REVISION QUESTIONS AUGUST 30 LESSON

ANS WERS

Question 5

Wendy buys one type of flower each day.

She chooses from tulips (T), roses (R), carnations (C), irises (I) and daisies (D).

The type of flower she buys on one day depends on the type of flower she bought the previous day, according to a transition matrix.

Today, Wendy bought tulips.

The transition matrix that, starting tomorrow, ensures Wendy buys flowers in alphabetical order (C, D, I, R, T) is

A.

$$\begin{array}{cc} \text{today} & \\ \begin{matrix} T & R & C & I & D \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} & \begin{matrix} T \\ R \\ C \\ I \\ D \end{matrix} \end{array} \text{tomorrow}$$

B.

$$\begin{array}{cc} \text{today} & \\ \begin{matrix} T & R & C & I & D \end{matrix} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} & \begin{matrix} T \\ R \\ C \\ I \\ D \end{matrix} \end{array} \text{tomorrow}$$

C.

$$\begin{array}{cc} \text{today} & \\ \begin{matrix} T & R & C & I & D \end{matrix} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} T \\ R \\ C \\ I \\ D \end{matrix} \end{array} \text{tomorrow}$$

D.

$$\begin{array}{cc} \text{today} & \\ \begin{matrix} T & R & C & I & D \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} & \begin{matrix} T \\ R \\ C \\ I \\ D \end{matrix} \end{array} \text{tomorrow}$$

E.

$$\begin{array}{cc} \text{today} & \\ \begin{matrix} T & R & C & I & D \end{matrix} & \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} & \begin{matrix} T \\ R \\ C \\ I \\ D \end{matrix} \end{array} \text{tomorrow}$$

Question 6

A carpenter can make four coffee tables and seven stools in a total of 33 hours.

The carpenter can make two coffee tables and three pencil boxes in a total of 12 hours.

The carpenter can make five stools and one pencil box in a total of 10 hours.

The time, in hours, that it takes to make one coffee table is closest to

A. 2
B. 3
C. 4
D. 5
E. 6

Let x = no. of hours for table
y = no. " " " stool
z = no. " " " pencil box

$4x + 7y = 33$
$2x + 3z = 12$
$5y + z = 10$

$$\begin{bmatrix} 4 & 7 & 0 \\ 2 & 0 & 3 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 33 \\ 12 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 & 7 & 0 \\ 2 & 0 & 3 \\ 0 & 5 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 33 \\ 12 \\ 10 \end{bmatrix} = \begin{bmatrix} 4.99 \\ 1.86 \\ 0.68 \end{bmatrix}$$

Question 7

A school has three computer classes, *A*, *B* and *C*. There are 15 students in each class.
Each student is given a mark out of 100 based on their performance in a test.
Matrix *M* below displays the marks obtained by these 45 students, listed by class.

$$M = \begin{bmatrix} 56 & 78 & 79 & 43 & 67 & 56 & 80 & 85 & 75 & 89 & 55 & 64 & 95 & 34 & 63 \\ 90 & 45 & 56 & 65 & 76 & 79 & 27 & 45 & 69 & 73 & 70 & 63 & 65 & 34 & 59 \\ 76 & 76 & 89 & 47 & 50 & 66 & 68 & 89 & 88 & 90 & 45 & 67 & 78 & 45 & 87 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \text{ class}$$

Two other matrices, *S* and *R*, are defined below.

$$S = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Which one of the following matrix expressions can be used to generate a matrix that displays the mean mark obtained for each class?

A. $\frac{1}{45}M$

B. $\frac{1}{3}R \times M$

C. $\frac{1}{3}R \times M \times S$

D. $\frac{1}{15}M \times S$

E. $\frac{1}{15}S \times R \times M$

Average of class A:

$\frac{1}{15}\left(56\times1 + 78\times1 + 79\times1 + 43\times1 \ldots + 63\times1\right)$

$\therefore$ Multiply row 1 of M by the column matrix S and the scalar $\frac{1}{15}$

$\therefore \frac{1}{15} M.S$ is the required product

(Again, a good strategy is to do one calculation to see what matrices you need to multiply in order to get it.)

Question 7

Each night, a large group of mountain goats sleep at one of two locations, A or B.
On the first night, equal numbers of goats are observed to be sleeping at each location.
From night to night, goats change their sleeping locations according to a transition matrix T.
It is expected that, in the long term, more goats will sleep at location A than at location B.

Assuming the total number of goats remains constant, a transition matrix T that would predict this outcome is

A.

$$T = \begin{array}{c c} & \textit{this night} \\ & \begin{array}{cc} A & B \end{array} \\ & \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix} \begin{array}{c} A \\ B \end{array} \end{array} \textit{next night}$$

B.

$$T = \begin{array}{c c} & \textit{this night} \\ & \begin{array}{cc} A & B \end{array} \\ & \begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} \begin{array}{c} A \\ B \end{array} \end{array} \textit{next night}$$

C.

$$T = \begin{array}{c c} & \textit{this night} \\ & \begin{array}{cc} A & B \end{array} \\ & \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \begin{array}{c} A \\ B \end{array} \end{array} \textit{next night}$$

D.

$$T = \begin{array}{c c} & \textit{this night} \\ & \begin{array}{cc} A & B \end{array} \\ & \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} \begin{array}{c} A \\ B \end{array} \end{array} \textit{next night}$$

E.

$$T = \begin{array}{c c} & \textit{this night} \\ & \begin{array}{cc} A & B \end{array} \\ & \begin{bmatrix} 0.1 & 0.8 \\ 0.9 & 0.2 \end{bmatrix} \begin{array}{c} A \\ B \end{array} \end{array} \textit{next night}$$

Question 9

$T = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$ is a transition matrix.

$S_3 = \begin{bmatrix} 1150 \\ 850 \end{bmatrix}$ is a state matrix.

If $S_3 = TS_2$, then S_2 equals

NOTE: When going backwards, the inverse matrix takes us back to the previous state.

A. $\begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$

B. $\begin{bmatrix} 1090 \\ 940 \end{bmatrix}$

C. $\begin{bmatrix} 1100 \\ 900 \end{bmatrix}$

D. $\begin{bmatrix} 1150 \\ 850 \end{bmatrix}$

E. $\begin{bmatrix} 1175 \\ 825 \end{bmatrix}$

$S_3 = T.S_2$

$\therefore T^{-1}S_3 = T^{-1}T.S_2$

$\therefore T^{-1}S_3 = I.S_2$

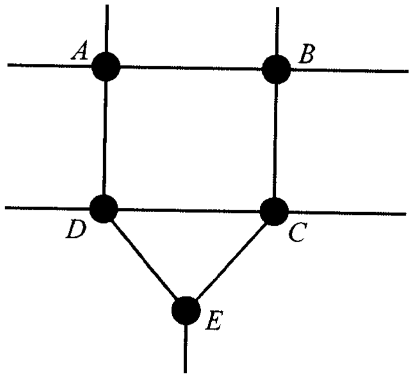
$\therefore S_2 = T^{-1}S_3$

$= \begin{bmatrix} 1100 \\ 900 \end{bmatrix}$

END OF MULTIPLE-CHOICE QUESTION BOOK

Question 5

A, B, C, D and E are five intersections joined by roads as shown in the diagram below.
Some of these roads are one-way only.



The matrix below indicates the direction that cars can travel along each of these roads.
In this matrix

- 1 in column A and row B indicates that cars can travel directly from A to B
- 0 in column B and row A indicates that cars cannot travel directly from B to A (either it is a one-way road or no road exists).

$$\begin{array}{c@{}c}
\text{from intersection} & \\
\begin{matrix} A & B & C & D & E \end{matrix} & \\
\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} & \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \text{ to intersection}
\end{array}$$

Cars can travel in both directions between intersections

A. A and D

B. B and C

C. C and D

D. D and E

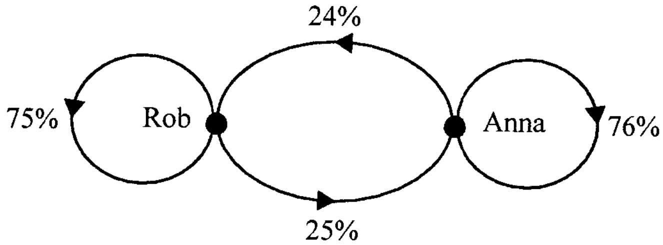
E. C and E

Use the following information to answer Questions 5 and 6.

Two politicians, Rob and Anna, are the only candidates for a forthcoming election. At the beginning of the election campaign, people were asked for whom they planned to vote. The numbers were as follows.

Candidate	Number of people who plan to vote for the candidate
Rob	5692
Anna	3450

During the election campaign, it is expected that people may change the candidate that they plan to vote for each week according to the following transition diagram.



Question 5

The total number of people who are expected to change the candidate that they plan to vote for one week after the election campaign begins is

A. 828
B. 1423
C. 2251
D. 4269
E. 6891

$0.25 \times 5692 + 0.24 \times 3450$

$= 2251$

Question 6

The election campaign will run for ten weeks.

If people continue to follow this pattern of changing the candidate they plan to vote for, the expected winner after ten weeks will be

A. Rob by about 50 votes.
B. Rob by about 100 votes.
C. Rob by fewer than 10 votes.
D. Anna by about 100 votes.
E. Anna by about 200 votes.

$S_{10} = T^{10} S_0$

$= T^{10} \begin{bmatrix} 5692 \\ 3450 \end{bmatrix}$

this week

$$T = \begin{array}{cc} & \begin{array}{cc} R & A \end{array} \\ & \begin{bmatrix} 0.75 & 0.24 \\ 0.25 & 0.76 \end{bmatrix} \begin{array}{c} R \\ A \end{array} \end{array} \text{next week}$$

$= \begin{bmatrix} 0.75 & 0.24 \\ 0.25 & 0.76 \end{bmatrix}^{10} \begin{bmatrix} 5692 \\ 3450 \end{bmatrix} = \begin{bmatrix} 4479 \\ 4663 \end{bmatrix}$

$4663 - \cancel{4}\,479$
$= 184$

Question 3

Four systems of simultaneous linear equations are shown below.

$$\begin{array}{cccc} 12x + 8y = 26 & 3x - 2y = 14 & -4x - 2y = 17 & x + 0.5y = 8 \\ 3x + 2y = 15 & -7x + 5y = 9 & -6x + 3y = 10 & 0.5x + y = 8 \end{array}$$

How many of these systems of simultaneous linear equations do **not** have a unique solution?

A. 0
B. 1
C. 2
D. 3
E. 4

$\begin{bmatrix} 12 & 8 \\ 3 & 2 \end{bmatrix}$ det $= 12\times2 - 8\times3 = 0$ ∴ No unique solution

$\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$ det $= 3\times5 - -2\times7 = 15 - 14 = 1$ ∴ unique solution

$\begin{bmatrix} -4 & -2 \\ -6 & 3 \end{bmatrix}$ det $= -4\times3 - -2\times-6 = -12 - 12 = -24$ ∴ Unique solution

$\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$ det $= 1 - \frac{1}{4} = \frac{3}{4}$ ∴ Unique solution

Question 4

The numbers of adult and child tickets purchased for five performances of a stage show are shown in the table below.

Performance	Adult	Child
1	142	24
2	128	31
3	89	24
4	104	18
5	115	23

Which one of the following matrix calculations can be used to determine both the total number of adult tickets and the total number of child tickets purchased for all five performances?

A. $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 142 & 128 & 89 & 104 & 115 \\ 24 & 31 & 24 & 18 & 23 \end{bmatrix}$ 5×1 2×5 undefined

We require either a 1×2 matrix:

$\begin{bmatrix} 142+128+89+104+115 & 24+31+24+18+23 \end{bmatrix}$

or a 2×1

$\begin{bmatrix} 142+128+89+104+115 \\ 24+31+24+18+23 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 142 & 128 & 89 & 104 & 115 \\ 24 & 31 & 24 & 18 & 23 \end{bmatrix}$ 5×2 2×5 gives a 5×5

C. $\begin{bmatrix} 142 & 128 & 89 & 104 & 115 \\ 24 & 31 & 24 & 18 & 23 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ 2×5 2×5 undefined

D. $\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 142 & 128 & 89 & 104 & 115 \\ 24 & 31 & 24 & 18 & 23 \end{bmatrix}$ 1×2 2×5

E. $\begin{bmatrix} 142 & 128 & 89 & 104 & 115 \\ 24 & 31 & 24 & 18 & 23 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 142+128+89+104+115 \\ 24+31+24+18+23 \end{bmatrix}$ 2×5 5×1

what we want

Question 5

A company makes Regular (R), Queen (Q) and King (K) size beds. Each bed comes in either the Classic style or the more expensive Deluxe style.

The price of each style of bed, in dollars, is listed in a price matrix P, where

$$\begin{array}{cc} & \begin{array}{ccc} R & Q & K \end{array} \\ P = & \begin{bmatrix} 145 & 210 & 350 \\ 185 & 270 & 410 \end{bmatrix} \begin{array}{l} \text{Classic} \\ \text{Deluxe} \end{array} \end{array}$$

The company wants to increase the price of all beds.

A new price matrix, listing the increased prices of the beds, can be generated from P by forming a **matrix product** with the matrix, M, where

$$M = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.35 \end{bmatrix}$$

This new price matrix is

A. $\begin{bmatrix} 145 & 210 & 350 \\ 185 & 270 & 410 \end{bmatrix}$

B. $\begin{bmatrix} 234.90 & 340.20 & 567 \\ 299.70 & 437.40 & 664.20 \end{bmatrix}$

C. $\begin{bmatrix} 174 & 252 & 420 \\ 222 & 324 & 492 \end{bmatrix}$

D. $\begin{bmatrix} 174 & 252 & 420 \\ 249.75 & 364.50 & 553.50 \end{bmatrix}$

E. $\begin{bmatrix} 195.75 & 283.50 & 472.50 \\ 249.75 & 364.50 & 553.50 \end{bmatrix}$

$$\begin{bmatrix} 1.2 & 0 \\ 0 & 1.35 \end{bmatrix}\begin{bmatrix} 145 & 210 & 350 \\ 185 & 270 & 410 \end{bmatrix}$$
$$= \begin{bmatrix} 1.2\times145 & 1.2\times210 & 1.2\times350 \\ 1.35\times185 & 1.35\times270 & 1.35\times410 \end{bmatrix}$$
$$= \begin{bmatrix} 174 & 252 & 420 \\ 249.75 & 364.5 & 553.5 \end{bmatrix}$$

Question 6

If $A = \begin{bmatrix} 1 & 3 \\ 6 & 4 \\ 0 & 0 \end{bmatrix}$ and the matrix product $XA = \begin{bmatrix} 4 & 1 \\ 1 & 4 \\ 3 & 5 \end{bmatrix}$, then the order of matrix X is

A. (2×2)
B. (2×3)
C. (3×1)
D. (3×2)
E. (3×3)

$3\times3 \quad 3\times2$
$X \quad A =$ $\begin{array}{c} 3\times2 \\ \begin{bmatrix} 4 & 1 \\ 1 & 4 \\ 3 & 5 \end{bmatrix} \end{array}$

Question 6

The order of matrix X is 3×2.

The element in row i and column j of matrix X is x_{ij} and it is determined by the rule

$$x_{ij} = i + j$$

The matrix X is

A. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2 \\ 3 & 3 \\ 4 & 4 \end{bmatrix}$

E. $\begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix}$

$x_{11} = 1+1 = 2$
$x_{12} = 1+2 = 3$
$x_{21} = 2+1 = 3$
$x_{22} = 2+2 = 4$
$x_{31} = 3+1 = 4$
$x_{32} = 3+2 = 5$

$\begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix}$

Question 7

A transition matrix, T, and a state matrix, S_2, are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$$

If $S_2 = TS_1$, the state matrix S_1 is

A. $\begin{bmatrix} 200 \\ 250 \\ 150 \end{bmatrix}$

B. $\begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$

C. $\begin{bmatrix} 300 \\ 0 \\ 300 \end{bmatrix}$

D. $\begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$

E. undefined

$S_2 = T.S_1$

$\therefore T^{-1}S_2 = T^{-1}T.S_1$

$T^{-1}S_2 = I.S_1$

$\therefore S_1 = T^{-1}S_2$

$S_1 = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix} = \begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$

Question 8

The matrix S_{n+1} is determined from the matrix S_n using the rule $S_{n+1} = TS_n - C$ where T, S_0 and C are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix}, \quad S_0 = \begin{bmatrix} 100 \\ 250 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

Given this information, the matrix S_2 equals

A. $\begin{bmatrix} 100 \\ 250 \end{bmatrix}$

B. $\begin{bmatrix} 148 \\ 122 \end{bmatrix}$

C. $\begin{bmatrix} 170 \\ 140 \end{bmatrix}$

D. $\begin{bmatrix} 180 \\ 130 \end{bmatrix}$

E. $\begin{bmatrix} 190 \\ 160 \end{bmatrix}$

$S_1 = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix}\begin{bmatrix} 100 \\ 250 \end{bmatrix} - \begin{bmatrix} 20 \\ 20 \end{bmatrix}$

$\therefore S_1 = \begin{bmatrix} 180 \\ 130 \end{bmatrix}$

$S_2 = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix}\begin{bmatrix} 180 \\ 130 \end{bmatrix} - \begin{bmatrix} 20 \\ 20 \end{bmatrix}$

$= \begin{bmatrix} 148 \\ 122 \end{bmatrix}$

Question 9

P, Q, R and S are matrices such that the matrix product $P = QRS$ is defined.
Matrix Q and matrix S are square, non-zero matrices for which $Q + S$ is **not defined**.
Which one of the following matrix expressions **is defined**?

~~A.~~ $R - S$

~~B.~~ $Q + R$

~~C.~~ P^2

~~D.~~ R^{-1}

E. $P \times S$

Q and S are NOT the same order

$\begin{matrix} n \times n & n \times m & m \times m \\ P = Q & R & S \end{matrix}$ $n \neq m$

Q order $n \times n$

R order $n \times m$

S order $m \times m$

P order $n \times m$

$n \times m \quad m \times m$

P. S is defined

END OF MULTIPLE-CHOICE QUESTION BOOK