ANSWERS

Question 3

Consider the following four statements.

A permutation matrix is always:

- I a square matrix False
- II a binary matrix True
- III a diagonal matrix False
- IV equal to the transpose of itself. False

How many of the statements above are **true**?

A. 0
B. 1
C. 2
D. 3
E. 4

Question 4

Four people, Ash (A), Binh (B), Con (C) and Dan (D), competed in a table tennis tournament.

In this tournament, each competitor played each of the other competitors once.

The results of the tournament are summarised in the matrix below.

A 1 in the matrix shows that the player named in that row defeated the player named in that column. For example, the 1 in row 3 shows that Con defeated Ash.

$$D = \text{winner}\;\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \textit{loser} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

In the tournament, each competitor was given a ranking that was determined by calculating the sum of their one-step and two-step dominances.

The competitor with the highest sum is ranked number one (1). The competitor with the second-highest sum was ranked number two (2), and so on.

Using this method, the rankings of the competitors in this tournament were

A. Dan (1), Ash (2), Con (3), Binh (4).
B. Dan (1), Ash (2), Binh (3), Con (4).
C. Con (1), Dan (2), Ash (3), Binh (4).
D. Ash (1), Dan (2), Binh (3), Con (4).
E. Ash (1), Dan (2), Con (3), Binh (4).

$$T = D + D^2$$

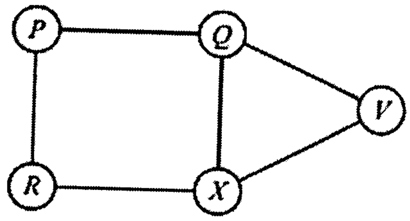
$$= \text{from}\;\begin{array}{c|cccc|c} & A & B & C & D & \text{TOTAL} \\ \hline A & 0 & 2 & 2 & 1 & 5 \\ B & 1 & 0 & 1 & 0 & 2 \\ C & 1 & 1 & 0 & 1 & 3 \\ D & 1 & 1 & 2 & 0 & 4 \end{array}$$

to

Ash (1), Dan (2), Con (3) Binh (4)

Question 1 (2 marks)

Five trout-breeding ponds, P, Q, R, X and V, are connected by pipes, as shown in the diagram below.



The matrix W is used to represent the information in this diagram.

$$W = \begin{array}{c} \begin{array}{ccccc} P & Q & R & X & V \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ P \\ Q \\ R \\ X \\ V \end{array}$$

In matrix W:

- the 1 in row 2, column 1, for example, indicates that pond P is directly connected by a pipe to pond Q
- the 0 in row 5, column 1, for example, indicates that pond P is not directly connected by a pipe to pond V.

a. In terms of the breeding ponds described, what does the sum of the elements in row 3 of matrix W represent? 1 mark

The number of pipes connected to R

The matrix W^2 is shown below.

$$W^2 = \begin{array}{c} \begin{array}{ccccc} P & Q & R & X & V \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 2 & 1 \\ 0 & 3 & 2 & 1 & 1 \\ 0 & 2 & 2 & 0 & 1 \\ 2 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} \\ P \\ Q \\ R \\ X \\ V \end{array}$$

b. Matrix W^2 has a 2 in row 2 (Q), column 3 (R).

Explain what this number tells us about the pipe connections between Q and R. 1 mark

There are 2 two-step connection links from Q to R.

Question 4

The numbers of adult and child tickets purchased for five performances of a stage show are shown in the table below.

Performance	Adult	Child
1	142	24
2	128	31
3	89	24
4	104	18
5	115	23

Which one of the following matrix calculations can be used to determine both the total number of adult tickets and the total number of child tickets purchased for all five performances?

A. $\underset{5\times 1}{\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}}\overset{2\times 5}{\begin{bmatrix}142 & 128 & 89 & 104 & 115\\ 24 & 31 & 24 & 18 & 23\end{bmatrix}}$ Multiplication undefined

B. $\underset{5\times 2}{\begin{bmatrix}1 & 1\\1 & 1\\1 & 1\\1 & 1\\1 & 1\end{bmatrix}}\overset{2\times 5}{\begin{bmatrix}142 & 128 & 89 & 104 & 115\\ 24 & 31 & 24 & 18 & 23\end{bmatrix}}$ This gives a 5×5 matrix, is not what we want.

C. $\overset{2\times 5}{\begin{bmatrix}142 & 128 & 89 & 104 & 115\\ 24 & 31 & 24 & 18 & 23\end{bmatrix}}\overset{2\times 5}{\begin{bmatrix}1 & 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1 & 1\end{bmatrix}}$ Multiplication is undefined

D. $\overset{1\times 2}{\begin{bmatrix}1 & 1\end{bmatrix}}\overset{2\times 5}{\begin{bmatrix}142 & 128 & 89 & 104 & 115\\ 24 & 31 & 24 & 18 & 23\end{bmatrix}}$ $\begin{bmatrix}142+24 & 128+31 & 89+24 & 104+18 & 115+23\end{bmatrix}$

E. $\overset{2\times 5}{\begin{bmatrix}142 & 128 & 89 & 104 & 115\\ 24 & 31 & 24 & 18 & 23\end{bmatrix}}\overset{5\times 1}{\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}} = \begin{array}{l}\begin{bmatrix}142+128+89+104+115\\ 24+31+24+18+23\end{bmatrix}\begin{array}{l}\text{Adults}\\ \text{Children}\end{array}\end{array}$

Question 1

Matrix B below shows the number of photography (P), art (A) and cooking (C) books owned by Steven (S), Trevor (T), Ursula (U), Veronica (V) and William (W).

$$B = \begin{array}{c} \begin{array}{ccc} P & A & C \end{array} & \\ \begin{bmatrix} 8 & 5 & 4 \\ 1 & 4 & 5 \\ 3 & 3 & 4 \\ 4 & 2 & 2 \\ 1 & 4 & 1 \end{bmatrix} & \begin{array}{c} S \\ T \\ U \\ V \\ W \end{array} \end{array}$$

The element in row i and column j of matrix B is b_{ij}.

The element b_{32} is the number of

A. art books owned by Trevor.
B. art books owned by Ursula.
C. art books owned by Veronica.
D. cooking books owned by Ursula.
E. cooking books owned by Trevor.

Question 2

Four matrices are shown below.

3×1 2×3 1×2 3×3

$$W = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 1 & 5 \\ 2 & 0 & 6 \end{bmatrix} \quad Y = \begin{bmatrix} 7 & 1 \end{bmatrix} \quad Z = \begin{bmatrix} 8 & 5 & 0 \\ 1 & 9 & 3 \\ 4 & 2 & 7 \end{bmatrix}$$

Which one of the following matrix products is **not** defined?

A. $W \times Y$
B. $X \times W$
C. $Y \times X$
D. $Z \times W$
E. $Z \times Y$

3×1 1×2
W Y Defined

2×3 3×1
X W Defined

1×2 2×3
Y X Defined

3×3 3×1
Z W Defined

3×3 1×2
Z Y Undefined

Module 6: Matrices

Question 1 (5 marks)

Students in a music school are classified according to three ability levels: beginner (B), intermediate (I) or advanced (A).

Matrix S_0, shown below, lists the number of students at each level in the school for a particular week.

$$S_0 = \begin{array}{c} \begin{bmatrix} 20 \\ 60 \\ 40 \end{bmatrix} \end{array} \begin{array}{l} B \\ I \\ A \end{array}$$

a. How many students in total are in the music school that week? 1 mark

$20 + 60 + 40 = 120$

The music school has four teachers, David (D), Edith (E), Flavio (F) and Geoff (G).

Each teacher will teach a proportion of the students from each level, as shown in matrix P below.

$$P = \begin{array}{c} \begin{array}{cccc} D & E & F & G \end{array} \\ \begin{bmatrix} 0.25 & 0.5 & 0.15 & 0.1 \end{bmatrix} \end{array}$$

The matrix product, $Q = S_0P$, can be used to find the number of students from each level taught by each teacher.

b. i. Complete matrix Q, shown below, by writing the missing elements in the shaded boxes. 1 mark

$$Q = \begin{bmatrix} 5 & \boxed{10} & 3 & 2 \\ 15 & 30 & \boxed{9} & 6 \\ 10 & 20 & 6 & 4 \end{bmatrix}$$

ii. How many intermediate students does Edith teach? 1 mark

30

3×1

1×4

$$\begin{bmatrix} 20 \\ 60 \\ 40 \end{bmatrix} \begin{bmatrix} 0.25 & 0.5 & 0.15 & 0.1 \end{bmatrix}$$

$$= \begin{array}{c} \\ B \\ I \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} D & E & F & G \end{array} \\ \begin{bmatrix} 20 \times 0.25 & 20 \times 0.5 & 20 \times 0.15 & 20 \times 0.1 \\ 60 \times 0.25 & 60 \times 0.5 & 60 \times 0.15 & 60 \times 0.1 \\ 40 \times 0.25 & 40 \times 0.5 & 40 \times 0.15 & 40 \times 0.1 \end{bmatrix} \end{array}$$

The music school pays the teachers \$15 per week for each beginner student, \$25 per week for each intermediate student and \$40 per week for each advanced student.

These amounts are shown in matrix C below.

$$C = \begin{array}{c} \begin{matrix} B & I & A \end{matrix} \\ \begin{bmatrix} 15 & 25 & 40 \end{bmatrix} \end{array}$$

The amount paid to each teacher each week can be found using a matrix calculation.

c. **i.** Write down a matrix calculation in terms of Q and C that results in a matrix that lists the amount paid to each teacher each week. 1 mark

CQ

ii. How much is paid to Geoff each week? 1 mark

$$\begin{array}{ccc} B & I & A \\ \end{array} \begin{array}{cccc} D & E & F & G \end{array}$$

$$\begin{bmatrix} 15 & 25 & 40 \end{bmatrix} \begin{bmatrix} 5 & 10 & 3 & 2 \\ 15 & 30 & 9 & 6 \\ 10 & 20 & 6 & 4 \end{bmatrix}$$

Geoff: $15 \times 2 + 25 \times 6 + 40 \times 4$

$= 30 + 150 + 160$

$= \$340.$