $(x_1,y_1)=(-1,8)$, $(x_2,y_2)=(3,5)$

$d=\sqrt{(5-8)^2+(3--1)^2}=\sqrt{3^2+4^2}=5$

$\overline{AC}$ $(x_1,y_1)=(1,4)$, $(x_2,y_2)=(1,9)$

$d=\sqrt{(9-4)^2+(1-1)^2}=\sqrt{25}=5$

d. What kind of quadrilateral is it?

It is a parallelgoram

Question 4

Find an expression for the distance between the two points with co-ordinates: (*a*, *b*) and (2*a*, - *b*)

$$(x_1,y_1) = (a,b),\ (x_2,y_2) = (2a,-b)$$

$$d = \sqrt{(2a-a)^2+(-b-b)^2}$$

$$= \sqrt{a^2+(-2b)^2} = \sqrt{a^2+4b^2}$$

Question 5 (Multiple Choice)

If the distance between the points (3, *b*) and (−5, 2) is 10 units, then the value of *b* is:

A. −8
[✓ B. −4]
C. 4
D. 0
E. 2

Try A: if $b=-8$ $(x_1,y_1)=(3,-8)$ $(x_2,y_2)=(-5,2)$

$$d=\sqrt{(-5-3)^2+(2--8)^2} = \sqrt{(-8)^2+(10)^2} = \sqrt{164}$$

$$\neq 10$$

Try B: if $b=-4$, $(x_1,y_1) = (3,-4)$, $(x_2,y_2)=(-5,2)$

$$d=\sqrt{(-5-3)^2+(2-4)^2} = \sqrt{(-8)^2+6^2} = \sqrt{100}$$

$$=10$$

∴ B is correct

Ex 3D: Questions

Question 1

Determine the co-ordinates of the midpoint of the line segment joining the following pairs of points:

a. (-5, -1) and (-1, -8)	b. (4, 2) and (-11, -2)	c. (3, 4) and (-5, 16)
$(x_1,y_1)=(-5,-1)$ $(x_2,y_2)=(-1,-8)$ $M=\left(\frac{-5+-1}{2},\frac{-1+-8}{2}\right)$ $=\left(-\frac{6}{2},-\frac{9}{2}\right)$ $=(-3,-4.5)$	$(x_1,y_1)=(4,2)$ $(x_2,y_2)=(-11,-2)$ $M=\left(\frac{4+-11}{2},\frac{2+-2}{2}\right)$ $=\left(\frac{-7}{2},\frac{0}{2}\right)$ $=(-3.5,0)$	$(x_1,y_1)=(3,4)$ $(x_2,y_2)=(-5,16)$ $M=\left(\frac{3+-5}{2},\frac{4+16}{2}\right)$ $=\left(\frac{-2}{2},\frac{20}{2}\right)$ $=(-1,10)$

Question 2

Find in terms of *a* and *b* the midpoint of the line segment joining the points: (*a*, 2*b*) and (5*a*, -7*b*).

(x_1,y_1) (x_2,y_2)
$(a,2b)$ $(5a,-7b)$

$$M = \left(\frac{a+5a}{2},\frac{2b-7b}{2}\right) = \left(\frac{6a}{2},\frac{-5b}{2}\right)$$

$$= \left(3a,\frac{-5b}{2}\right)$$

Question 3

The coordinates of the midpoint, M, of the line segment AB are (2, −3). If the coordinates of A are (7, 4), find the coordinates of B.

(x_1,y_1) M (x_2,y_2)
? ? $(2,-3)$ $(7,4)$

$$M = \left(\frac{x_1+7}{2},\frac{y_1+4}{2}\right)$$

$$\therefore \frac{x_1+7}{2} = 2$$

$$\therefore x_1 = 2\times2-7 = -3$$

$$\frac{y_1+4}{2} = -3$$

$$\therefore y_1 = 2\times-3\,\overline{\cancel{+}}4 = -\cancel{2}\,10$$

$B = \cancel{(-7,8),(-2)}$ $(-3,-10)$

Question 4

Find in terms of p and q the midpoint of the line segment joining the points: $(p-2q, 2p+q)$ and $(3p+6q, -2p+8q)$

(x_1, y_1) $(p-2q, 2p+q)$ (x_2, y_2) $(3p+6q, -2p+8q)$

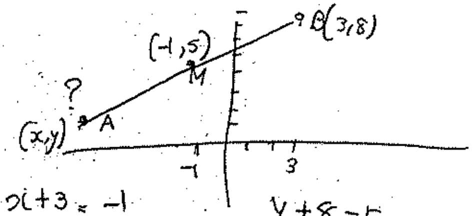
$$\text{Midpoint} = \left(\frac{p-2q+3p+6q}{2}, \frac{2p+q-2p+8q}{2}\right)$$

$$= \left(\frac{4p+4q}{2}, \frac{9q}{2}\right) = \left(2p+2q, \frac{9q}{2}\right)$$

Question 5 (multiple choice)

If the midpoint of AB is (−1, 5) and the coordinates of B are (3, 8), then A has coordinates:
If the midpoint of AB is (−1, 5) and the coordinates of B are (3, 8), then A has coordinates:

A. (1, 6.5)
B. (2, 13)
[✓ C. (−5, 2)]
D. (4, 3)
E. (7, 11)



$$\frac{x+3}{2} = -1 \qquad \frac{y+8}{2} = 5$$

$\therefore x = -5$ $\quad y = 2$

$\therefore A = (-5, 2)$

Question 6

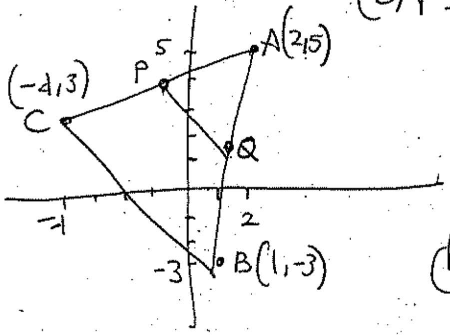
The vertices of a triangle are A(2, 5), B(1, −3) and C(−4, 3).

i. Find:
  a. the coordinates of P, the midpoint of AC
  b. the coordinates of Q, the midpoint of AB
  c. the length of PQ
  d. the length of BC.

ii. Hence show that BC = 2PQ.

HINT: Sketch a diagram first!!

(i)



(a) $P = \left(\frac{-4+2}{2}, \frac{3+5}{2}\right)$
$= (-1, 4)$

(b) $Q = \left(\frac{2+1}{2}, \frac{5+-3}{2}\right)$
$= (1.5, 1)$

(c) ~~PQ~~ =

$P = \overset{(x_1, y_1)}{(-1, 4)}$ $\quad Q = \overset{(x_2, y_2)}{(1.5, 1)}$

$d(PQ)$

$$= \sqrt{(1.5 - -1)^2 + (1-4)^2}$$

$$= \sqrt{(2.5)^2 + (-3)^2}$$

$$= \sqrt{9 + 6.25}$$

$$= \sqrt{15.25}$$

$$= \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$$

$d(BC)$ $\quad B = \overset{x_1, y_1}{(1, -3)}$ $\quad C = \overset{x_2, y_2}{(-4, 3)}$

$$d(B, C) = \sqrt{(-4-1)^2 + (3--3)^2}$$

$$= \sqrt{(-5)^2 + (6)^2}$$

$$= \sqrt{25+36} = \sqrt{61}$$

(ii) $d(\overline{BC}) = \sqrt{61}$ and $d(\overline{PQ}) = \frac{\sqrt{61}}{2}$

Therefore, $\overline{PQ}$ is half the length of $\overline{BC}$.