ANSWERS

BASIC PROBABILITY

Question 1

Jo and Em are often caught for talking in class.

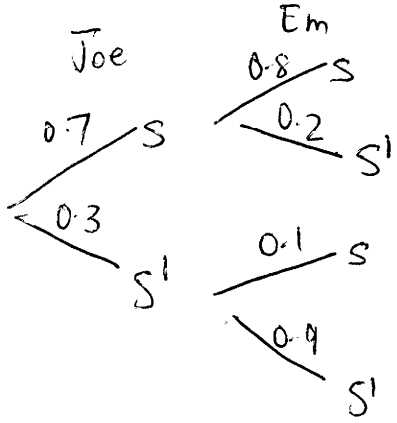
The probability that Jo starts a conversation is 70%. If Jo starts talking, the chance that Em responds is 80%. If Jo doesn't start a conversation, the chance that Em does is 10%.

a. Draw a tree diagram to represent the outcomes in this situation.

b. Determine the probability that Em was talking to Jo in class.

c. Given that Em didn't talk in class, what was the chance that Jo also didn't talk in class? Give your answer in simplified fraction form.

(a)



S = start

S' = doesn't start

(b) $Pr(\text{Em talking to Jo}) = Pr(SS) + Pr(S'S)$

$= 0.7 \times 0.8 + 0.3 \times 0.1$

$= 0.59$

(c) Let A = Em didn't talk

B = Jo didn't talk

$Pr(B|A) = \frac{Pr(B \cap A)}{Pr(A)}$

$= \frac{0.27}{0.41}$

$= \frac{27}{41}$

$Pr(B \cap A) = Pr(S'S') = 0.3 \times 0.9$

$= 0.27$

$Pr(A) = Pr(SS') + Pr(S'S')$

$= 0.7 \times 0.2 + 0.3 \times 0.9$

$= 0.14 + 0.27$

$= 0.41$

Question 2

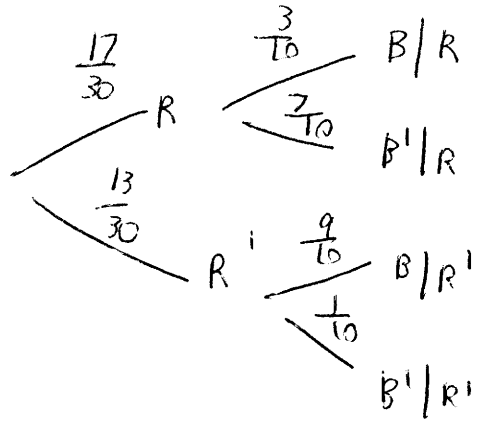
In the month of June, it rains on average 17 days of the month in Perth. It if rains, the chance that Peter catches the bus to work is $\frac{3}{10}$. If it doesn't rain, the chance that Peter catches the bus to work is $\frac{9}{10}$.

a. Draw a Tree diagram to represent the outcomes in this situation.

b. Determine the probability that on a given day in June, Peter catches the bus to work.

c. Given that Peter caught the bus to work, what was the chance it was raining that day?

(a)



(b) $Pr(B) = Pr(R) \times Pr(B|R) + Pr(R') \times Pr(B|R')$

$= \frac{17}{30} \times \frac{3}{10} + \frac{13}{30} \times \frac{9}{10}$

$= \frac{51}{300} + \frac{117}{300}$

$= \frac{168}{300}$

$= \frac{56}{100} = \frac{14}{25}$

(c) $Pr(R|B) = \frac{Pr(R \cap B)}{Pr(B)} = \frac{\frac{17}{30} \times \frac{3}{10}}{\frac{14}{25}} = \frac{\frac{17}{100}}{\frac{14}{25}}$

$= \frac{17}{56}$

Question 3

$Pr(A)=0.6 \quad Pr(B)=0.8$

Suppose $P(A \cup B) = 0.9$. $P(A') = 0.4$ and $P(B') = 0.2$.

Find $P(A \cap B')$.

$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

$\therefore 0.9 = 0.6 + 0.8 - Pr(A \cap B)$

$\therefore Pr(A \cap B) = 0.5$

Find $P(B \cap A')$.

Hence determine $P(A \cap B)$.

	B	B'	
A	0.5	0.1	0.6
A'	0.3	0.1	0.4
	0.8	0.2	1

Are events A and B mutually exclusive?

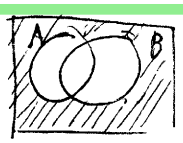
$Pr(A \cap B') = 0.1$

$Pr(B \cap A') = 0.3$

$Pr(A \cap B) = 0.5$

Since $Pr(A \cap B) \neq 0$, A and B are not mutually exclusive.

Question 4



$(A \cup B)' = A' \cap B'$

Suppose $P(A \mid B) = 0.4$ and $P((A \cup B)') = 0.15$, where A and B are independent.

(a) Find $P(A)$.

(b) Determine $P(A' \cap B)$.

(c) Given that $P(A \cap B) = x$, write an expression for $P(B)$ in terms of x.

Use the above results to solve for x.

Hence state the value of $P(B)$.

(a) Since A, B are independent

$Pr(A) = Pr(A \mid B) = 0.4$

(b)

	B	B'	
A	x		0.4
A'	0.45	0.15	0.6

$Pr(A' \cap B) = 0.45$

(c) $Pr(B) = 0.45 + x$

Since A, B are independent $Pr(A \cap B) = Pr(A) \times Pr(B)$

$\therefore x = 0.4(0.45 + x)$

$\therefore x = 0.18 + 0.4x$

$\therefore x = \frac{18}{60} = \frac{3}{10}$ and $Pr(B) = 0.75$

Question 5

In a factory, machines A, B and C are all producing metal rods of the same length. Machine A produces 35% of the rods, machine B produces 25% and the rest are produced by machine C. Of their production of rods, machines A, B and C produce 3%, 6% and 5% defective rods respectively.

(a) Draw a tree diagram to represent this information.

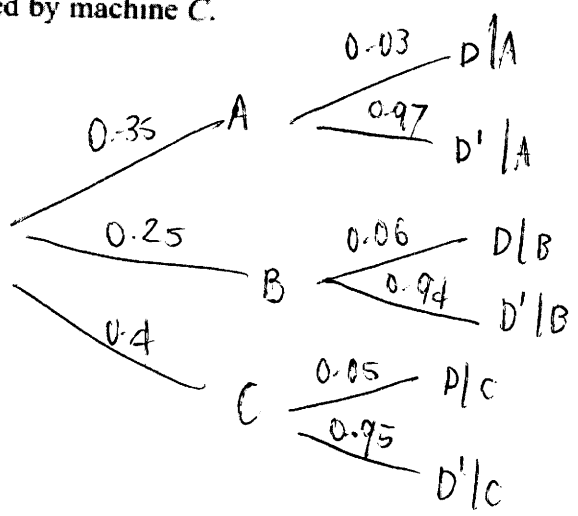
(b) Find the probability that a randomly selected rod is

(i) produced by machine A and is defective,

(ii) is defective.

(c) Given that a randomly selected rod is defective, find the probability that it was produced by machine C.

(a)



(b) (i) $Pr(A \cap D) = Pr(A) \times Pr(D|A) = 0.35 \times 0.03 = 0.0105$

(ii) $Pr(D) = Pr(A \cap D) + Pr(B \cap D) + Pr(C \cap D)$

$= 0.15 \times 0.03 + 0.25 \times 0.06 + 0.4 \times 0.05$

$= 0.0395$

(iii) $Pr(C|D) = \frac{Pr(C \cap D)}{Pr(D)} = \frac{0.4 \times 0.05}{0.0395} = \frac{0.02}{0.0395}$

$= \frac{200}{395} = \frac{40}{79}$

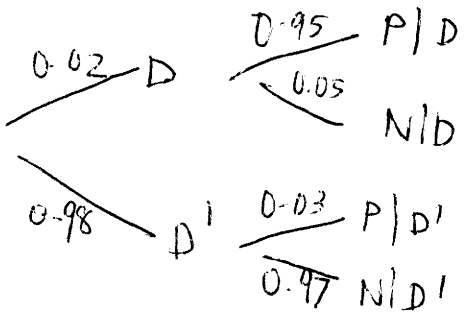
Question 6

A disease is known to be present in 2% of a population. A test is developed to help determine whether or not someone has the disease.

Given that a person has the disease, the test is positive with probability 0.95

Given that a person does not have the disease, the test is positive with probability 0.03

(a) Draw a tree diagram to represent this information.



P = positive
N = negative
D = has disease
D' = does not have disease

A person is selected at random from the population and tested for this disease.

(b) Find the probability that the test is positive.

$$Pr(P) = 0.02 \times 0.95 + 0.98 \times 0.03 = 0.0484$$

A doctor randomly selects a person from the population and tests him for the disease. Given that the test is positive,

(c) find the probability that he does not have the disease.

$$Pr(D' | P) = \frac{Pr(D' \cap P)}{Pr(P)}$$

$$= \frac{0.98 \times 0.03}{0.0484}$$

$$= \frac{0.0294}{0.0484} = \frac{294}{484} = \frac{147}{242}$$

Question 7

i. Jake and Kamil are sometimes late for school.
The events J and K are defined as follows

J = the event that Jake is late for school
K = the event that Kamil is late for school

$P(J) = 0.25$, $P(J \cap K) = 0.15$ and $P(J' \cap K') = 0.7$

On a randomly selected day, find the probability that

(a) at least one of Jake or Kamil are late for school,

$= 1 - Pr(\text{no one is late}) = 1 - 0.7 = 0.3$

(b) Kamil is late for school.

$Pr(K) = 0.2$

Given that Jake is late for school,

(c) find the probability that Kamil is late.

	K	K'	
J	0.15	0.1	0.25
J'	0.05	0.7	0.75
	0.2	0.8	1

The teacher suspects that Jake being late for school and Kamil being late for schoc linked in some way.

(d) Determine whether or not J and K are statistically independent.

(e) Comment on the teacher's suspicion in the light of your calculation in (d).

(c) $Pr(K|J) = \frac{Pr(K \cap J)}{Pr(J)} = \frac{0.15}{0.25} = \frac{3}{5}$

(d) $Pr(J) = 0.25$, $Pr(K) = 0.2$

$Pr(J) \times Pr(K) = 0.25 \times 0.2 = 0.05$

Since $Pr(J \cap K) = 0.15$, we see that $Pr(J \cap K) \neq Pr(J) \times Pr(K)$, so they are <u>not</u> statistically independent.

(e) She is right to be suspicious

Question 8

(a) Given that $P(A) = a$ and $P(B) = b$ express $P(A \cup B)$ in terms of a and b when

(i) A and B are mutually exclusive,
(ii) A and B are independent.

Two events R and Q are such that

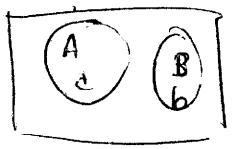
$P(R \cap Q') = 0.15$, $P(Q) = 0.35$ and $P(R|Q) = 0.1$

Find the value of

(b) $P(R \cup Q)$,

(c) $P(R \cap Q)$,

(d) $P(R)$.

(i) 

If mutually exclusive,
$Pr(A \cup B) = a + b$

(ii) If independent, $Pr(A \cap B) = Pr(A) \times Pr(B) = ab$

$\therefore Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

$= a + b - ab$

(b) $Pr(R|Q) = \frac{Pr(R \cap Q)}{Pr(Q)}$ $\therefore 0.1 = \frac{Pr(R \cap Q)}{0.35}$

$\therefore Pr(R \cap Q) = 0.035$

	Q	Q'	
R	0.035	0.15	0.185
R'	0.315	0.5	0.815
	0.35	0.65	1

$Pr(R \cup Q) = Pr(R) + Pr(Q) - Pr(R \cap Q)$

$= 0.185 + 0.35 - 0.035$

$= 0.5$

(c) $Pr(R \cap Q) = 0.035$

(d) $Pr(R) = 0.185$