

DISTANCE CALCULATIONS SOLUTIONS.

Remember, the first step is to work out whether it involves a Great Circle or a Small Circle.

Same longitude \longrightarrow Great circle $\longrightarrow R = 6400$

Equator \longrightarrow Great circle

Same latitude \longrightarrow Small circle $\longrightarrow r = 6400\cos(\theta)$

Same Hemisphere: Subtract angles

Opposite Hemispheres: Add angles

Question 2 (5 marks)

Daniel lives in Mildura (34° S, 142° E). He will fly to Sydney (34° S, 151° E) and then fly on to Rome (42° N, 12° E) to compete in the discus event at an international athletics competition.

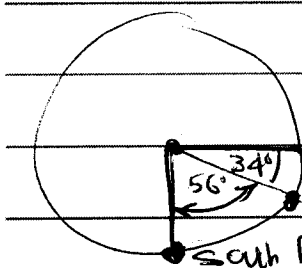
In this question, assume that the radius of Earth is 6400 km.

- a. Find the shortest great circle distance to the South Pole from Mildura (34° S, 142° E).

Round your answer to the nearest kilometre.

1 mark

$$90^\circ - 34^\circ = 56^\circ$$



$$s = \frac{\pi R \theta}{180}$$

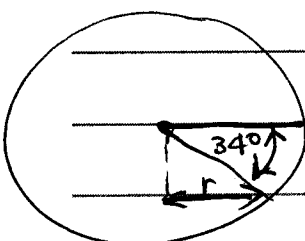
$$s = \frac{\pi \times 6400 \times 56}{180} \approx 6255 \text{ km}$$

- b. The flight from Mildura (34° S, 142° E) to Sydney (34° S, 151° E) travels along a small circle.

- i. Find the radius of this small circle.

Round your answer to two decimal places.

1 mark



$$r = 6400 \cos(34^\circ)$$

$$r = 5305.84 \text{ km}$$

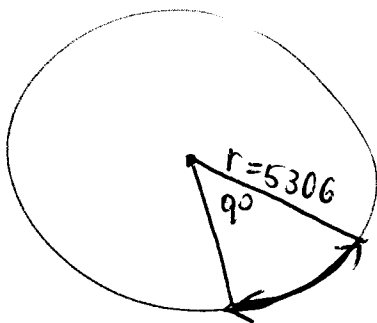
$$r \approx 5306 \text{ km}$$

- ii. Find the distance the plane travels between Mildura (34° S, 142° E) and Sydney (34° S, 151° E).

Round your answer to the nearest kilometre.

1 mark

$$151^\circ - 142^\circ = 9^\circ \quad \text{Subtract because they are the same hemisphere}$$



$$s = \frac{\pi r \theta}{180}$$

$$s = \frac{\pi \times 5305.84 \times 9}{180}$$

$$s \approx 833.44 \text{ km}$$

$$\approx 833 \text{ km}$$

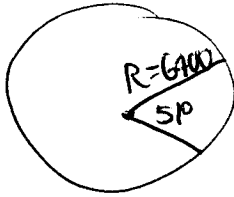
Question 1

Calculate the distance between Washington DC USA (39°N, 77°W) and Lima, Peru (12°S, 77°W).
Give your answer to the nearest km.

same longitude

$$R = 6400$$

$$39^\circ + 12^\circ = 51^\circ$$



$$S = \frac{\pi R \theta}{180}$$

$$S = \frac{\pi \times 6400 \times 51}{180}$$

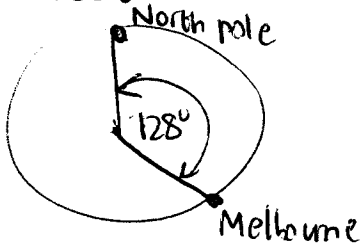
$$= 5697 \text{ km}$$

$$S = 5696.8 \text{ km}$$

Question 2

Calculate the shortest great circle distance between Melbourne (38°S, 145°E) and the North pole.

Great circle along meridian



$$38^\circ + 90^\circ = 128^\circ$$

$$S = \frac{\pi R \theta}{180}$$

$$S = \frac{\pi \times 6400 \times 128}{180}$$

$$= 14,298 \text{ km}$$

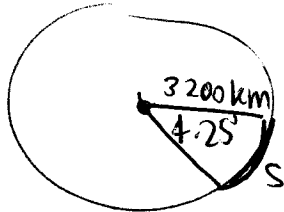
Question 2

A cruise boat travels from King George Island (60°S, 60°W) due west along a small circle to visit a penguin colony on the South Shetland Island (60°S, 64.25°W). Calculate how far the ship travelled, correct to the nearest km.

Same latitude → small circle

$$r = 6400 \cos(60^\circ) = 3,200 \text{ km}$$

$$64.25^\circ - 60^\circ = 4.25^\circ$$



$$S = \frac{\pi r \theta}{180}$$

$$S = \frac{\pi \times 3200 \times 4.25}{180} = 237.4 \text{ km}$$

Question 3

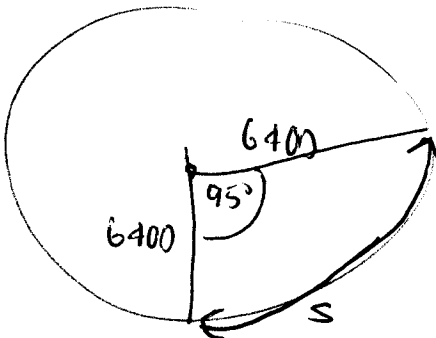
Calculate the shortest distance, to the nearest km between Macapa, Brazil (0°, 52°W) and Kismayo, Somalia (0°, 43°E)

$$= 237 \text{ km}$$

Equator → Great circle

$$43^\circ + 52^\circ = 95^\circ$$

Add because they are in different hemispheres



$$S = \frac{\pi R \theta}{180}$$

$$S = \frac{\pi \times 6400 \times 95}{180}$$

$$S = 10611.60 \text{ km}$$

$$= 10,612 \text{ km}$$

Question 4

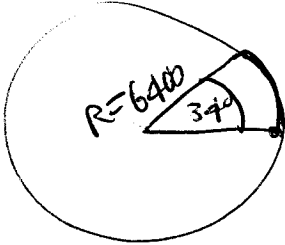
same longitude → great circle

A ship sails due South from Channel-Port-aux-Basques, Canada, $47^{\circ}\text{N } 59^{\circ}\text{W}$ to Barbados, $13^{\circ}\text{N } 59^{\circ}\text{W}$.

2

How far did the ship sail, to the nearest kilometre? Assume that the radius of Earth is 6400 km.

$$47^{\circ} - 13^{\circ} = 34^{\circ}$$



$$S = \frac{\pi R \theta}{180}$$

$$S = \frac{\pi \times 6400 \times 34}{180} \approx 3797.84 \text{ km}$$

$$\approx \boxed{3798 \text{ km}}$$

Question 5

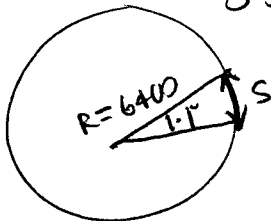
A light aircraft is scheduled to fly directly from Wagga Wagga to Sydney. Due to bad weather, the aircraft must fly another route.

- a. The first leg of the journey has the aircraft flying from Wagga Wagga ($35^{\circ}\text{S}, 147.4^{\circ}\text{E}$) to point A ($33.9^{\circ}\text{S}, 147.4^{\circ}\text{E}$).

Same longitude → great circle

Calculate the great circle distance from Wagga Wagga to point A, correct to the nearest km.

$$35^{\circ} - 33.9^{\circ} = 1.1^{\circ}$$



$$S = \frac{\pi R \theta}{180}$$

$$S = \frac{\pi \times 6400 \times 1.1}{180} \approx 122.9 \text{ km} \approx \boxed{123 \text{ km}}$$

- b. The second leg of the journey has the aircraft flying due east along a small circle from point A ($33.9^{\circ}\text{S}, 147.4^{\circ}\text{E}$) to Sydney ($33.9^{\circ}\text{S}, 151.2^{\circ}\text{E}$). Calculate the distance from A to Sydney, to the nearest km.

same latitude → small circle

$$r = 6400 \cos(33.9^{\circ}) = 5312.08 \text{ km}$$

$$151.2^{\circ} - 147.4^{\circ} = 3.8^{\circ}$$

$$S = \frac{\pi r \theta}{180}$$

$$S = \frac{\pi \times 5312.08 \times 3.8}{180}$$

$$S = 352.3 \text{ km}$$

$$= \boxed{352 \text{ km}}$$

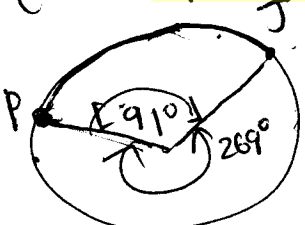
Question 6

(b) Pontianak has a longitude of 109°E , and Jarvis Island has a longitude of 160°W . Both places lie on the Equator.

- (i) Find the shortest distance between these two places, to the nearest kilometre. You may assume that the radius of the Earth is 6400 km. 2
- (ii) The position of Rabaul is 4° to the south and 48° to the west of Jarvis Island. What is the latitude and longitude of Rabaul? 2

(i) equator \rightarrow great circle $109^\circ + 160^\circ = 269^\circ$

Shortest distance would be: the minor arc where central angle = 91° .

$$S = \frac{\pi R \theta}{180} = \frac{\pi \times 6400 \times 91}{180} = 10,164.8 \text{ km}$$


(ii) Rabaul: $(4^\circ\text{S}, 152^\circ\text{E}) \approx \boxed{10165 \text{ km}}$

Question 7

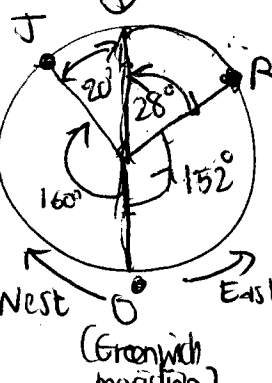
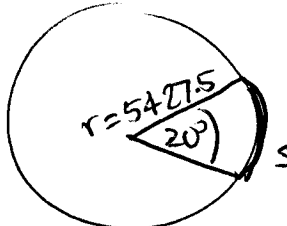
Neville departs San Diego ($32^\circ\text{N}, 117^\circ\text{W}$) flies due east to Dallas ($32^\circ\text{N}, 97^\circ\text{W}$).

Determine the distance he flies east, to the nearest km.

Same latitude \rightarrow small circle

$$r = 6400 \cos(32^\circ) \approx 5427.51 \text{ km}$$

$$117^\circ - 97^\circ = 20^\circ$$

$$S = \frac{\pi r \theta}{180} = \frac{\pi \times 5427.51 \times 20}{180} \approx \boxed{1895 \text{ km}}$$



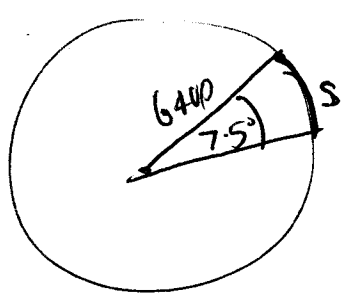
Question 8

A plane departed Brisbane ($27.5^\circ\text{S}, 153^\circ\text{E}$) and first headed due north from Brisbane until it reached the position ($20^\circ\text{S}, 153^\circ\text{E}$).

a. Calculate this distance, to the nearest km.

Same longitude \rightarrow great circle

$$27.5^\circ - 20^\circ = 7.5^\circ$$

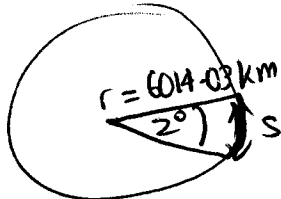
$$S = \frac{\pi R \theta}{180} = \frac{\pi \times 6400 \times 7.5}{180} \approx 837.76 \text{ km} \approx \boxed{838 \text{ km}}$$


- b. The plane then headed **due west** from $(20^{\circ}\text{S}, 153^{\circ}\text{E})$ to Hamilton Island, $(20^{\circ}\text{S}, 151^{\circ}\text{E})$. Calculate this distance to the nearest km.

Same latitude \rightarrow small circle

$$r = 6400 \cos(20^{\circ}) = 6014.03$$

$$153^{\circ} - 151^{\circ} = 2^{\circ}$$



Question 9

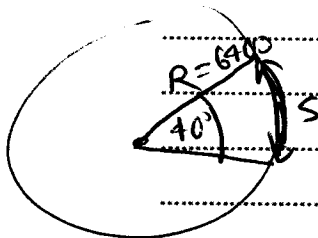
$$S = \frac{\pi r \theta}{180}$$

$$S = \frac{\pi \times 6014.03 \times 2}{180} \approx 209.9 \text{ km}$$

$$\approx \boxed{210 \text{ km}}$$

- (c) Two cities lie on the same meridian of longitude. One is 40° north of the other.

What is the distance between the two cities, correct to the nearest kilometre?



$$S = \frac{\pi R \theta}{180}$$

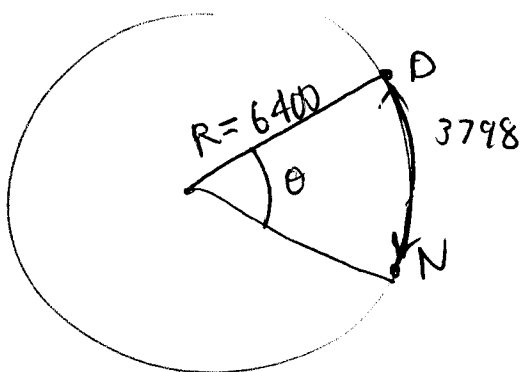
$$S = \frac{\pi \times 6400 \times 40}{180}$$

$$S = 4468.04$$

$$= \boxed{4468 \text{ km}}$$

Question 10

Damascus is located at $(33^{\circ}\text{N}, 36^{\circ}\text{E})$. The great circle distance along the meridian from Damascus to Nairobi is 3798 km. Nairobi is due south of Damascus. Determine the coordinates of Nairobi.



$$S = \frac{\pi \times R \theta}{180}$$

$$\therefore 3798 = \frac{\pi \times 6400 \times \theta}{180}$$

$$\therefore \theta = \frac{3798 \times 180}{6400 \pi}$$

$$\theta = 34^{\circ}$$

$$\therefore \text{Nairobi is } 1^{\circ}\text{S} \quad (33 - 34 = -1 = 1^{\circ}\text{S})$$

$$\therefore \text{Co-ordinates for Nairobi:}$$

$$\boxed{(1^{\circ}\text{S}, 36^{\circ}\text{E})}$$