# Parade College

**Year 10 Mathematical Methods Enhancement**

**Topic Test: Circular Functions**

## STRUCTURE OF TEST

| *Number of Questions* | *Number of Questions to be answered* | *Number of marks* |
|---|---|---|
| 8 | 8 | 30 |

*No calculator is permitted for this test.*

*No theory notes or bound reference is allowed for this test.*

**Question 1**

Write down the exact value of each of the following:

i) $\sin(\frac{11\pi}{6})$

$$\sin\left(\frac{11\pi}{6}\right) = \sin\left(2\pi - \frac{\pi}{6}\right) = -\frac{1}{2}$$

ii) $\cos(\frac{-7\pi}{4})$

$$\cos\left(\frac{-7\pi}{4}\right) = \cos\left(-2\pi + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

iii) $\tan(\frac{8\pi}{3})$

$$\tan\left(\frac{8\pi}{3}\right) = \tan\left(2\pi + \frac{2\pi}{3}\right) = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$$

iv) $\sin(2n\pi + \frac{\pi}{3})$ where $n$ is any integer.

$$\sin\left(2n\pi + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

4 marks

**Question 2**

Simplify the expression:

$\sin(\pi + \theta) + 2\cos(\frac{\pi}{2} + \theta)$

$$\sin(\pi + \theta) + 2\cos\left(\frac{\pi}{2} + \theta\right)$$

$$= -\sin\theta - 2\sin\theta$$

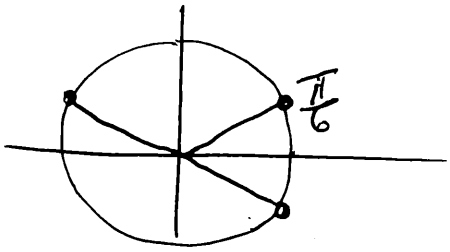
2 marks

$$= -3\sin\theta$$

**Question 3**

Solve the equation: $\sqrt{3}\tan(x) + 1 = 0$ over the domain: $-\pi < x < 2\pi$

$$\tan(x) = -\frac{1}{\sqrt{3}} \qquad -\pi < x < 2\pi$$




$$x = -\frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{11\pi}{6}$$

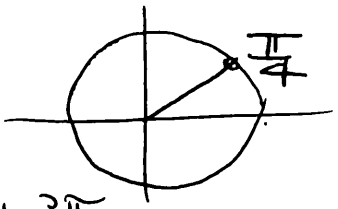
3 marks

**Question 4**

Solve the equation:

$2\sin(4x) - \sqrt{2} = 0$ for the domain: $0 \leq x \leq \pi$

$$\sin(4x) = \frac{\sqrt{2}}{2} \qquad 0 \leq 4x \leq 4\pi$$




$$4x = \frac{\pi}{4}, \frac{3\pi}{4}, 2\pi + \frac{\pi}{4}, 2\pi + \frac{3\pi}{4}$$

$$4x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$$

$$\therefore x = \frac{\pi}{16}, \frac{3\pi}{16}, \frac{9\pi}{16}, \frac{11\pi}{16}$$

4 marks

**Question 5**

For the function: $y = -8\cos\left(\frac{\pi t}{16}\right) + 15$, state:

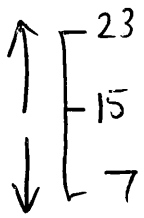
a. the period

$$\frac{2\pi}{\frac{\pi}{16}} = 2\pi \times \frac{16}{\pi}$$

$$= 32$$

1 mark

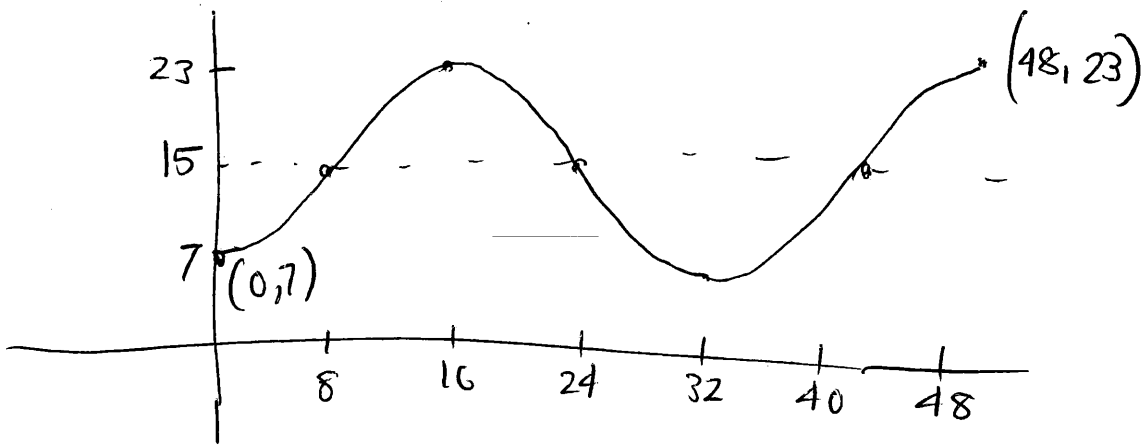
b. the range




Range: $[7, 23]$

1 mark

c. Sketch the graph of this function for the domain $0 \leq x \leq 48$. Label the co-ordinates of the endpoints.




3 marks

**Question 6**

If $\sin(\alpha) = -\frac{3}{4}$ and $\pi < \alpha < \frac{3\pi}{2}$, find the values of:

a. $\cos(\alpha)$

$\cos^2\alpha = 1 - \sin^2\alpha$

$= 1 - \left(\frac{-3}{4}\right)^2$

$= 1 - \frac{9}{16}$

$= \frac{7}{16}$ $\therefore \cos(\alpha) = \pm\frac{\sqrt{7}}{4}$

Since $\pi < \alpha < \frac{3\pi}{2}$,

$\cos(\alpha) = -\frac{\sqrt{7}}{4}$

b. $\tan(\alpha)$

$\tan(\alpha) = \frac{\sin\alpha}{\cos\alpha}$

$= \frac{-\frac{3}{4}}{-\frac{\sqrt{7}}{4}}$ $= \frac{3}{\sqrt{7}}$

$= \frac{3\sqrt{7}}{7}$

4 marks

**Question 7**

A Ferris wheel rotates at the rate of 3 revolutions per minute. When $t = 0$, a chair is at the highest point, which is 32 m above the ground. The lowest point on the wheel is 2 m above the ground.

a. Show that the period of the Ferris wheel's rotation is 20 seconds.

$60 \div 3 = 20$ sec

b. The height $h$ of the chair at time $t$ seconds after the chair leaves the highest point can be written as a function of the form: $h(t) = a\cos(nt) + b$
Determine the values of $a$, $n$ and $b$.

15 ↑ 32
c = 17
15 ↓ 2

$b = \frac{(32+2)}{2} = 17$

$a = 15$

$20 = \frac{2\pi}{n}$

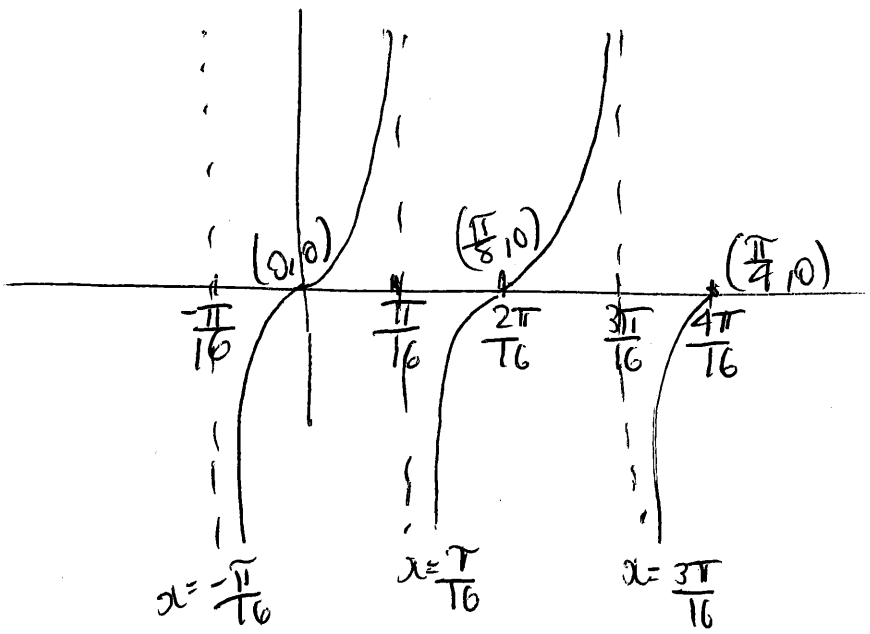
$\therefore n = \frac{\pi}{10}$

1+3 = 4 marks

**Question 8**

Sketch the graph of the function: $y = 9\tan(8x)$ for the domain: $-\frac{\pi}{16} \leq x \leq \frac{\pi}{4}$.
Include the co-ordinates of all axis intercepts and the equations of any asymptotes.

Period $= \frac{\pi}{8}$




4 marks