Question 5

At the Enhaut Secondary School, enrolments are growing. The number of enrolments N has been modelled as a function of t, the number of years after the opening of the school, according to the rule: $N = N_0(2.5)^{kt}$ where N_0 and k are positive constants. At the time of opening, there were 300 students enrolled. Two years after opening, there were 360 enrolments.

a. Calculate the value of N_0.

$N(0) = N_0 \quad \therefore 300 = N_0 \times 2.5^0 \quad \therefore N_0 = 300$

b. Calculate the value of k. Give your answer as

i. an exact value

ii. a decimal correct to four decimal places.

When $t = 2$, $N(2) = 360$

$\therefore 360 = 300 \times (2.5)^{2k}$

$\frac{360}{300} = (2.5)^{2k}$

$\therefore 1.2 = (2.5)^{2k}$

$\log_{10}(1.2) = \log_{10}(2.5^{2k})$

$\therefore \log_{10}(1.2) = 2k\log_{10}(2.5)$

$k = \frac{\log_{10}(1.2)}{2\log_{10}(2.5)}$

Use CAS to solve:

$(2.5)^{2k} = 1.2$

$k \approx 0.0995$

c. According to the model, how many students will be enrolled at Enhaut after 7 years?

$N = N_0(2.5)^{0.099489t}$ When $t = 7$, $N = 568$ (nearest whole number)

At the Enbaisse Secondary School, enrolments began to decline as soon as the Enhaut Secondary School opened. The enrolment M at Enbaisse t years after Enhaut opened can be modelled by the function: $M = M_0(2.5)^{-rt}$

Initially the enrolment at Enbaisse was 1250, but 3 years after Enhaut was established the enrolment was 1000.

d. Calculate the value of M_0.

$M_0 = 1250$

e. Calculate the value of r, giving your answer as:

i. an exact value

ii. a decimal, correct to four decimal places.

$M = 1250 \times (2.5)^{-rt} \quad M(3) = 1000$

$\therefore 1000 = 1250 \times (2.5)^{-3r}$

$\frac{1000}{1250} = (2.5)^{-3r}$

$\frac{4}{5} = (2.5)^{-3r}$

$\log_{10}(0.8) = -3r\log_{10}(2.5)$

$-3r = \frac{\log_{10}(0.8)}{\log_{10}(2.5)}$

$\therefore r = -\frac{\log_{10}(0.8)}{3\log_{10}(2.5)}$

Solving

$0.8 = (2.5)^{-3r}$,

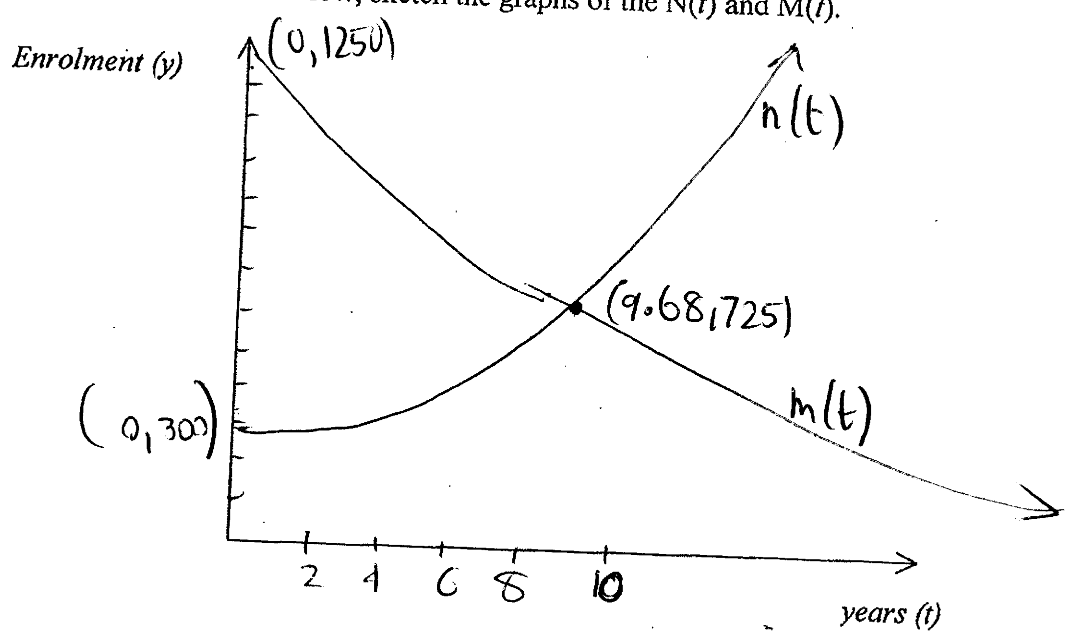
$r \approx 0.0812$

f. Calculate the number of students who will be attending Enbaisse Secondary School after 7 years according to this model.

$M = 1250 \times (2.5)^{-0.081176\ldots t}$

$M(7) \approx 843$ students

g. On the set of axes below, sketch the graphs of the $N(t)$ and $M(t)$.



h. Find the time at which the enrolments of both schools will be equal. Give your answer correct to two decimal places.

$t = 9.68$ (use ~~2nd~~ intersection on CAS)

OR:

solve: ~~solve~~ $n(t) = m(t)$

i. Express the function $M(t) = M_0\,(2.5)^{-rt}$ in the form: $M(t) = M_0 \times b^{t}$ and the function: $N(t) = N_0(2.5)^{kt}$ in the form: $N(t) = N_0 \times a^{t}$. Hence state the percentage by which the school attendance decreases each year at Enbaisse and increases each year at Enhaut.

$$M(t) = 1250 \times 2.5^{-0.081176\ldots t}$$

$$M(t) = 1250 \times \left(2.5^{-0.081176\ldots}\right)^{t}$$

$$= 1250 \times (0.92832)^{t}$$

$\therefore$ Loses 7.17% each year

$$N(t) = 300 \times (2.5)^{0.099489t}$$

$$= 300 \times (2.5^{0.099489})^{t}$$

$$= 300 \times (1.09545)^{t}$$

$\therefore$ Increases by 9.55% per year