VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

SOLUTIONS

Victorian Certificate of Education
Year

FURTHER MATHEMATICS

Written examination 1

Day Date

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (1 hour 30 minutes)**

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A – Core	24	24			24
B – Modules	32	16	4	2	16
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question book of 32 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

Instructions
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Version 4 – September 2016

SOLUTIONS – MULTIPLE CHOICE

SECTION A – Core

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1

The following stem plot shows the areas, in square kilometres, of 27 suburbs of a large city.

key: $1|6 = 1.6\ \text{km}^2$

1	5	6	7	8				
2	1	2	4	5	6	8	9	9
3	0	1	1	2	2	8	9	
4	0	4	7					
5	0	1						
6	1	9						
7								
8	4							

$n = 27$

$\frac{n+1}{2}$ th $=$ 14th

The median area of these suburbs, in square kilometres, is

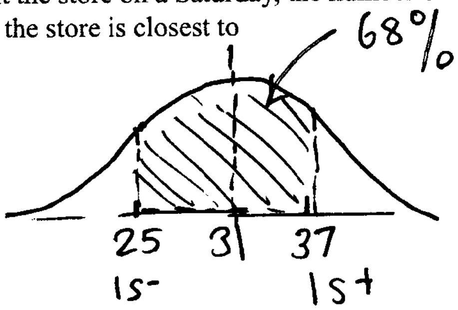
A. 3.0
B. 3.1
C. 3.5
D. 30.1
E. 30.5

Question 2

The time spent by shoppers at a hardware store on a Saturday is approximately normally distributed with a mean of 31 minutes and a standard deviation of 6 minutes.

If 2850 shoppers are expected to visit the store on a Saturday, the number of shoppers who are expected to spend between 25 and 37 minutes in the store is closest to

A. 16
B. 68
C. 460
D. 1900
E. 2400



0.68×2850
$= 1938$

Use the following information to answer Questions 3–6.

The following table shows the data collected from a random sample of seven drivers drawn from the population of all drivers who used a supermarket car park on one day. The variables in the table are:

- *distance* – the distance that each driver travelled to the supermarket from their home
- *sex* – the sex of the driver (female, male)
- *number of children* – the number of children in the car
- *type of car* – the type of car (sedan, wagon, other)
- *postcode* – the postcode of the driver's home.

***Distance* (km)**	***Sex* (F = female, M = male)**	***Number of children***	***Type of car* (1 = sedan, 2 = wagon, 3 = other)**	***Postcode***
4.2	F	2	1	8148
0.8	M	3	2	8147
3.9	F	3	2	8146
5.6	F	1	3	8245
0.9	M	1	3	8148
1.7	F	2	2	8147
2.5	M	2	2	8145

Question 3

The mean, $\bar{x}$, and the standard deviation, s_x, of the variable, distance, for these drivers are closest to

A. $\bar{x} = 2.5 \quad s_x = 3.3$

B. $\bar{x} = 2.8 \quad s_x = 1.7$

C. $\bar{x} = 2.8 \quad s_x = 1.8$

D. $\bar{x} = 2.9 \quad s_x = 1.7$

E. $\bar{x} = 3.3 \quad s_x = 2.5$

CAS

distance
4.2
0.8
3.9
5.6
⋮

Then use Stat Calculations One-Variable Statistics

$\bar{x} = 2.8$

$S_x = 1.822$

Question 4

The number of discrete numerical variables in this data set is

A. 0

B. 1

C. 2

D. 3

E. 4

Only # children is a discrete numerical variable

Postcode is a categorical variable.

Question 5

The number of ordinal variables in this data set is

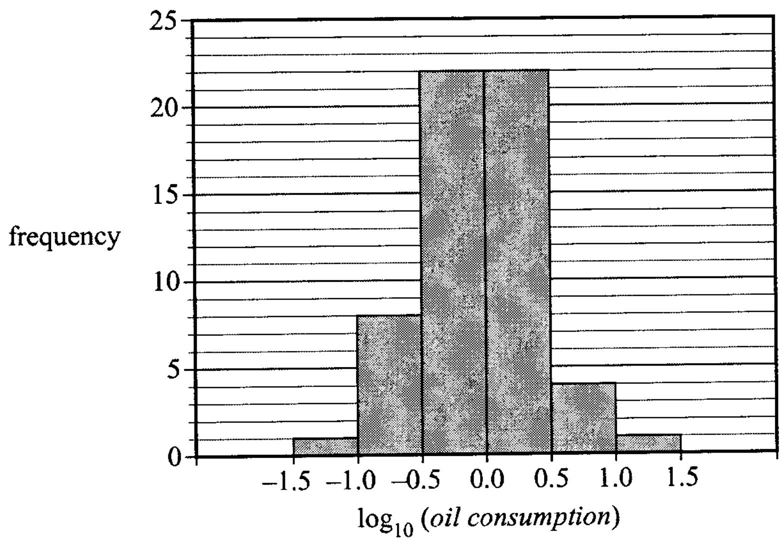
A. 0
B. 1
C. 2
D. 3
E. 4

Question 6

The number of female drivers with three children in the car is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 7



The histogram above displays the distribution of the annual per capita *oil consumption* (tonnes) for 58 countries plotted on a **log** scale.

The percentage of countries with an annual per capita *oil consumption* of more than 10 tonnes is closest to

A. 1%
B. 2%
C. 27%
D. 57%
E. 98%

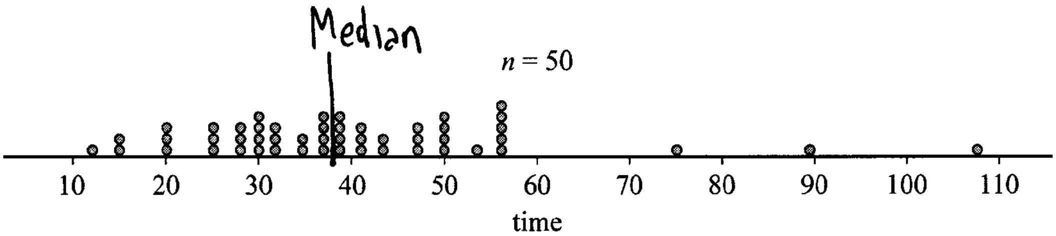
$10 = 10^1$

so $\log_{10}(10) = 1$

We are looking at the column from 1.0 to 1.5, which has a percentage frequency of 1, which is a percentage frequency of $\frac{1}{58} \times \frac{100}{1} \approx 1.7\%$

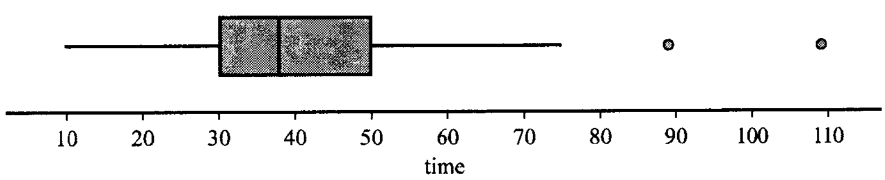
Question 8

The dot plot below shows the distribution of the time, in minutes, that 50 people spent waiting to get help from a call centre.

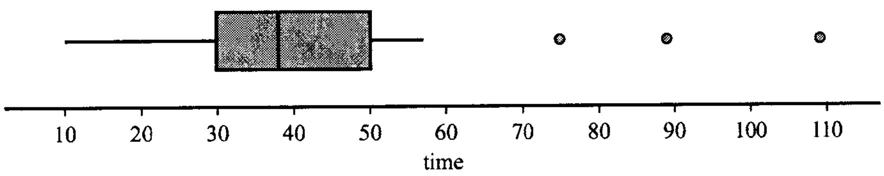


Which one of the following boxplots best represents the data?

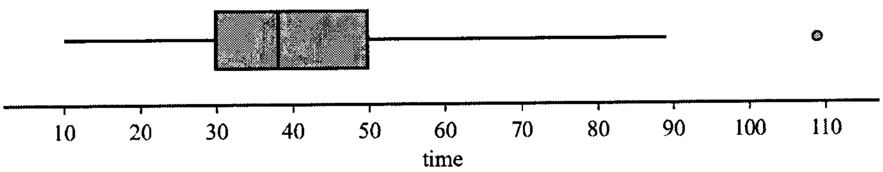
A.



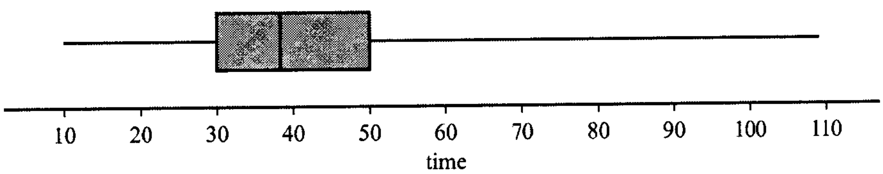
B.



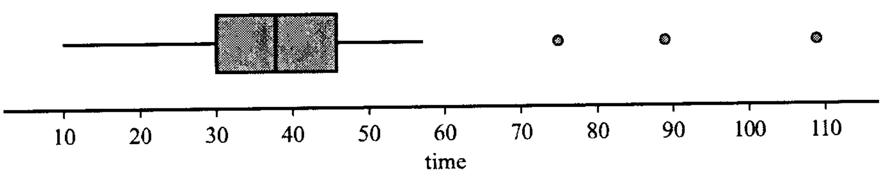
C.



D.



E.



Median : $\frac{n+1}{2}$ th

$= 25\frac{1}{2}$ th

∴ Halfway between 25th and 26th

∴ ≈ 38

Q_1 is 13th data point

= 30

Q_3 is 13th after mean

≈ 50

∴ IQR ≈ 20

Upper fence

$= Q_3 + 1.5 \times 20$

$\approx 50 + 30$

≈ 80

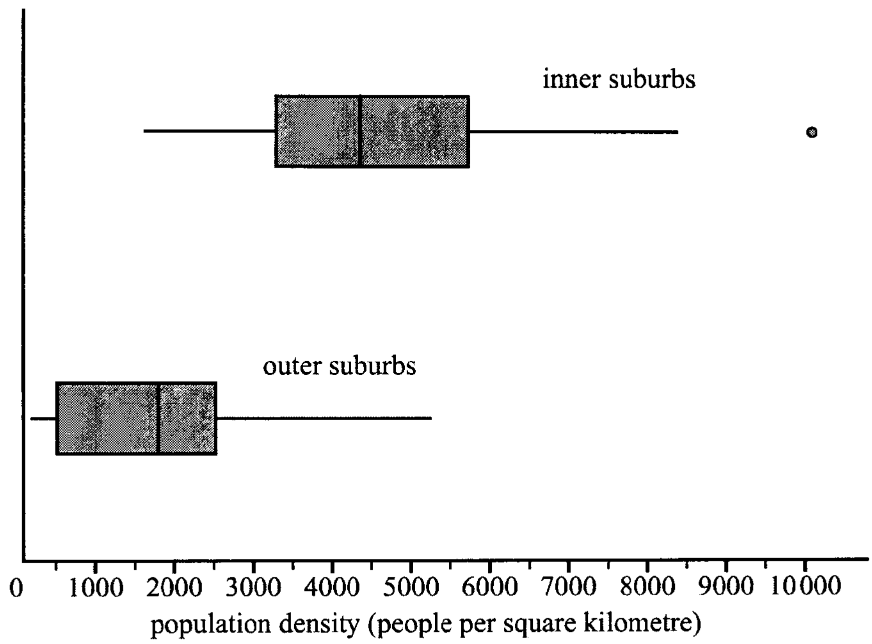
This means that 74 is not an outlier but 90, 110 are.

∴ A is correct option

(it is the only boxplot with two upper outliers)

Question 9

The parallel boxplots below summarise the distribution of population density, in people per square kilometre, for the inner suburbs and the outer suburbs of a large city.



Which one of the following statements is **not** true?

A. More than 50% of the outer suburbs have population densities below 2000 people per square kilometre.

B. More than 75% of the inner suburbs have population densities below 6000 people per square kilometre.

C. Population densities are more variable in the outer suburbs than in the inner suburbs.

D. The median population density of the inner suburbs is approximately 4400 people per square kilometre.

E. Population densities are, on average, higher in the inner suburbs than in the outer suburbs.

IQR and range are both higher for inner city suburbs ∴ Variability higher for inner.

Question 10

A single back-to-back stem plot would be an appropriate graphical tool to investigate the association between a car's speed, in kilometres per hour, and the

A. driver's age, in years.

B. car's colour (white, red, grey, other).

C. car's fuel consumption, in kilometres per litre.

D. average distance travelled, in kilometres.

E. driver's sex (female, male).

Back to back Stemplot is used where we have a categorical variable with two categories (M and F) and a numerical variable (speed)

Question 11

The equation of a least squares regression line is used to predict the fuel consumption, in kilometres per litre of fuel, from a car's weight, in kilograms.

This equation predicts that a car weighing 900 kg will travel 10.7 km per litre of fuel, while a car weighing 1700 kg will travel 6.7 km per litre of fuel.

The slope of this least squares regression line is closest to

A. –200.0
B. –0.005
C. –0.004
D. 0.005
E. 200.0

explanatory variable: x = weight
response variable: y = fuel consumption
(900, 10.7) (1700, 6.7)

$$m = \frac{6.7 - 10.7}{1700 - 900} = \frac{-4}{800} = -\frac{1}{200} = -0.005$$

Question 12

A large study of secondary-school male students shows that there is a negative association between the time spent playing sport each week and the time spent playing computer games.

From this information, it can be concluded that

A. male students who spend a lot of time playing computer games do not play sport.
B. encouraging male students to spend less time playing sport will increase the time they spend playing computer games.
C. encouraging male students to spend more time playing sport will reduce the time they spend playing computer games.
D. male students who tend to spend more time playing sport tend to spend less time playing computer games.
E. male students who tend to spend more time playing sport tend to spend more time playing computer games.

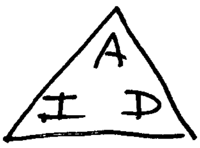
a correlation only shows the tendency of the response variable to move in either a positive or negative direction as the explanatory variable increases. It NEVER suggests any causal relationship between the variables.

Question 13

The seasonal index for heaters in winter is 1.25

To correct for seasonality, the actual heater sales in winter should be

A. reduced by 20%
B. increased by 20%
C. reduced by 25%
D. increased by 25%
E. reduced by 75%



$$D = \frac{A}{I} = \frac{A}{1.25}$$
$$= 0.8A$$

$\therefore$ A must be reduced by 20% to get D.

Use the following information to answer Questions 14 and 15.

The seasonal indices for the first 11 months of the year for sales in a sporting equipment store are shown in the table below.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Seasonal index	1.23	0.96	1.12	1.08	0.89	0.98	0.86	0.76	0.76	0.95	1.12	

Question 14

The seasonal index for December is

A. 0.89

B. 0.97

C. 1.02

D. 1.23

E. 1.29

$12 - (1.23 + 0.96 + 1.12 + 1.08 + 0.89 + 0.98 + 0.86 + 0.76 + 0.76 + 0.95 + 1.12)$

Question 15

In May, the store sold \$213 956 worth of sporting equipment.

The deseasonalised value of these sales was closest to

A. \$165 857

B. \$190 420

C. \$209 677

D. \$218 322

E. \$240 400

$$D = \frac{A}{I} = \frac{213956}{0.89} = 240400$$

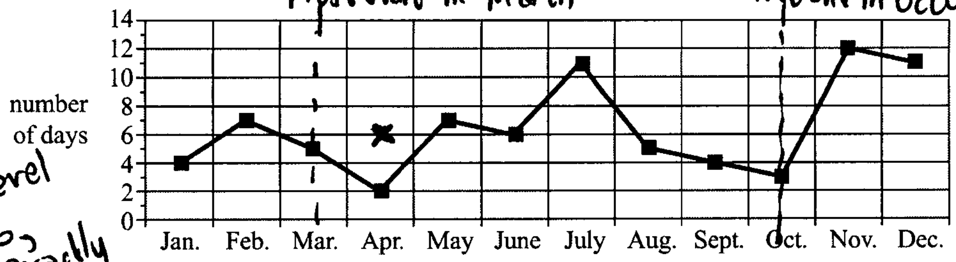
Question 16

The time series plot below shows the number of days that it rained in a town each month during 2011.

Must start in March

Must end in October

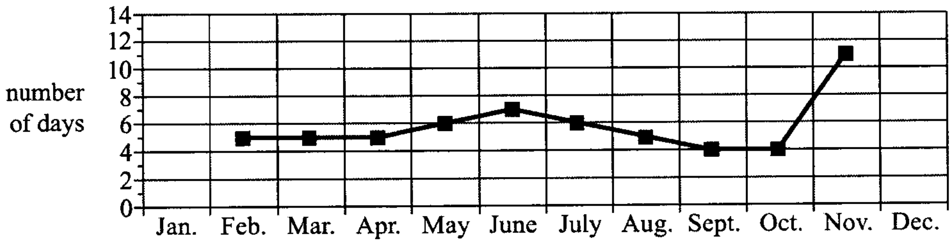
April value must be on the level at 6, exactly



Using five-median smoothing, the smoothed time series plot will look most like

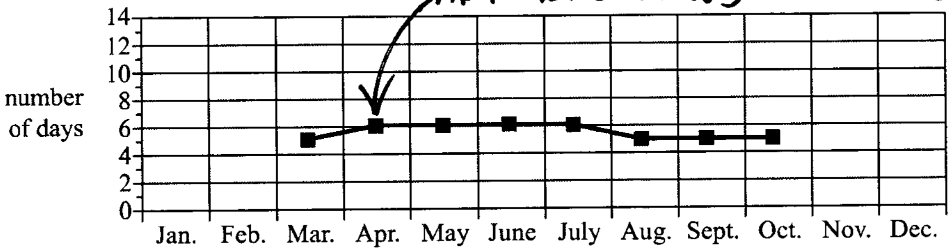
~~A.~~

Doesn't start in March



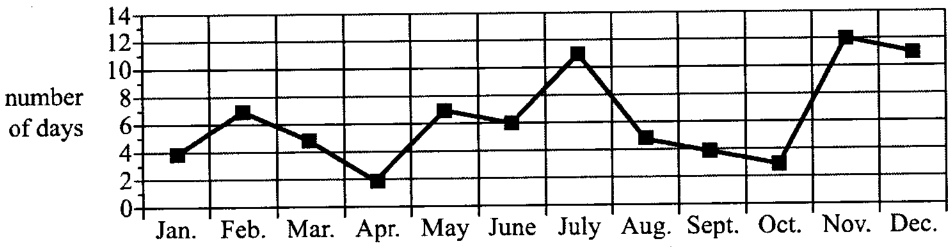
B.

April value directly on the value $y=6$



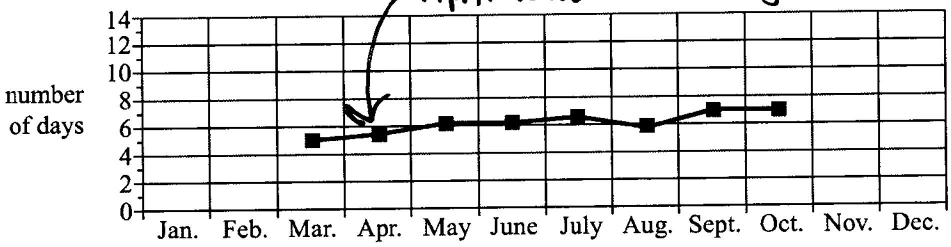
~~C.~~

Doesn't start in March



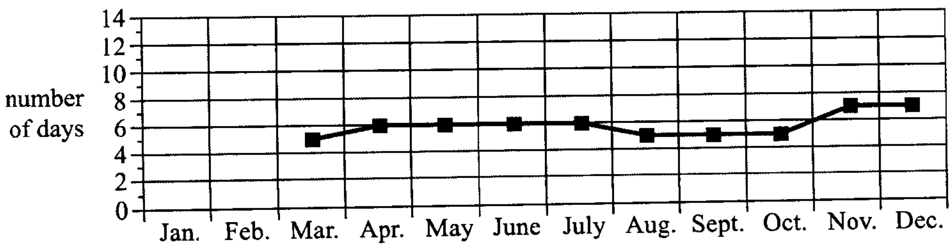
~~D.~~

April value below $y=6$.



~~E.~~

Doesn't end in October



Note the process of elimination used to solve this problem.

Recursion and financial modelling

Question 17

$$P_0 = 2000,\ P_{n+1} = 1.5P_n - 500$$

The first three terms of a sequence generated by the recurrence relation above are

A. 500, 2500, 2000 …
B. 2000, 1500, 1000 …
C. 2000, 2500, 3000 …
D. 2000, 2500, 3250 …
E. 2000, 3000, 4500 …

$P_1 = 1.5 \times 2000 - 500 = 2500$
$P_2 = 1.5 \times 2500 - 500 = 3250$
$\therefore$ 2000, 2500, 3250

Question 18

Which of the following recurrence relations will generate a sequence whose values decay geometrically?

A. $L_0 = 2000,\ L_{n+1} = L_n - 100$
B. $L_0 = 2000,\ L_{n+1} = L_n + 100$
C. $L_0 = 2000,\ L_{n+1} = 0.65L_n$
D. $L_0 = 2000,\ L_{n+1} = 6.5L_n$
E. $L_0 = 2000,\ L_{n+1} = 0.85L_n - 100$

This recurrence relation says: the next value is 0.65 × current value.

so the next value is always less than the previous value. Hence we have decay. It is geometrical becuase the decrease is not constant, but each time is a percentage (65%) of a reducing value.

Question 19

Eva has $1200 that she plans to invest for one year.
One company offers to pay her interest at the rate of 6.75% per annum compounding daily.
The effective annual interest rate for this investment would be closest to

A. 6.75%
B. 6.92%
C. 6.96%
D. 6.98%
E. 6.99%

CAS: eff(6.75, 365) ≈ 6.98

Question 20

Rohan invests $15 000 at an annual interest rate of 9.6% compounding monthly.
Let V_n be the value of the investment after n months.
A recurrence relation that can be used to model this investment is

A. $V_0 = 15\,000,\ V_{n+1} = 0.96V_n$
B. $V_0 = 15\,000,\ V_{n+1} = 1.008V_n$
C. $V_0 = 15\,000,\ V_{n+1} = 1.08V_n$
D. $V_0 = 15\,000,\ V_{n+1} = 1.0096V_n$
E. $V_0 = 15\,000,\ V_{n+1} = 1.096V_n$

$R = 1 + \frac{r/n}{100}$
$R = 1 + \frac{9.6/12}{100}$
$R = 1 + 0.008$
$= 1.008$

$\therefore V_{n+1} = 1.008 V_n,$
$V_0 = 15000$

Use the following information to answer Questions 21–23.

Kim invests \$400 000 in an annuity paying 3.2% interest per annum.

The annuity is designed to give her an annual payment of \$47 372 for 10 years.

The amortisation table for this annuity is shown below.

Some of the information is missing.

Payment number (n)	Payment made	Interest earned	Reduction in principal	Balance of annuity
0	0	0.00	0.00	400 000.00
1	47 372.00	12 800.00	34 572.00	
2	47 372.00	11 693.70	35 678.30	329 749.70
3	47 372.00	10 551.99	36 820.01	292 929.69
4	47 372.00	9 373.75	37 998.25	254 931.44
5	47 372.00	8 157.81		215 717.24
6	47 372.00	6 902.95	40 469.05	175 248.19
7	47 372.00	5 607.94	41 764.06	133 484.14
8	47 372.00			90 383.63
9	47 372.00	2 892.28	44 479.72	45 903.90
10	47 372.00	1 468.92	45 903.08	0.83

Question 21

The balance of the annuity after one payment has been made is

A. \$339 828.00
B. \$352 628.00
C. \$365 428.00
D. \$387 200.00
E. \$400 000.00

$400000 - 34572$

$= 365{,}428$

Question 22

The reduction in the principal of the annuity after payment number 5 is

A. \$36 820.01
B. \$37 998.25
C. \$39 214.19
D. \$40 469.05
E. \$41 764.06

$47372 - 8157.81$

$= 39214.19$

Question 23

The amount of payment number 8 that is the interest earned is closest to

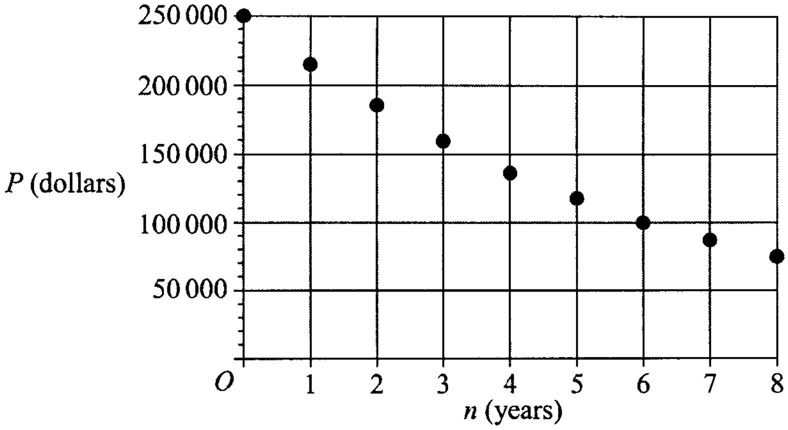
A. \$3799.82
B. \$4074.67
C. \$4271.49
D. \$4836.57
E. \$5607.94

$133484.14 \times \frac{3.2}{100}$

$= 4271.49$

Question 24

The following graph shows the decreasing value of an asset over eight years.



Let P_n be the value of the asset after n years, in dollars.

A rule for evaluating P_n could be

~~A.~~ $P_n = 250\,000 \times (1 + 0.14)^n$ This would give growth, not decay

~~B.~~ $P_n = 250\,000 \times 1.14 \times n$ This is linear growth

~~C.~~ $P_n = 250\,000 \times (1 - 0.14) \times n$ This is linear growth

D. $P_n = 250\,000 \times (0.14)^n$

E. $P_n = 250\,000 \times (1 - 0.14)^n$

After 1 year, the value has fallen from 250000 to $\approx$ 220000

$P_1 \approx 225000$

$P_0 = 250000$

$\therefore P_1 = 250000 \times \left(1 - \frac{r}{100}\right)^1$

$215000 = 250000 \times \left(1 - \frac{r}{100}\right)$

$\frac{215}{250} = 1 - \frac{r}{100}$

$0.86 \approx 1 - \frac{r}{100}$

$\therefore \frac{r}{100} \approx 0.14$

$\therefore$ E is the correct option

Module 1 – Matrices

Before answering these questions, you must **shade** the 'Matrices' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Matrix B, below, shows the number of photography (P), art (A) and cooking (C) books owned by Steven (S), Trevor (T), Ursula (U), Veronica (V) and William (W).

$$B = \begin{array}{c} \begin{array}{ccc} P & A & C \end{array} \\ \begin{bmatrix} 8 & 5 & 4 \\ 1 & 4 & 5 \\ 3 & 3 & 4 \\ 4 & 2 & 2 \\ 1 & 4 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ S \\ T \\ U \\ V \\ W \end{array}$$

$b_{32} = 3$ which is in row U and column A

$\therefore$ No. of art books owned by Ursula.

The element in row i and column j of matrix B is b_{ij}.

The element b_{32} is the number of

A. art books owned by Trevor.

B. art books owned by Ursula.

C. art books owned by Veronica.

D. cooking books owned by Ursula.

E. cooking books owned by Trevor.

Question 2

The total cost of one ice-cream and three soft drinks at Catherine's shop is \$9.

The total cost of two ice-creams and five soft drinks is \$16.

Let x be the cost of an ice-cream and y be the cost of a soft drink.

The matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ is equal to

A. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

B. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

D. $\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

E. $\begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

$x + 3y = 9$

$2x + 5y = 16$

$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 16 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

Question 3

Consider the following four statements.

A permutation matrix is always:

I a square matrix True

II a binary matrix True

III a diagonal matrix ← not necessarily true

IV equal to the transpose of itself. ← only a symmetrical matrix is the transpose of itself

How many of the statements above are **true**?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 4

Four people, Ash (A), Binh (B), Con (C) and Dan (D), competed in a table tennis tournament.

In this tournament, each competitor played each of the other competitors once.

The results of the tournament are summarised in the matrix below.

A 1 in the matrix shows that the player named in that row defeated the player named in that column. For example, the 1 in row 3 shows that Con defeated Ash.

$$D = \begin{array}{cc} & \textit{loser} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ \textit{winner} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

In the tournament, each competitor was given a ranking that was determined by calculating the sum of their one-step and two-step dominances.

The competitor with the highest sum is ranked number one (1). The competitor with the second-highest sum was ranked number two (2), and so on.

Using this method, the rankings of the competitors in this tournament were

A. Dan (1), Ash (2), Con (3), Binh (4).

B. Dan (1), Ash (2), Binh (3), Con (4).

C. Con (1), Dan (2), Ash (3), Binh (4).

D. Ash (1), Dan (2), Binh (3), Con (4).

E. Ash (1), Dan (2), Con (3), Binh (4).

$$D + D^2 = \begin{array}{cc} & \\ \textit{winner} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 2 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 0 \end{bmatrix} \begin{array}{c} 5 \\ 2 \\ 3 \\ 4 \end{array} \end{array}$$

Ash (1), Dan (2), Con (3), Binh (4)

Question 5

The matrix S_{n+1} is determined from the matrix S_n using the rule $S_{n+1} = T S_n - C$, where T, S_0 and C are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix}, \; S_0 = \begin{bmatrix} 100 \\ 250 \end{bmatrix} \text{ and } C = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

Must do it sequentially, step by step.

Given this information, the matrix S_2 equals

A. $\begin{bmatrix} 100 \\ 250 \end{bmatrix}$

B. $\begin{bmatrix} 148 \\ 122 \end{bmatrix}$

C. $\begin{bmatrix} 170 \\ 140 \end{bmatrix}$

D. $\begin{bmatrix} 180 \\ 130 \end{bmatrix}$

E. $\begin{bmatrix} 190 \\ 160 \end{bmatrix}$

$$S_1 = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix}\begin{bmatrix} 100 \\ 250 \end{bmatrix} - \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$
$$= \begin{bmatrix} 180 \\ 130 \end{bmatrix}$$
$$S_2 = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix}\begin{bmatrix} 180 \\ 130 \end{bmatrix} - \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$
$$= \begin{bmatrix} 148 \\ 122 \end{bmatrix}$$

Question 6

A and B are square matrices such that $AB = BA = I$, where I is an identity matrix.

Which one of the following statements is **not** true?

A. $ABA = A$

B. $AB^2A = I$

C. B must equal A

D. B is the inverse of A

E. both A and B have inverses

$AB = BA$

$\therefore B = A^{-1}$

Test A:

$ABA = A.A^{-1}A = I.A = A$ true

Test B:

$A.B^2.A$

$= AA^{-1}.A^{-1}.A$

$= I.I = I$ true

Test C: false (A is not necessarily equal to its own inverse)

(D and E are both true statements)

Question 7

The order of matrix X is 3×2.

The element in row i and column j of matrix X is x_{ij} and it is determined by the rule

$$x_{ij} = i + j$$

The matrix X is

A. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ **B.** $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix}$ **C.** $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2 \\ 3 & 3 \\ 4 & 4 \end{bmatrix}$ **E.** $\begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix}$

$x_{11} = 1+1 = 2$
$x_{12} = 1+2 = 3$
$x_{21} = 2+1 = 3$
$x_{22} = 2+2 = 4$
$x_{31} = 3+1 = 4$
$x_{32} = 3+2 = 5$

$X = \begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix}$

Question 8

A transition matrix, T, and a state matrix, S_2, are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$$

If $S_2 = TS_1$, the state matrix S_1 is

A. $\begin{bmatrix} 200 \\ 250 \\ 150 \end{bmatrix}$ **B.** $\begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$ **C.** $\begin{bmatrix} 300 \\ 0 \\ 300 \end{bmatrix}$

D. $\begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$ **E.** undefined

$S_2 = T S_1$

$\therefore \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix} = T \cdot S_1$

$\therefore T^{-1} \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix} = T^{-1} \cdot T \cdot S_1$

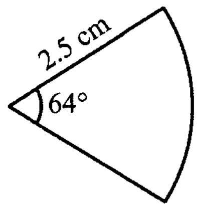
$\therefore T^{-1} \cdot \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix} = I \cdot S_1$

$\therefore S_1 = T^{-1} \cdot \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$ $\rightarrow S_1 = \begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$

Module 3 – Geometry and measurement

Before answering these questions, you must **shade** the 'Geometry and measurement' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1



A sector of a circle of radius 2.5 cm subtends an angle of 64° at the centre of the circle.

The area of the sector, in square centimetres, is closest to

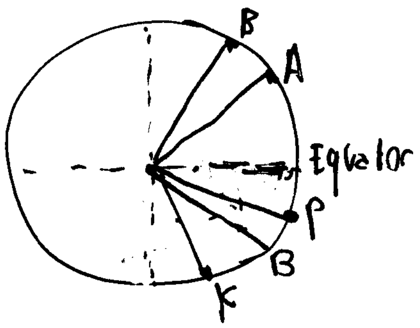
A. 2.8
B. 3.5
C. 7.0
D. 88.4
E. 110.5

$$A = \frac{\pi r^2 \theta}{360} = \frac{\pi \times 2.5^2 \times 64}{360} \simeq 3.49\,cm^2$$

Question 2

The city that is closest to the equator is

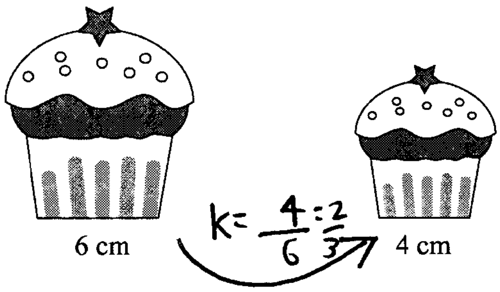
A. Athens, latitude 38.0° N
B. Belgrade, latitude 44.8° N
C. Kingston, latitude 45.3° S
D. Pretoria, latitude 25.7° S
E. Brisbane, latitude 27.5° S



Question 3

A cafe sells two sizes of cupcakes with a similar shape.

The large cupcake is 6 cm wide at the base and the small cupcake is 4 cm wide at the base.



$k = \frac{4}{6} = \frac{2}{3}$

The price of a cupcake is proportional to its volume.

If the large cupcake costs \$5.40, then the small cupcake will cost

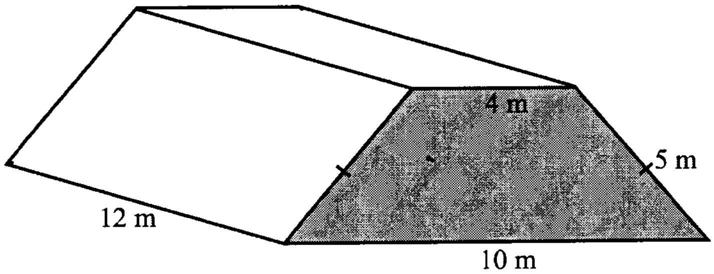
A. \$1.60
B. \$2.32
C. \$2.40
D. \$3.40
E. \$3.60

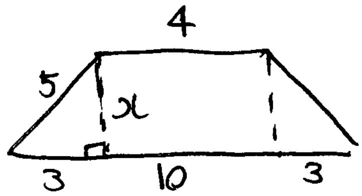
$k = \frac{2}{3}$

$\therefore k^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

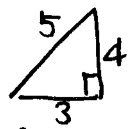
$\therefore \text{Price} = 5.40 \times \frac{8}{27} = \$1.60$

Question 4





$x = 4$



(3, 4, 5 RAT)

A greenhouse is built in the shape of a trapezoidal prism, as shown in the diagram above.

The cross-section of the greenhouse (shaded) is an isosceles trapezium. The parallel sides of this trapezium are 4 m and 10 m respectively. The two equal sides are each 5 m.

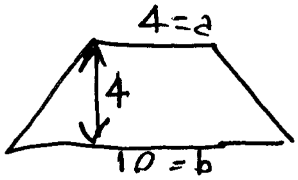
The length of the greenhouse is 12 m.

The five exterior surfaces of the greenhouse, **not** including the base, are made of glass.

The total area of the glass surfaces of the greenhouse, in square metres, is

A. 196
B. 212
C. 224
D. 344
E. 672

$A = A_{trapezium} \times 2 + A_{top} + A_{side} \times 2$

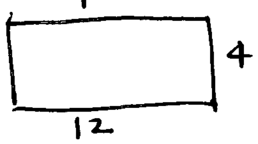


$A = \frac{h}{2}(a + b)$

$= \frac{4}{2} \times (4 + 10)$

$= 28\ m^2$

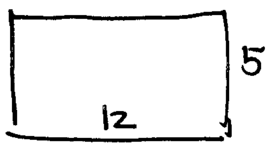
Top



$A = 4 \times 12$

$= 48\ m^2$

Side



$A = 5 \times 12$

$= 60\ m^2$

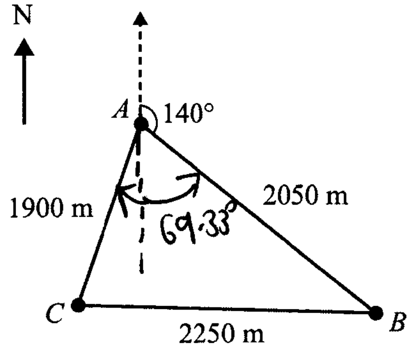
SECTION B –

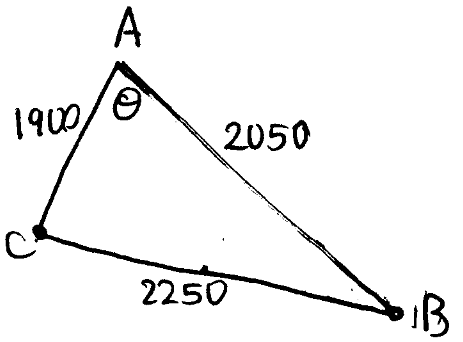
$\therefore A = 28 \times 2 + 48 + 60 \times 2 = 224\ m^2$

Use the following information to answer Questions 5 and 6.

Question 5

A cross-country race is run on a triangular course. The points *A*, *B* and *C* mark the corners of the course, as shown below.





The distance from *A* to *B* is 2050 m.
The distance from *B* to *C* is 2250 m.
The distance from *A* to *C* is 1900 m.
The bearing of *B* from *A* is 140°.
The bearing of *C* from *A* is closest to

$$\cos(\theta) = \frac{1900^2 + 2050^2 - 2250^2}{2 \times 1900 \times 2050}$$

$$\therefore \theta = \cos^{-1}\left(\frac{1900^2 + 2050^2 - 2250^2}{2 \times 1900 \times 2050}\right)$$

$$\theta = 69.33^\circ$$

A. 032°
B. 069°
C. 192°
D. 198°
E. 209°

$\therefore$ Bearing of C from A $= 140^\circ + 69.33^\circ$
$\simeq 209.3^\circ$

Question 6

The area within the triangular course *ABC*, in square metres, can be calculated by evaluating

A. $\sqrt{3100 \times 1200 \times 1050 \times 850}$

B. $\sqrt{3100 \times 2250 \times 2050 \times 1900}$

C. $\sqrt{6200 \times 4300 \times 4150 \times 3950}$

D. $\frac{1}{2} \times 2050 \times 2250 \times \sin(140^\circ)$

E. $\frac{1}{2} \times 2050 \times 2250 \times \sin(40^\circ)$

$a = 1900,\ b = 2050,\ c = 2250$

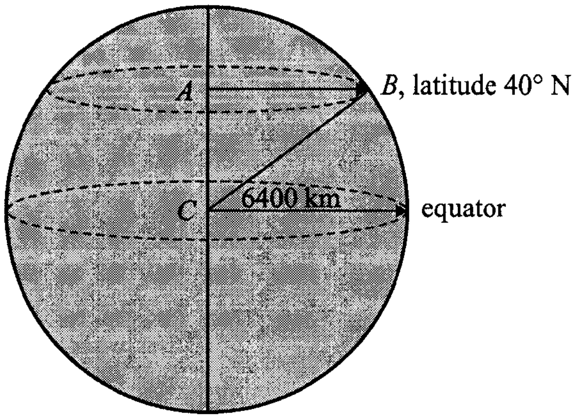
$$s = \frac{1900 + 2050 + 2250}{2} = 3100$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{3100(3100-1900)(3100-2050)(3100-2250)}$$

$$= \sqrt{3100 \times 1200 \times 1050 \times 850}$$

Question 7

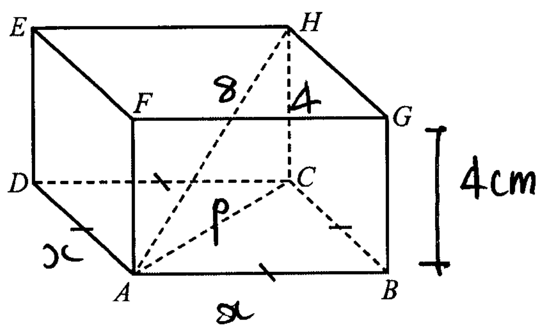


Assume that the radius of Earth is 6400 km.
The diagram above shows a small circle of Earth, with centre at *A*, whose latitude is 40° N.
The radius of this small circle, in kilometres, is closest to

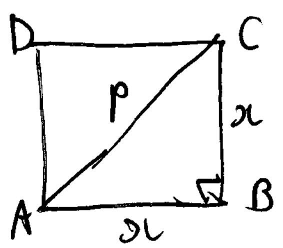
A. 4114
B. 4903
C. 5543
D. 6400
E. 7390

$r = 6400\cos(40°)$
≈ 4903 km

Question 8

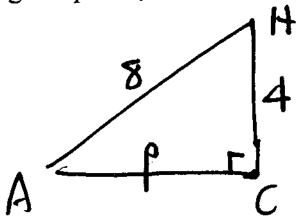


$V = x \times x \times 4$
$= 4x^2$



A right rectangular prism with a square base, *ABCD*, is shown above.
The diagonal of the prism, *AH*, is 8 cm.
The height of the prism, *HC*, is 4 cm.
The volume of this rectangular prism, in cubic centimetres, is

A. 64
B. 96
C. 128
D. 192
E. 256



$p^2 = 8^2 - 4^2$
$\therefore p^2 = 64 - 16$
$p^2 = 48$

$p^2 = x^2 + x^2$
$\therefore 48 = 2x^2$
$x^2 = 24$

$\therefore \text{Volume} = 4x^2$
$= 4 \times 24$
$= 96\text{ cm}^3$