ANSWERS

Section A: Multiple Choice

Question 1

The function with rule $f(x) = 4\tan\left(\frac{x}{3}\right)$ has period

A. $\frac{\pi}{3}$

B. 6π

C. 3

D. 3π

E. $\frac{2\pi}{3}$

Question 2

166° is closest to

A. 2.8^c
B. 2.9^c
C. 3.0^c
D. 3.1^c
E. 3.2^c

$$\frac{166 \times \pi}{180} \simeq 2.9^c$$

Question 3

The function: $f: R \to R, f(x) = -2\cos(3x) - 1$ has amplitude, period and range :

	Amplitude	Period	Range
A.	-2	$\frac{2\pi}{3}$	$[-3,1]$
~~B.~~	2	$\frac{\pi}{3}$	$[-1,3]$
~~C.~~	3	2π	$[-3,1]$
~~D.~~	2	3π	$[-3,1]$
E.	2	$\frac{2\pi}{3}$	$[-3,1]$

1

+2

−1

−2

−3

Range: $[-3, 1]$ Period: $\frac{2\pi}{3}$

Amplitude $= 2$

Question 4

The function $y = \cos(x)$ undergoes the following transformations:

1) Its period is tripled
2) It is then dilated by a factor of 5 away from the x-axis.
3) It is then translated 7 units in the positive direction of the y-axis.

The equation of the resulting curve would be:

A. $y = 5\cos(3x) + 7$

B. $y = \frac{1}{5}\cos(3x) + 7$

C. $y = 5\cos\left(\frac{x}{3}\right) + 7$

D. $y = 5\cos\left(\frac{x}{3}\right) - 7$

E. $y = 7\cos\left(\frac{x}{3}\right) + 5$

$\cos(x)$
$\downarrow$
$\cos\left(\frac{x}{3}\right)$
$\downarrow$
$5\cos\left(\frac{x}{3}\right)$
$\downarrow$
$5\cos\left(\frac{x}{3}\right) + 7$

Question 5

The equation: $2\cos(5x) = 3\sin(5x)$ has n solutions over the domain $-\frac{\pi}{5} < x < \frac{2\pi}{5}$. The value of n is:

A. 6

B. 5

C. 4

D. 3

E. 2

Use CAS to find the solutions using Solve for $-\frac{\pi}{5} < x < \frac{2\pi}{5}$

Question 6

The temperature $T(°C)$ inside a building on a given day is given by the function $T = 5\sin\left(\frac{\pi t}{10}\right) + 20$, where t is the number of hours after 12 am. At 8 am, T is approximately equal to:

A. 20°C

B. 25°C

C. 18°C

D. 16°C

E. 23°C

At $t=8$,

$$T = 5\sin\left(\frac{8\pi}{10}\right) + 20$$

$$\simeq 22.94$$

Question7

The tidal height of the water in metres at a pier can be modelled by the function: $H(t) = 9 - 3\cos(\frac{\pi t}{6})$. The time in hours between a low tide and the very next high tide would be:

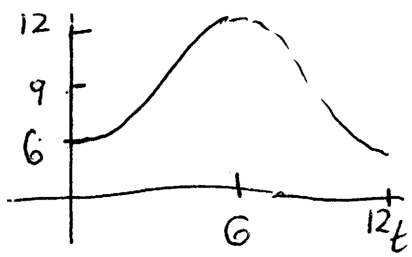
A. 24

B. 12

C. 6

D. 3

E. 12π

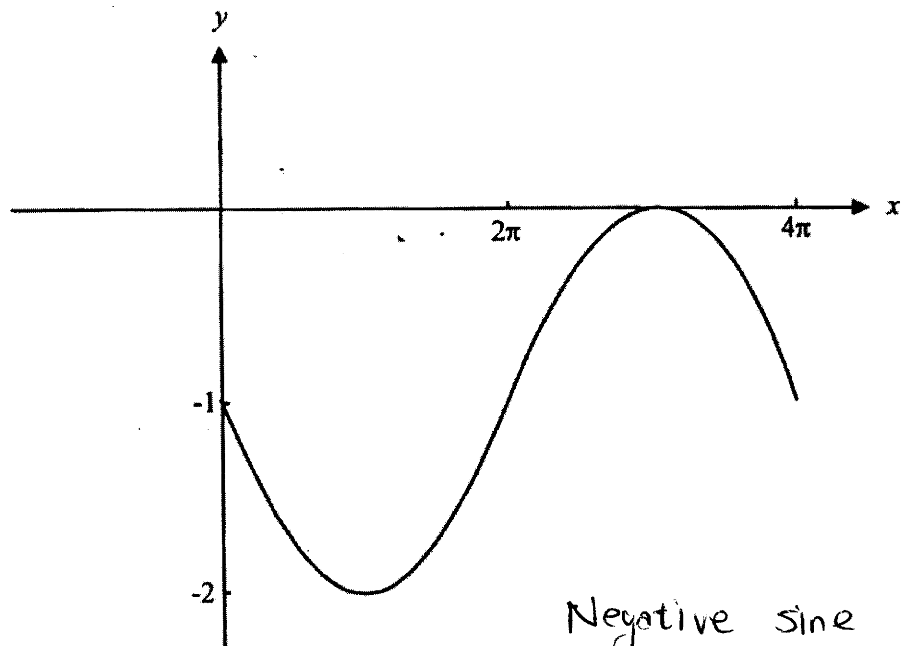


$$\text{Period} = \frac{2\pi}{\frac{\pi}{6}} = 12$$

∴ High tide occurs 6 hours after low tide.

Question 8

One period of the graph of the function f is shown below.



Negative sine

Period = 4π

$\therefore \frac{2\pi}{n} = 4\pi$

$n = \frac{1}{2}$

Amplitude = 1

$\therefore y = -\sin\left(\frac{x}{2}\right) - 1$

The rule for the function f could be

A. $y = \cos(2x) - 1$

B. $y = \cos\left(\frac{x}{2}\right) - 1$

C. $y = -\sin\left(\frac{x}{2}\right) - 1$

D. $y = -\sin(2x) - 1$

E. $y = \sin\left(\frac{x}{2}\right) - 1$

Question 9

The function $h: [0, b] \rightarrow R, h(x) = 5\sin(\frac{x}{4})$ has an inverse function. The largest possible value of b is: For h to have an inverse function, h must be one to one

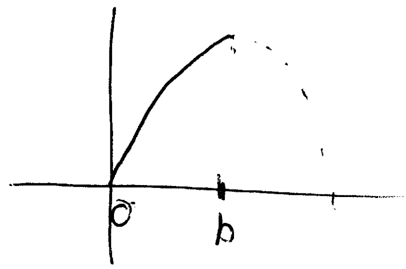
A. 2π

B. 3π

C. 4π

D. 5π

E. 6π



(b is the x value of the first turning point)

$$\text{Period} = \frac{2\pi}{\frac{1}{4}} = 8\pi$$

$$\therefore b = 2\pi$$

Question 10

If $\sin(\beta) = \frac{8}{17}$ and $\cos(\beta) = \frac{15}{17}$, then $\cos(\frac{3\pi}{2}+\beta)$ is equal to:

A. $\frac{3\pi}{2} + \frac{15}{17}$

B. $\frac{15}{17}$

C. $-\frac{15}{17}$

D. $-\frac{8}{17}$

E. $\frac{8}{17}$

$$\cos\left(\frac{3\pi}{2}+\beta\right) = \sin(\beta) = \frac{8}{17}$$

Question 11

The smallest positive value of x for which the graph of $y = 5\tan(2x)$ intersects the line with equation $y = 5$ is:

A. $\frac{\pi}{16}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{5}$

D. $\frac{5\pi}{8}$

E. 0.1

$$5\tan(2x) = 5$$

$$\tan(2x) = 1$$

$$2x = \frac{\pi}{4}$$

$$x = \frac{\pi}{8}$$

Question 12

The range of the function $y = a\sin(bx) + c$ where a, b and c are real constants, and $a > 0$, is:

A. $[a-c, a+c]$

B. $[a-b, a+b]$

C. $[b-c, b+c]$

D. $[c-a, c+a]$

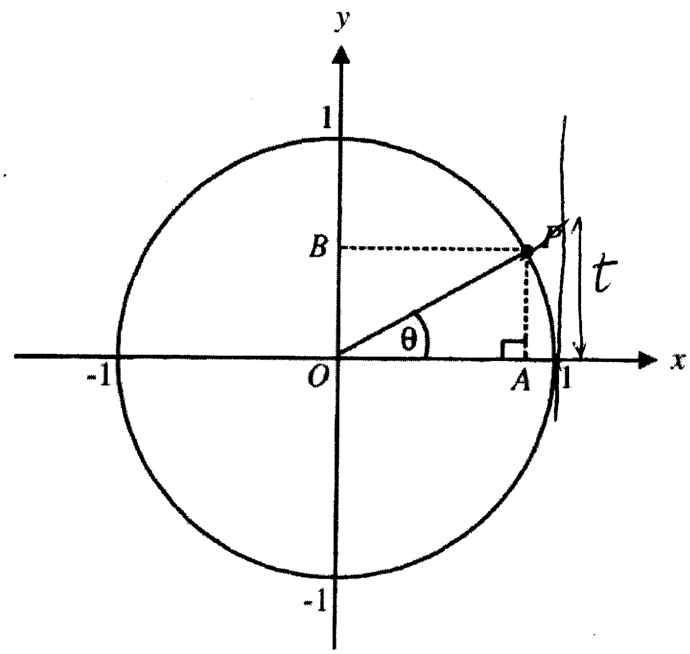
E. R

$c+a$

c

$c-a$

Question 13



The diagram above shows a unit circle which the point P lies on. The angle made between OP and the positive branch of the x-axis is θ. Which one of the following is true?

A. $\sin(\theta) = OP$

B. $\sin(\theta) = 1$

C. $\cos(\theta) = AP$

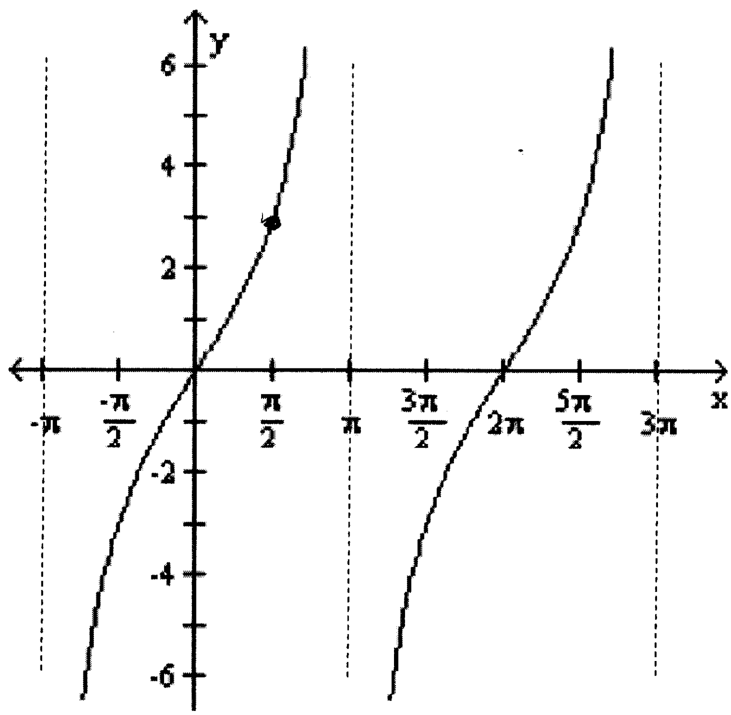
D. $\cos(\theta) = 1$

E. $\tan(\theta) = \dfrac{AP}{BP}$

$$\tan(\theta) = \frac{t}{1} = \frac{PA}{OA} = \frac{\overline{AP}}{\overline{BP}}$$

Question 14

A possible equation for the graph and its vertical asymptotes shown below would be:



A. $y = \tan(x)$

B. $y = \tan(2x)$

C. $y = 6\tan(\pi x)$

D. $y = 6\tan(\frac{x}{2})$

E. $y = \tan(\frac{x}{2})$

$y = 3\tan\left(\frac{x}{2}\right)$

Period $= 2\pi$

$\therefore \frac{\pi}{h} = 2\pi$

$\therefore h = \frac{1}{2}$

$\therefore y = a\tan\left(\frac{x}{2}\right)$

When $\frac{x}{2} = \frac{\pi}{4}$ so $x = \frac{\pi}{2}$

$y = a\tan\left(\frac{\pi}{4}\right) = a$

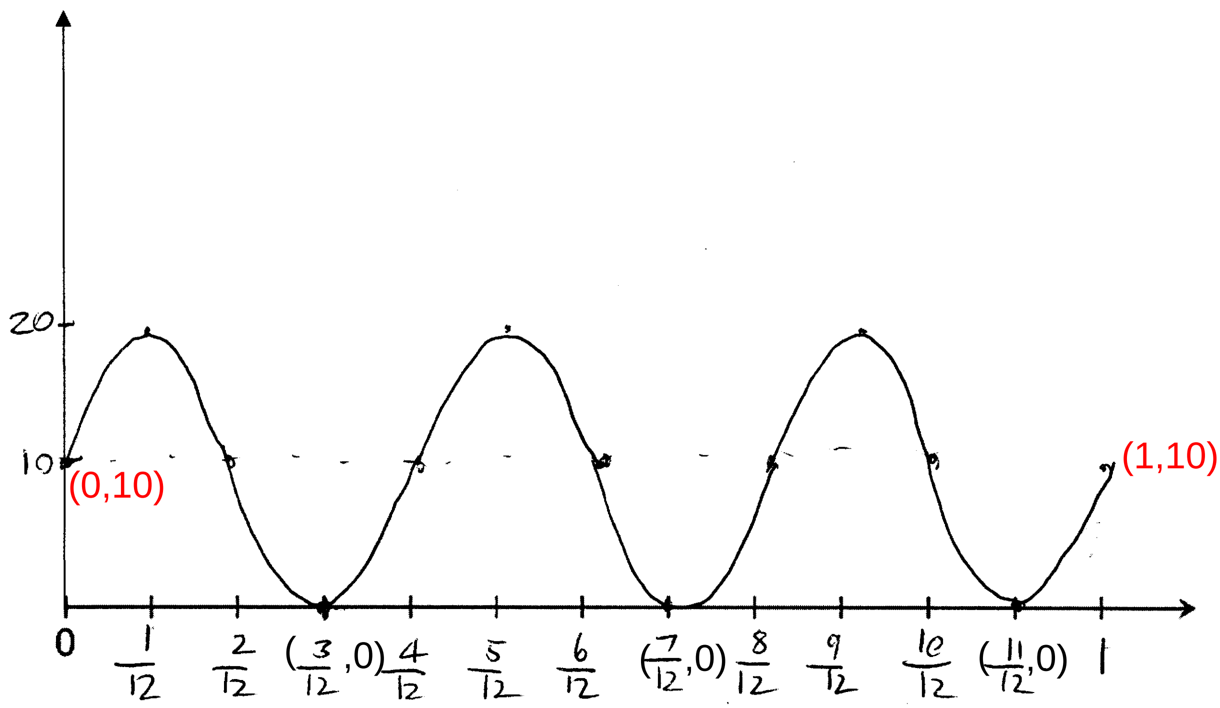
From graph, $y = 3$ when $x = \frac{\pi}{2}$ $\therefore a = 3$

Section B: Extended Response Question

A particle moves along a straight line and its distance, x cm from O at time t (seconds) is given by:

$x = 10 + 10\sin(6\pi t)$

a. Sketch the graph of x for $0 \leq t \leq 1$. Ensure that you include on your graph the co-ordinates of all intercepts with the axes and endpoints.



3 marks

b. What is the maximum distance of the particle from O ?

20 cm

1 mark

c. At what times, for $0 \leq t \leq 1$, is the particle at O ?

$t = \frac{3}{12}, \frac{7}{12}, \frac{11}{12} \quad \therefore t = \frac{1}{4}, \frac{7}{12}, \frac{11}{12}$

1 mark

d. For what values of t, where $0 \leq t \leq \frac{1}{3}$, is the particle moving **towards** O?

Look where gradient is negative.

$\frac{1}{12} < t < \frac{1}{4}$

1 mark

d. What is the **exact** displacement of the particle from O at time $t = \frac{5}{18}$?

$$x\left(\frac{5}{18}\right) = 10 - 5\sqrt{3}$$

1 mark

e. To determine the values of t during the first second of motion when the particle is 5 cm from O, we would need to solve a trigonometric equation.

i. Write down this equation.

$$10 + 10\sin(6\pi t) = 5 \quad , \quad 0 < t < 1$$

1 mark

ii. Determine the values of t within the first second for which the particle is 5 cm from O.

$$t = \frac{7}{36}, \frac{11}{36}, \frac{19}{36}, \frac{23}{36}, \frac{31}{36}, \frac{35}{36}$$

1 mark

iii. State the values of t, where $0 \le t \le 1$, for which the particle is less than 5 cm from O

$$t \in \left(\frac{7}{36}, \frac{11}{36}\right) \cup \left(\frac{19}{36}, \frac{23}{36}\right) \cup \left(\frac{31}{36}, \frac{35}{36}\right)$$

1 mark

f. Calculate the percentage of the time during the particle's motion that it spends more than 18 cm from O. Give your answer correct to one decimal place.

Solving $x(t) = 18$ for $0 < t < \frac{1}{3}$

$t = 0.04919454, \quad 0.11747213$

$\therefore$ Required percentage $= \frac{(0.11747213 - 0.04919454)}{1/3} \times 100\%$

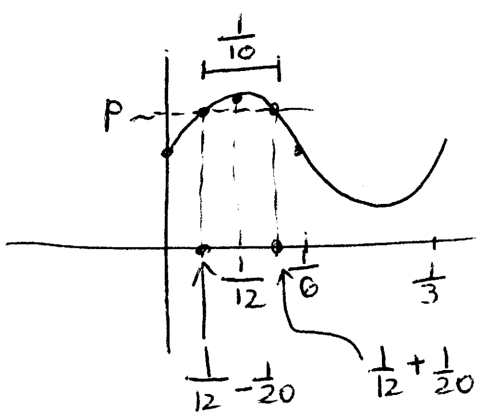
$= 20.48\%$

$\simeq 20.5\%$

3 marks

g. The particle is more than p cm from O for a **continuous** period of exactly 0.1 seconds during each cycle of its motion. Determine the value of p, correct to two decimal places.

2 marks



$t = \frac{1}{12} - \frac{1}{20} = \frac{5}{60} - \frac{3}{60} = \frac{1}{30}$

$t = \frac{1}{12} + \frac{1}{20} = \frac{5}{60} + \frac{3}{60} = \frac{8}{60} = \frac{2}{15}$

$x\left(\frac{1}{30}\right) \simeq 15.88 \qquad x\left(\frac{2}{15}\right) = 15.88$

$\therefore p = 15.88$