SOLUTIONS FOR EXAM 2

SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided. Write using blue or black pen.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

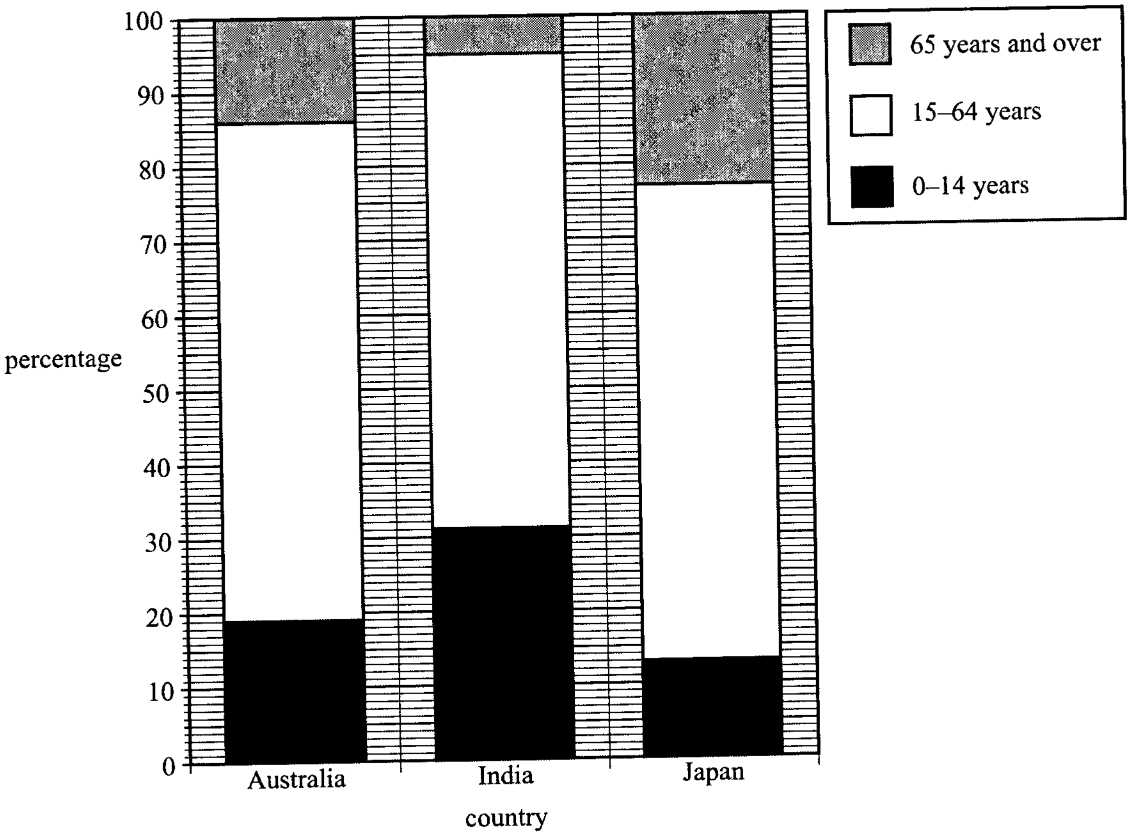
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (3 marks)

The segmented bar chart below shows the age distribution of people in three countries, Australia, India and Japan, for the year 2010.



Source: Australian Bureau of Statistics, 3201.0 – *Population by Age and Sex, Australian States and Territories*, June 2010

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a. Write down the percentage of people in Australia who were aged 0–14 years in 2010. 1 mark

19%

b. In 2010, the population of Japan was 128 000 000.

How many people in Japan were aged 65 years and over in 2010? 1 mark

23% of 128,000,000 = 29,440,000

c. From the graph on page 2, it appears that there is no association between the percentage of people in the 15–64 age group and the country in which they live.

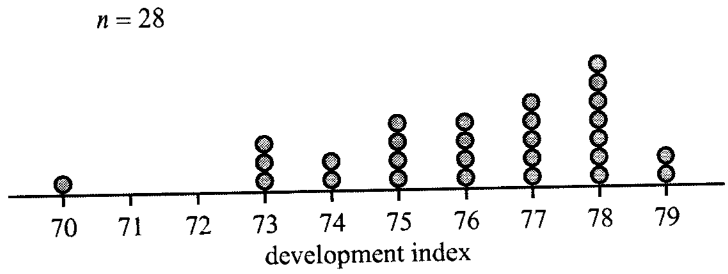
Explain why, quoting appropriate percentages to support your explanation. 1 mark

It seems that the percentage of people in the 15–64 age grap is not associabed with the country they live in because this percentage is very similar across all three contries:
in Australia: 67%, in India 64% and in Japan 64%

Question 2 (3 marks)

The development index for a country is a whole number between 0 and 100.

The dot plot below displays the values of the development indices for 28 countries.



a. Using the information in the dot plot, determine each of the following. 1 mark

The mode $\boxed{78}$ The range $\boxed{9}$

b. Write down an appropriate calculation and use it to explain why the country with a development index of 70 is an outlier for this group of countries. 2 marks

Median is halfway between 14th and 15th = 76.5

Q_1 is halfway between 7th and 8th

$\therefore Q_1 = 75$

$Q_3 = 78$

$\therefore IQR = 78 - 75 = 3$

Lower fence $= Q_1 - 1.5 \times IQR$

$= 75 - 1.5 \times (78 - 75)$

$= 75 - 1.5 \times 3$

$= 70.5$

Since $70 < 70.5$, it is an outlier.

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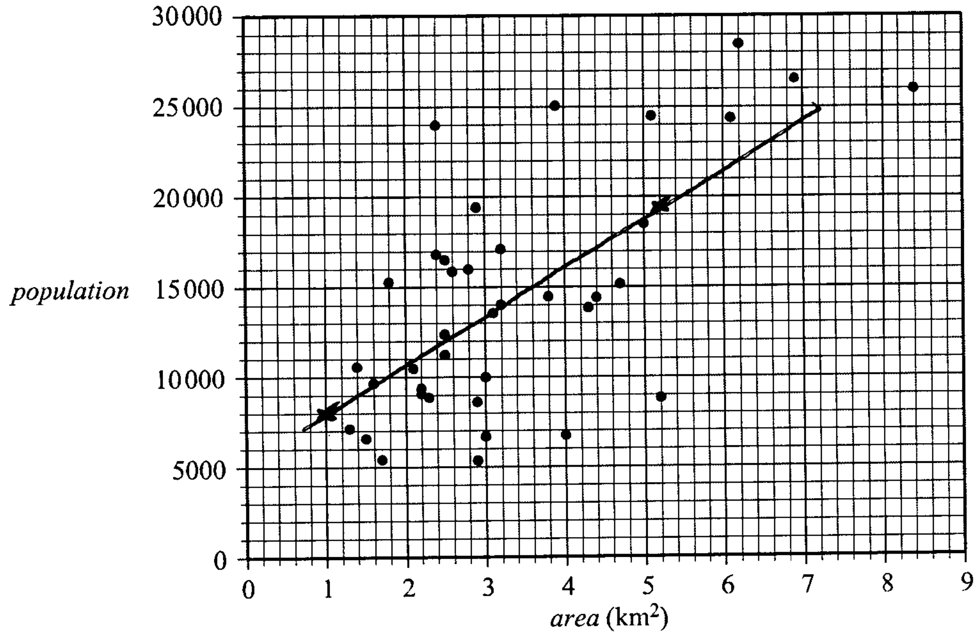
Question 3 (6 marks)

The scatterplot below shows the *population* and *area* (in square kilometres) of a sample of inner suburbs of a large city.



The equation of the least squares regression line for the data in the scatterplot is

Find the co-ordinates of two points on the line

$$population = 5330 + 2680 \times area$$

Let area = 1
pop = 5330 + 2680×1 = 8010
∴ (5330, 8010)

a. Write down the response variable. 1 mark

Population

Let area = 5
pop = 5330 + 2680×5
= 18730 (5, 18730)

b. Draw the least squares regression line on the **scatterplot above**. 1 mark

(*Answer on the scatterplot above.*)

c. Interpret the slope of this least squares regression line in terms of the variables *area* and *population*. 2 marks

On average, population increases by 2680 per 1 km² increase in area

Template answer!

d. Wiston is an inner suburb. It has an area of 4 km^2 and a population of 6690.

The correlation coefficient, r, is equal to 0.668

i. Calculate the residual when the least squares regression line is used to predict the population of Wiston from its area. 1 mark

Prediction: Pop = 5330 + 2680 × 4 = 16050

Residual = 6690 − 16050 = −9360

ii. What percentage of the variation in the population of the suburbs is explained by the variation in area?

Round your answer to one decimal place. 1 mark

$r^2 = (0.668)^2 = 0.4462$

∴ 44.6% of the variation in population can be explained by the variation in area.

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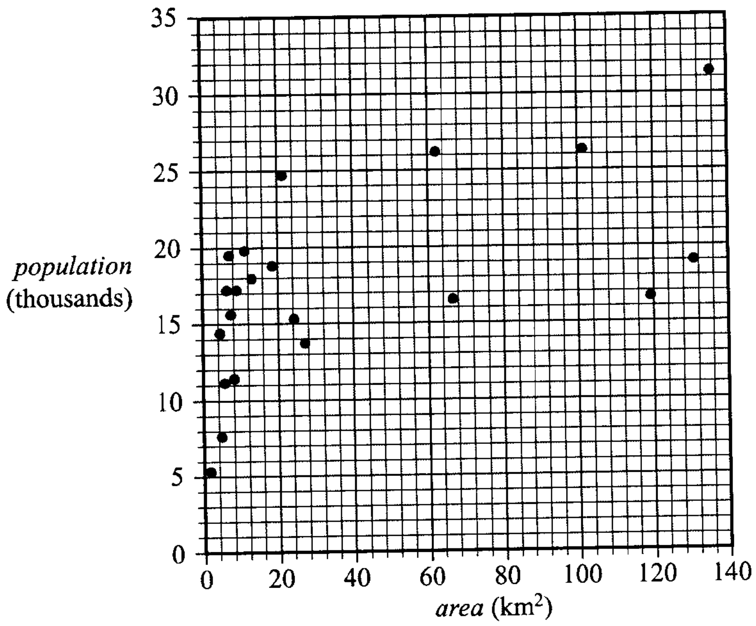
Question 4 (3 marks)

The scatterplot and table below show the *population*, in thousands, and the *area*, in square kilometres, for a sample of 21 outer suburbs of the same city.

CAS
log(area)



Area (km^2)	*Population* (thousands)
1.6	5.2
4.4	14.3
4.6	7.5
5.6	11.0
6.3	17.1
7.0	19.4
7.3	15.5
8.0	11.3
8.8	17.1
11.1	19.7
13.0	17.9
18.5	18.7
21.3	24.6
24.2	15.2
27.0	13.6
62.1	26.1
66.5	16.4
101.4	26.2
119.2	16.5
130.7	18.9
135.4	31.3

In the outer suburbs, the relationship between *population* and *area* is non-linear.

A **log** transformation can be applied to the variable *area* to linearise the scatterplot.

a. Apply the **log** transformation to the data and determine the equation of the least squares regression line that allows the population of an outer suburb to be predicted from the logarithm of its area.

Write the slope and intercept of this least squares regression line in the boxes provided below.

Round your answers to two significant figures. 2 marks

population = $\boxed{7.7}$ + $\boxed{7.7}$ log (*area*)

b. Use the equation of the least squares regression line in **part a.** to predict the population of an outer suburb with an area of 90 km^2.

Round your answer to the nearest one thousand people. 1 mark

pop = 7.7 + 7.7 log(90) = 22.75

∴ Predicted population

= 23,000

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Question 5 (4 marks)

There is a negative association between the variables *population density*, in people per square kilometre, and *area*, in square kilometres, of 38 inner suburbs of the same city.

For this association, $r^2 = 0.141$

a. Write down the value of the correlation coefficient for this association between the variables *population density* and *area*.

Round your answer to three decimal places. 1 mark

$r = -\sqrt{0.141} \doteq -0.37549\underline{9}667\ldots$

$\simeq -0.375$

Be careful of rounding !

b. The mean and standard deviation of the variables *population density* and *area* for these 38 inner suburbs are shown in the table below.

	Population density **(people per km^2)**	***Area*** **(km^2)**
Mean	4370	3.4
Standard deviation	1560	1.6

One of these suburbs has a population density of 3082 people per square kilometre.

i. Determine the standard *z*-score of this suburb's population density.

Round your answer to one decimal place. 1 mark

$z = \dfrac{x - \bar{x}}{s} = \dfrac{3082 - 4370}{1560} \simeq -0.8$

ii. Interpret the *z*-score of this suburb's population density with reference to the mean population density. 1 mark

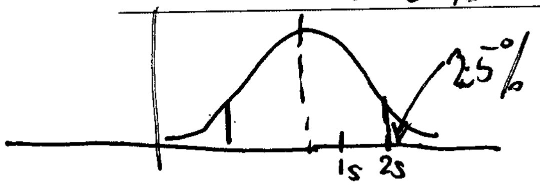
This suburb's population density is 0.8 standard deviations below the mean.

iii. Assume the areas of these inner suburbs are approximately normally distributed.

How many of these 38 suburbs are **expected** to have an area that is two standard deviations or more above the mean?

Round your answer to the nearest whole number. 1 mark

2.5% of $38 = \dfrac{2.5}{100} \times 38 \simeq 1$ suburb



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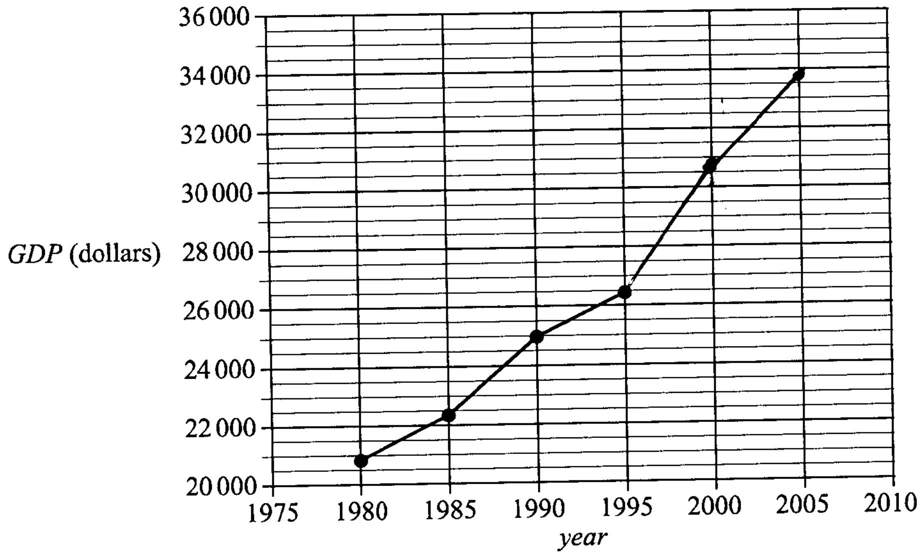
Question 6 (5 marks)

Table 1 shows the Australian gross domestic product (*GDP*) per person, in dollars, at five yearly intervals (*year*) for the period 1980 to 2005.

Table 1

Year	1980	1985	1990	1995	2000	2005
GDP	20900	22300	25000	26400	30900	33800



a. Complete the **time series plot above** by plotting the *GDP* for the years 2000 and 2005. 1 mark

(*Answer on the time series plot above.*)

b. Briefly describe the general trend in the data. 1 mark

An increasing trend

c. In Table 2, the variable year has been rescaled using 1980 = 0, 1985 = 5, and so on. The new variable is *time*.

Table 2

Year	1980	1985	1990	1995	2000	2005
Time	0	5	10	15	20	25
GDP	20 900	22 300	25 000	26 400	30 900	33 800

i. Use the variables *time* and *GDP* to write down the equation of the least squares regression line that can be used to predict *GDP* from *time*. Take *time* as the explanatory variable. 2 marks

GDP = 20000 + 524 × time

ii. The least squares regression line in **part c.i.** above has been used to predict the *GDP* in 2010. 1 mark

Explain why this prediction is unreliable.

It is unreliable because it is extrapolation (prediction outside the dataset).

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Recursion and financial modelling

Question 7 (4 marks)

Hugo is a professional bike rider.

The value of his bike will be depreciated over time using the flat rate method of depreciation.

The value of Hugo's bike, in dollars, after n years, V_n, can be modelled using the recurrence relation below.

$$V_0 = 8400, \qquad V_{n+1} = V_n - 1200$$

a. Using the recurrence relation, write down calculations to show that the value of Hugo's bike after two years is \$6000. 1 mark

$V_1 = V_0 - 1200 = 8400 - 1200 = 7200$

$V_2 = V_1 - 1200 = 7200 - 1200 = 6000$

$\therefore$ Value is \$6000

Hugo will sell his bike when its value reduces to \$3600.

b. After how many years will Hugo sell his bike? 1 mark

$V = 8400 - 1200n$

$3600 = 8400 - 1200n$

Solving: $n = 4$

$\therefore$ After 4 years

The unit cost method can also be used to depreciate the value of Hugo's bike.

A rule for the value of the bike, in dollars, after travelling n kilometres is

$$V_n = 8400 - 0.25n$$

c. What is the depreciation of the bike per kilometre? 1 mark

\$0.25

After two years, the value of the bike when depreciated by the unit cost method will be the same as the value of the bike when depreciated by the flat rate method.

d. How many kilometres has the bike travelled after two years? 1 mark

$8400 - 0.25n = 6000$

$\therefore 2400 = 0.25n$

$n = 9600$

$\therefore$ After 9,600 km

Question 8 (5 marks)

Hugo won \$5000 in a road race. He deposited this money into a savings account.

The value of Hugo's savings after n months, S_n, can be modelled by the recurrence relation below.

$$S_0 = 5000, \qquad S_{n+1} = 1.004\, S_n$$

a. What is the annual interest rate (compounding monthly) for Hugo's savings account? 1 mark

interest

$\frac{\frac{r}{12}}{100} = 0.004$ → Annual interest rate $= 4.8\%$

$\therefore \frac{r}{12} = 0.4 \quad \therefore r = 4.8$

b. What would be the value of Hugo's savings after 12 months? 1 mark

$S_n = 5000(1.004)^n$

$\therefore S_{12} = 5000 \times 1.004^{12}$

$= \$5245.35$

Using a different investment strategy, Hugo could deposit \$3000 into an account earning compound interest at the rate of 4.2% per annum, compounding monthly, and make additional payments of \$200 after every month.

Let T_n be the value of Hugo's investment after n months using this strategy.

The monthly interest rate for this account is 0.35%.

c. i. Write down a recurrence relation, in terms of T_{n+1} and T_n, that models the value of Hugo's investment using this strategy. 1 mark

$T_0 = 3000$ $\quad \therefore T_0 = 3000$

$T_{n+1} = \left(1 + \frac{0.35}{100}\right) T_n + 200 \quad T_{n+1} = 1.0035\, T_n + 200$

ii. What is the total interest Hugo would have earned after six months? 2 marks

$N = 6$

$I = 4.2$

$PV = -3000$ gives $FV = 4274.10$

$PMT = -200$ $\quad \therefore$ Interest

$FV = ?$

$P/Y = 12$ $\quad = 4274.11 - 3000 - 6 \times 200$

$C/Y = 12$ $\quad = \$74.11$

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Question 9 (3 marks)

Hugo needs to buy a new bike.

He borrowed $7500 to pay for the bike and will be charged interest at the rate of 5.76% per annum, compounding monthly.

Hugo will fully repay this loan with repayments of $430 each month.

a. How many repayments are required to fully repay this loan?
Round your answer to the nearest whole number. 1 mark

N = ? FV=0 gives N = 18.26
I = 5.76 P/y = 12 ≃ 18
PV = 7500 C/y = 12 It will take 18 payments
PMT = −430

After the fifth repayment, Hugo increased his monthly repayment so that the loan was fully repaid with a further seven repayments (that is, 12 repayments in total).

b. **i.** What is the minimum value of Hugo's new monthly repayment? 1 mark

N=5 P/y=12
I=5.76 C/y=12
PV=7500 gives FV=−5510.997
PMT=−430
FV=?

N=7 gives
I=5.76 PMT=−802.47
PMT=?
PV=5510.997
FV=0
P/y=C/y=12

New payment = $802.47

ii. What is the value of the final repayment required to ensure the loan is fully repaid after 12 repayments? 1 mark

N=7
I=5.76
PV=5510.997
PMT=−802.47
FV=?
P/y=12
C/y=12

gives FV=−0.0252

∴ Final repayment
= 802.47 + 0.025
= $802.50

[Alternatively:
N = 6
I = 5.76
PV = 5511
PMT = −802.47
gives FV = −798.66
∴ Final payment
$= 798.66 + \frac{\frac{5.76}{12}}{100} \times 798.66$
= 802.494
= $802.49

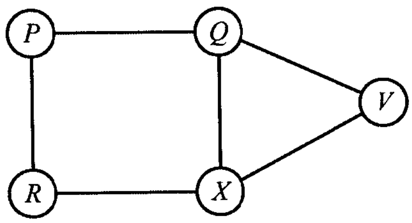
Either answer should be accepted — make sure you show your steps.

END OF SECTION A
TURN OVER

Module 1 – Matrices

Question 1 (2 marks)

Five trout-breeding ponds, P, Q, R, X and V, are connected by pipes, as shown in the diagram below.



The matrix W is used to represent the information in this diagram.

$$W = \begin{array}{cc} \begin{array}{ccccc} P & Q & R & X & V \end{array} & \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} P \\ Q \\ R \\ X \\ V \end{array} \end{array}$$

In matrix W:

- the 1 in row 2, column 1, for example, indicates that pond P is directly connected by a pipe to pond Q
- the 0 in row 5, column 1, for example, indicates that pond P is not directly connected by a pipe to pond V.

a. In terms of the breeding ponds described, what does the sum of the elements in row 3 of matrix W represent? 1 mark

There is a total of two pipe connections to R.

The matrix W^2 is shown below.

$$W^2 = \begin{array}{cc} \begin{array}{ccccc} P & Q & R & X & V \end{array} & \\ \begin{bmatrix} 2 & 0 & 0 & 2 & 1 \\ 0 & 3 & 2 & 1 & 1 \\ 0 & 2 & 2 & 0 & 1 \\ 2 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix} & \begin{array}{c} P \\ Q \\ R \\ X \\ V \end{array} \end{array}$$

b. Matrix W^2 has a 2 in row 2 (Q), column 3 (R).

Explain what this number tells us about the pipe connections between Q and R. 1 mark

There is a total of two 2-step connections between R and Q.

(They are: R–P–Q and R–X–Q)

Question 2 (10 marks)

10 000 trout eggs, 1000 baby trout and 800 adult trout are placed in a pond to establish a trout population.

In establishing this population:

- eggs (E) may die (D) or they may live and eventually become baby trout (B)
- baby trout (B) may die (D) or they may live and eventually become adult trout (A)
- adult trout (A) may die (D) or they may live for a period of time but will eventually die.

From year to year, this situation can be represented by the transition matrix T, where

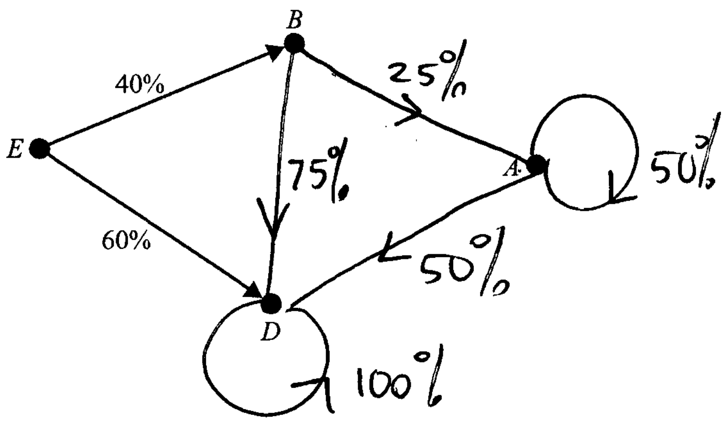
$$\begin{array}{c} \textit{this year} \\ \begin{array}{cc} T = \begin{array}{c} \begin{array}{cccc} E & B & A & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix} \end{array} & \begin{array}{c} \\ E \\ B \\ A \\ D \end{array} \end{array} \end{array} \quad \textit{next year} \quad \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} \begin{array}{c} E \\ B \\ A \\ D \end{array}$$

a. Use the information in the transition matrix T to

i. determine the number of eggs in this population that die in the first year — 1 mark

$0.6 \times 10000 = 6000$

ii. complete the transition diagram below, showing the relevant percentages. — 2 marks



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The initial state matrix for this trout population, S_0, can be written as

$$S_0 = \begin{array}{c} \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} \end{array} \begin{array}{l} E \\ B \\ A \\ D \end{array}$$

Let S_n represent the state matrix describing the trout population after n years.

b. Using the rule $S_{n+1} = T S_n$, determine

i. S_1 1 mark

$$S_1 = \begin{bmatrix} 0 \\ 4000 \\ 650 \\ 7150 \end{bmatrix}$$

ii. the number of adult trout predicted to be in the population after four years. Round your answer to the nearest whole number of trout. 1 mark

$$S_4 = T^4 S_0 = \begin{bmatrix} 0 \\ 0 \\ 331 \\ 11469 \end{bmatrix} \begin{array}{l} E \\ B \\ A \\ D \end{array}$$

∴ 331 adult trout

c. The transition matrix T predicts that, in the long term, all of the eggs, baby trout and adult trout will die.

i. How many years will it take for all of the adult trout to die (that is, when the number of adult trout in the population is first predicted to be less than one)? 1 mark

13 years (Trial and error: $T^{12}S_0 = \begin{bmatrix} 0 \\ 0 \\ 1.29 \\ 11798.7 \end{bmatrix}$ $T^{13}S_0 = \begin{bmatrix} 0 \\ 0 \\ 0.65 \\ 11799.4 \end{bmatrix}$

ii. What is the largest number of adult trout that is predicted to be in the pond in any one year? 1 mark

$TS_0 = \begin{bmatrix} 0 \\ 4000 \\ 650 \\ 7150 \end{bmatrix}$, $T^2S_0 = \begin{bmatrix} 0 \\ 0 \\ 1325 \\ 10075 \end{bmatrix}$, $T^3S_0 = \begin{bmatrix} 0 \\ 0 \\ 662.5 \\ 11137.5 \end{bmatrix}$, $T^4S_0 = \begin{bmatrix} 8 \\ 331.25 \\ 11468.8 \end{bmatrix}$

∴ Largest number of adults = 1325

d. Determine the number of eggs, baby trout and adult trout that, if added to or removed from the pond at the end of each year, will ensure that the number of eggs, baby trout and adult trout in the population remains constant from year to year. 2 marks

$$S_1 = \begin{bmatrix} 0 \\ 4000 \\ 650 \\ 7150 \end{bmatrix} \quad S_0 = \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix}$$

$$S_1 - S_0 = \begin{bmatrix} -10000 \\ 3000 \\ -150 \\ 7150 \end{bmatrix} \begin{array}{l} E \\ B \\ A \\ D \end{array}$$

We want $S_1 - S_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \end{bmatrix}$

∴ Add 10000 eggs, remove 3000 babies, add 150 adults

The rule $S_{n+1} = T S_n$ that was used to describe the development of the trout in this pond does not take into account new eggs added to the population when the adult trout begin to breed.

To take breeding into account, assume that every year 50% of the adult trout each lay 500 eggs.

The matrix describing the population after n years, S_n, is now given by the new rule

$$S_{n+1} = T S_n + 500\, M S_n$$

where

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.40 & 0 & 0 & 0 \\ 0 & 0.25 & 0.50 & 0 \\ 0.60 & 0.75 & 0.50 & 1.0 \end{bmatrix}, \quad M = \begin{bmatrix} 0 & 0 & 0.50 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } S_0 = \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix}$$

e. Use this new rule to determine S_2. 1 mark

$$S_1 = T S_0 + 500 M S_0 = \begin{bmatrix} 200000 \\ 4000 \\ 650 \\ 7150 \end{bmatrix}$$

$$S_2 = T S_1 + 500 M S_1 = \begin{bmatrix} 162500 \\ 80000 \\ 1325 \\ 130475 \end{bmatrix}$$

$$S_2 = T \begin{bmatrix} 200000 \\ 4000 \\ 650 \\ 7150 \end{bmatrix} + 500 M \begin{bmatrix} 200000 \\ 4000 \\ 650 \\ 7150 \end{bmatrix} =$$

$$\therefore S_2 = \begin{bmatrix} 162500 \\ 80000 \\ 1325 \\ 130475 \end{bmatrix}$$

Must take it step by step!

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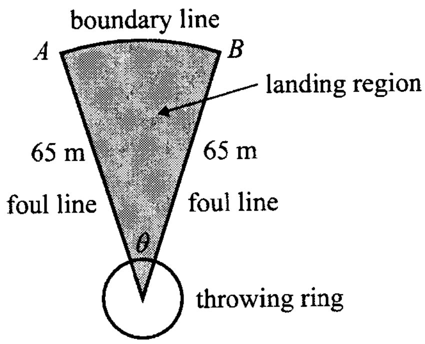
Module 3 – Geometry and measurement

Question 1 (3 marks)

One of the field events at athletics competitions is the discus.

The field markings for the discus event consist of a circular throwing ring, foul lines and the boundary line of the field, as shown in the diagram below. The shaded area on the diagram is the landing region for a discus throw.



The foul lines meet the boundary line at points A and B, 65 m from the centre of the throwing ring. The angle θ is 34.92°.

a. What is the length of the boundary line from point A to point B?
Write your answer in metres, rounded to two decimal places. 1 mark

Arc $\widehat{AB}$: $S = \dfrac{\pi r \theta}{180} = \dfrac{\pi \times 65 \times 34.92}{180}$

$\simeq 39.62$ m

b. Calculate the area of the landing region.
Round your answer to the nearest square metre. 2 marks

$A = \dfrac{\pi r^2 \theta}{360} = \dfrac{\pi \times 65^2 \times 34.92}{360}$

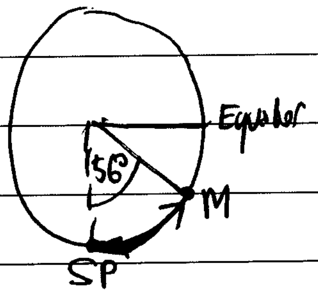
$= 1288 \text{ m}^2$

Question 2 (5 marks)

Daniel lives in Mildura (34° S, 142° E). He will fly to Sydney (34° S, 151° E) and then fly on to Rome (42° N, 12° E) to compete in the discus event at an international athletics competition.

In this question, assume that the radius of Earth is 6400 km.

a. Find the shortest great circle distance to the South Pole from Mildura (34° S, 142° E).
Round your answer to the nearest kilometre. 1 mark



$s = \frac{\pi R \theta}{180}$

$\therefore S = \frac{\pi \times 6400 \times 56}{180} \simeq 6255\text{ km}$

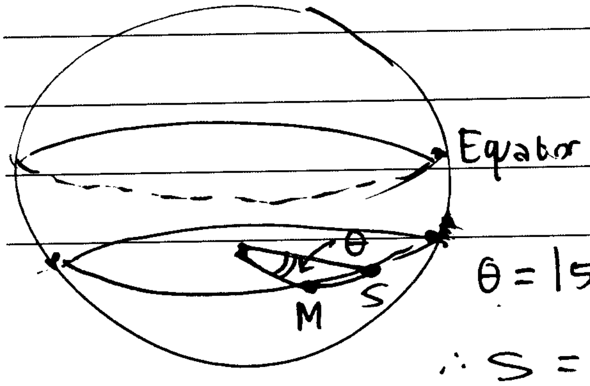
b. The flight from Mildura (34° S, 142° E) to Sydney (34° S, 151° E) travels along a small circle.

i. Find the radius of this small circle.
Round your answer to two decimal places. 1 mark

$r = 6400\cos(34°)$

$r = 5305.84\text{ km}$

ii. Find the distance the plane travels between Mildura (34° S, 142° E) and Sydney (34° S, 151° E).
Round your answer to the nearest kilometre. 1 mark



$\theta = 151° - 142° = 9°$

$\therefore s = \frac{\pi r \theta}{180}$

$r = 5305.84\text{ km}$

$\theta = 9°$

$s = \frac{\pi \times 5305.84 \times 9}{180}$

$s \approx 833\text{ km}$

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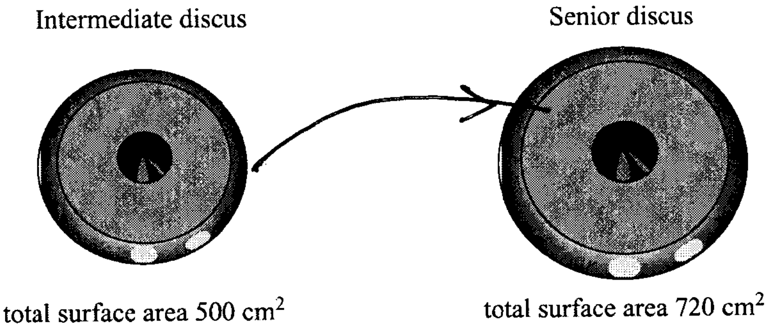
Question 3 (2 marks)

Daniel will compete in the intermediate division of the discus competition. Competitors in the intermediate division use a smaller discus than the one used in the senior division, but of a similar shape. The total surface area of each discus is given below.



By what value can the volume of the intermediate discus be multiplied to give the volume of the senior discus?

$k^2 = \frac{720}{500} = 1.44$

$\therefore k = \sqrt{1.44} = 1.2$

$\therefore k^3 = 1.728$

$\therefore$ Required volume factor $= 1.728$

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Question 4 (2 marks)

Daniel has qualified for the finals of the discus competition.

On his first throw, Daniel threw the discus to point A, a distance of 53.32 m on a bearing of 057°.

On his second throw from the same point, Daniel threw the discus a distance of 57.51 m.

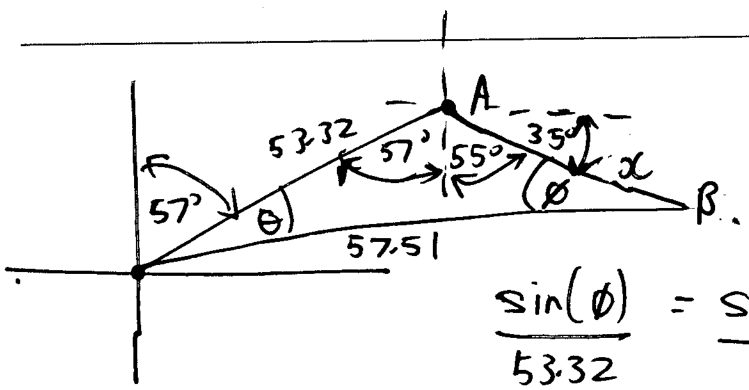
The second throw landed at point B, on a bearing of 125°, measured from point A.

Determine the distance, in metres, between points A and B.

Round your answer to one decimal place.

∴ Distance between A and B

$\simeq$ 9.4 m



$$\frac{\sin(\phi)}{53.32} = \frac{\sin(112°)}{57.51}$$

$$\phi = \sin^{-1}\left(\frac{53.32\sin(112°)}{57.51}\right)$$

$$\phi \simeq 59.28°$$

$$\therefore \theta = 180° - 112° - 59.28° = 8.72°$$

$$\frac{x}{\sin(8.72°)} = \frac{57.51}{\sin(112°)}$$

$$x = \frac{57.51\sin(8.72°)}{\sin(112°)}$$

$$x \simeq 9.40$$