BASIC PROBABILITY

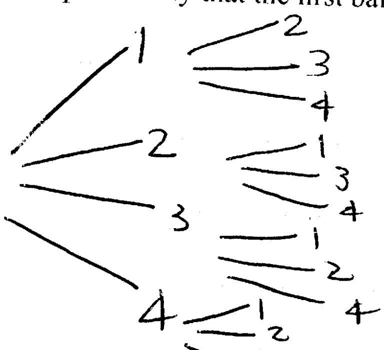
CAS FREE QUESTIONS

ANSWERS

Question 5

Four identical balls are numbered 1, 2, 3 and 4 and put into a box. A ball is randomly drawn from the box, and not returned to the box. A second ball is then randomly drawn from the box.

a. What is the probability that the first ball drawn is numbered 4 and the second ball drawn is numbered 1?



$$\Pr(4,1) = \frac{1}{12}$$

1 mark

b. What is the probability that the sum of the numbers on the two balls is 5?

$$\Pr(1,4) + \Pr(2,3) + \Pr(3,2) + \Pr(4,1) = \frac{4}{12}$$

$$= \frac{1}{3}$$

1 mark

c. Given that the sum of the numbers on the two balls is 5, what is the probability that the second ball drawn is numbered 1?

Let A = "Sum is 5"
B = "second ball is 1"

Conditional probability

$$\Pr(B|A) = \frac{\Pr(B \cap A)}{\Pr(A)} = \frac{\Pr(4,1)}{\frac{1}{3}} = \frac{1}{12} \times \frac{3}{1}$$

$$= \frac{1}{4}$$

2 marks

Question 8

Two events, A and B, are such that $\Pr(A) = \frac{3}{5}$ and $\Pr(B) = \frac{1}{4}$.

If A' denotes the complement of A, calculate $\Pr(A' \cap B)$ when

a. $\Pr(A \cup B) = \frac{3}{4}$

	B	B'	
A	$\frac{1}{10}$		$\frac{3}{5}$
A'			$\frac{2}{5}$
	$\frac{1}{4}$	$\frac{3}{4}$	1

If $\Pr(A \cup B) = \frac{3}{4}$ then

$\frac{3}{4} = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\therefore \frac{3}{4} = \frac{3}{5} + \frac{1}{4} - \Pr(A \cap B)$

$\therefore \Pr(A \cap B) = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$

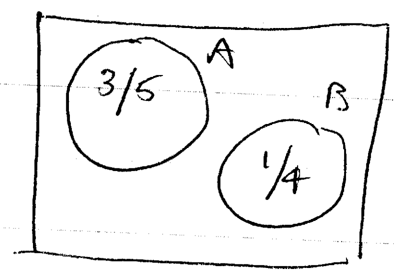
$\therefore \Pr(A' \cap B) = \frac{1}{4} - \frac{1}{10} = \frac{3}{20}$

2 marks

b. A and B are mutually exclusive.

$\Pr(A' \cap B) = \Pr(B)$

$= \frac{1}{4}$



1 mark

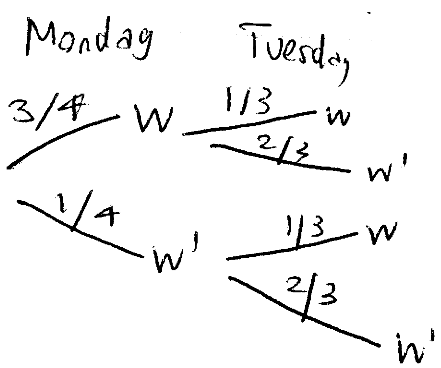
Question 9 (6 marks)

Sally aims to walk her dog, Mack, most mornings. If the weather is pleasant, the probability that she will walk Mack is $\frac{3}{4}$, and if the weather is unpleasant, the probability that she will walk Mack is $\frac{1}{3}$.

Assume that pleasant weather on any morning is independent of pleasant weather on any other morning.

a. In a particular week, the weather was pleasant on Monday morning and unpleasant on Tuesday morning.

Find the probability that Sally walked Mack on at least one of these two mornings. 2 marks



Pr(walked on at least 1 of these mornings)

$= 1 - \Pr(w' \, w')$

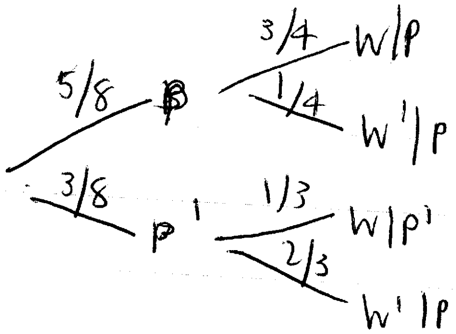
$= 1 - \frac{1}{4} \times \frac{2}{3}$

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$

b. In the month of April, the probability of pleasant weather in the morning was $\frac{5}{8}$.

i. Find the probability that on a particular morning in April, Sally walked Mack. 2 marks



$$\Pr(W) = \Pr(P) \times \Pr(W|P) + \Pr(P') \times \Pr(W|P')$$

$$= \frac{5}{8} \times \frac{3}{4} + \frac{3}{8} \times \frac{1}{3}$$

$$= \frac{15}{32} + \frac{1}{8} = \frac{19}{32}$$

ii. Using your answer from **part b.i.**, or otherwise, find the probability that on a particular morning in April, the weather was pleasant, given that Sally walked Mack that morning. 2 marks

Let A = "weather was pleasant on a particular morning"

B = Sally walked Mack

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$= \frac{\frac{5}{8} \times \frac{3}{4}}{\frac{19}{32}}$$

$$= \frac{15}{19}$$

Question 8 (3 marks)

For events A and B from a sample space, $\Pr(A|B) = \frac{3}{4}$ and $\Pr(B) = \frac{1}{3}$.

a. Calculate $\Pr(A \cap B)$.

1 mark

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\therefore \frac{3}{4} = \frac{\Pr(A \cap B)}{\frac{1}{3}}$$

$$\therefore \Pr(A \cap B) = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$$

b. Calculate $\Pr(A' \cap B)$, where A' denotes the complement of A.

1 mark

$$\Pr(A' \cap B) = \frac{1}{3} - \frac{1}{4}$$

$$= \frac{1}{12}$$

	B	B'	
A	$\frac{1}{4}$		
A'			
	$\frac{1}{3}$		

c. If events A and B are independent, calculate $\Pr(A \cup B)$.

1 mark

If A, B are independent,

$$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$$

$$\therefore \frac{1}{4} = \Pr(A) \times \frac{1}{3}$$

$$\therefore \Pr(A) = \frac{3}{4}$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\therefore \Pr(A \cup B) = \frac{3}{4} + \frac{1}{3} - \frac{1}{4}$$

$$= \frac{1}{2} + \frac{1}{3}$$

$$= \frac{5}{6}$$

(NOTE: A simpler solution comes from noticing that if A and B are independent, $\Pr(A) = \Pr(A|B) = \frac{3}{4}$ immediately)

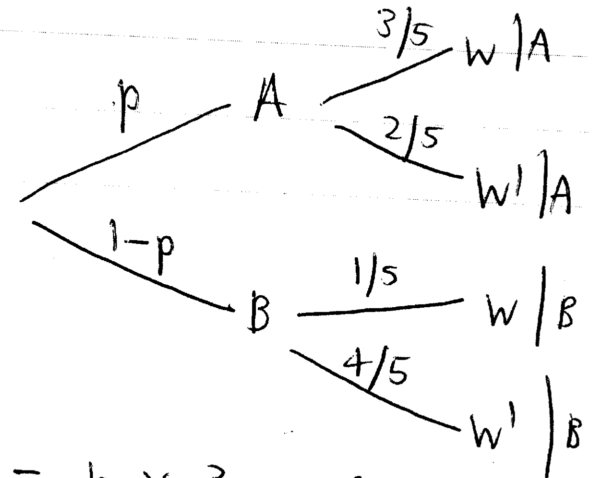
Question 9 (4 marks)

An egg marketing company buys its eggs from farm A and farm B. Let p be the proportion of eggs that the company buys from farm A. The rest of the company's eggs come from farm B. Each day, the eggs from both farms are taken to the company's warehouse.

Assume that $\frac{3}{5}$ of all eggs from farm A have white eggshells and $\frac{1}{5}$ of all eggs from farm B have white eggshells.

a. An egg is selected at random from the set of all eggs at the warehouse.

Find, in terms of p, the probability that the egg has a white eggshell. 1 mark



$$\Pr(W) = p \times \frac{3}{5} + (1-p) \times \frac{1}{5}$$

$$= \frac{3p}{5} + \frac{1}{5} - \frac{p}{5}$$

$$= \frac{2p}{5} + \frac{1}{5}$$

b. Another egg is selected at random from the set of all eggs at the warehouse.

i. Given that the egg has a white eggshell, find, in terms of p, the probability that it came from farm B.

2 marks

$$\Pr(B|W) = \frac{\Pr(B \cap W)}{\Pr(W)}$$

$$= \frac{(1-p)\times\frac{1}{5}}{\frac{2p+1}{5}} = \frac{1-p}{2p+1}$$

ii. If the probability that this egg came from farm B is 0.3, find the value of p.

1 mark

$$\frac{1-p}{2p+1} = \frac{3}{10}$$

$$10 - 10p = 6p + 3$$

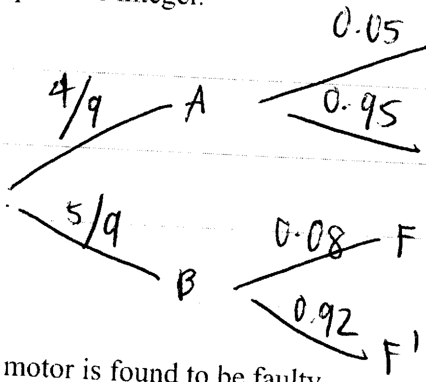
$$7 = 16p$$

$$p = \frac{7}{16}$$

Question 7 (3 marks)

A company produces motors for refrigerators. There are two assembly lines, Line A and Line B. 5% of the motors assembled on Line A are faulty and 8% of the motors assembled on Line B are faulty. In one hour, 40 motors are produced from Line A and 50 motors are produced from Line B. At the end of an hour, one motor is selected at random from all the motors that have been produced during that hour.

a. What is the probability that the selected motor is faulty? Express your answer in the form $\frac{1}{b}$, where b is a positive integer. 2 marks



$$Pr(F) = \frac{4}{9} \times \frac{5}{100} + \frac{5}{9} \times \frac{8}{100}$$

$$= \frac{20}{900} + \frac{40}{900}$$

$$= \frac{60}{900} = \frac{6}{90} = \frac{1}{15}$$

b. The selected motor is found to be faulty.

Conditional probability

What is the probability that it was assembled on Line A? Express your answer in the form $\frac{1}{c}$, where c is a positive integer. 1 mark

$$Pr(A|F) = \frac{Pr(A \cap F)}{Pr(F)}$$

$$= \frac{\frac{4}{9} \times \frac{5}{100}}{\frac{1}{15}}$$

$$= \frac{20}{900} \times \frac{15}{1}$$

$$= \frac{2}{90} \times \frac{15}{1}$$

$$= \frac{1}{3}$$

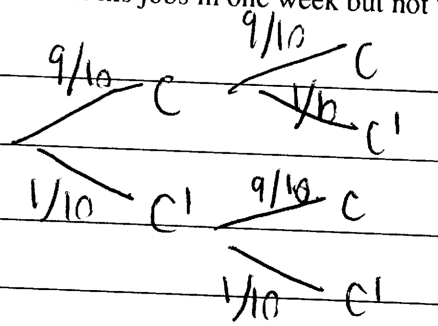
Question 9 (6 marks)

Geoff owns a lawn mowing business. If there is no rain during a week then the probability that he completes all his jobs is $\frac{9}{10}$, but if there is rain, then the probability that he completes all his jobs is $\frac{2}{5}$.

The probability that there is rain one week is assumed to be independent of there being rain during any other week.

a. Find the probability that during the last fortnight, when there was no rain, Geoff completed all of his jobs in one week but not the other.

2 marks



$Pr(CC') + Pr(C'C)$

$= \frac{9}{10} \times \frac{1}{10} + \frac{1}{10} \times \frac{9}{10}$

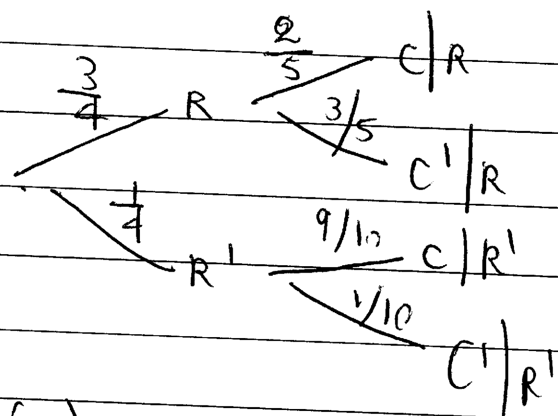
$= \frac{18}{100} = 0.18$

OR $\frac{9}{50}$

During winter, the probability of there being rain during a week is $\frac{3}{4}$.

b. i. Find the probability that during the first week of winter, Geoff completed all of his jobs.

2 marks



$Pr(C) = \frac{3}{4} \times \frac{2}{5} + \frac{1}{4} \times \frac{9}{10}$

$= \frac{6}{20} + \frac{9}{40}$

$= \frac{21}{40}$

ii. Find the probability that during a different week in winter there was rain, given that Geoff didn't complete all of his jobs that week. 2 marks

Conditional probability

$$\Pr(R|C') = \frac{\Pr(R \cap C')}{\Pr(C')}$$

$$= \frac{\frac{3}{4} \times \frac{3}{5}}{1 - \frac{21}{40}}$$

$$= \frac{\frac{9}{20}}{\frac{19}{40}}$$

$$= \frac{9}{20} \times \frac{40}{19}$$

$$= \frac{18}{19}$$

Question 10 **CAS QUESTIONS** **(10 marks)**

The events A and B have probabilities $P(A) = 0.3$, $P(\overline{B} \mid \overline{A}) = 0.2$ and $P(B \mid A) = 0.4$.

(a) Show that $P(A \cap B) = 0.12$. **NOTE: $\overline{A} = A'$** (1 mark)

$$\Pr(B \mid A) = \frac{\Pr(B \cap A)}{\Pr(A)}$$

$$\therefore \Pr(A \cap B) = \Pr(A) \times \Pr(B \mid A) = 0.3 \times 0.4 = 0.12$$

(b) Show that $P(A \cup B) = 0.86$. (2 marks)

$\Pr(B' \mid A') = 0.2$

$$\therefore 0.2 = \frac{\Pr(B' \cap A')}{\Pr(A')}$$

$$\therefore 0.2 = \frac{\Pr(B' \cap A')}{0.7}$$

$\therefore \Pr(B' \cap A') = 0.14$

	B	B'	
A	0.12	0.18	0.3
A'	0.56	0.14	0.7
	0.68	0.32	1

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\therefore \Pr(A \cup B) = 0.3 + 0.68 - 0.12$

$= 0.86$

(c) Determine $P(B)$.

$\Pr(B) = 0.68$

(d) Determine $P(A \mid B)$. (2 marks)

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{0.12}{0.68} = \frac{12}{68} = \frac{3}{17}$$

(e) Are events A and B independent? Justify your answer. (2 marks)

$\Pr(A) \times \Pr(B) = 0.3 \times 0.68 = 0.204$

$\Pr(A \cap B) = 0.12$

Since $\Pr(A \cap B) \neq \Pr(A) \times \Pr(B)$, they are not independent.

[OR $\Pr(A) = 0.3$ but $\Pr(A \mid B) = \frac{3}{17}$

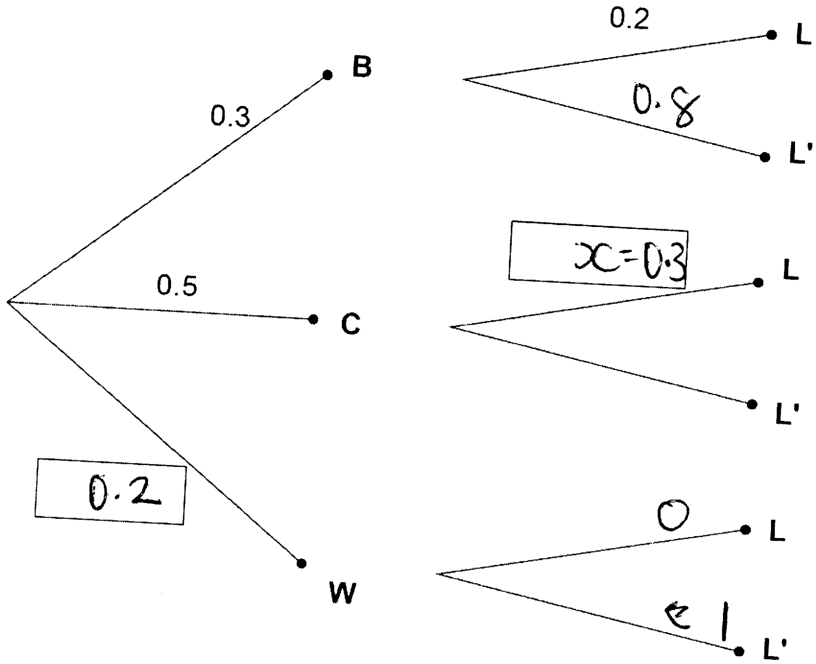
since Pr(A) is not equal to Pr(A|B)
A and B are not independent

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 13

(6 marks)

James travels to school in one of three ways. Thirty per cent of the time he rides his bicycle (**B**), 50% of the time his mother drives him (**C**) and the rest of the time he walks (**W**). When he rides his bicycle, there is a 20% chance of his having a puncture that will make him late for school (**L**). On the days he walks, he is never late for school.



Overall, James is late for school 21% of the time.

(a) Part of the tree diagram is shown above. Write the two unknown probabilities in the boxes above. (3 marks)

$Pr(L) = 0.3 \times 0.2 + 0.5x + 0.2 \times 0$

$\therefore 0.06 + \frac{x}{2} = 0.21$

$\frac{x}{2} = 0.15 \quad \therefore x = 0.3$

(b) On a day when he arrives late for school, what is the probability that he has ridden his bicycle? (3 marks)

Conditional probability

$Pr(B|L) = \frac{Pr(B \cap L)}{Pr(L)} = \frac{0.3 \times 0.2}{0.21} = \frac{6}{21} = \frac{2}{7}$

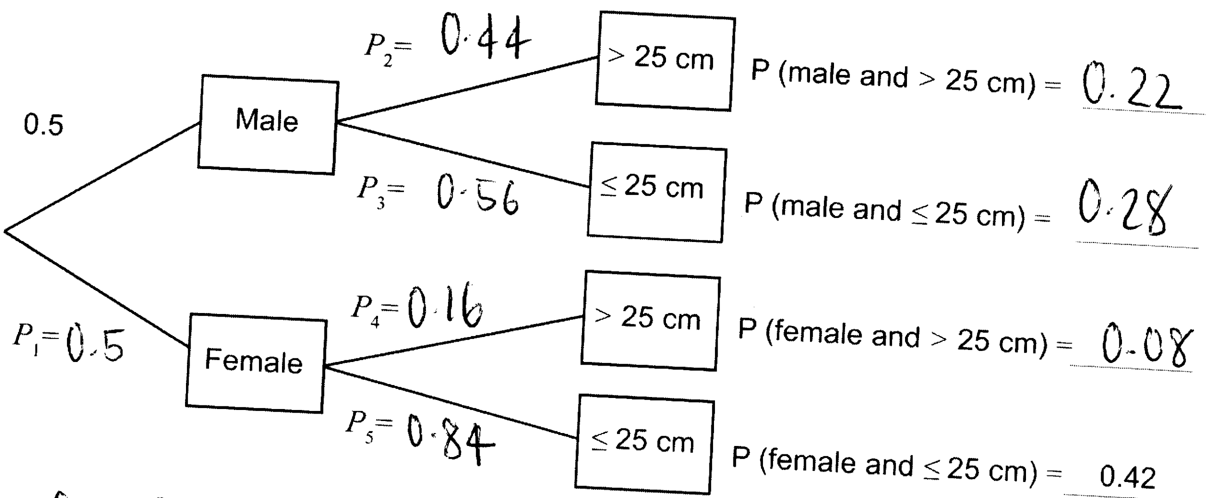
Question 15

(9 marks)

In a population of fish, 50% are male and 50% are female. Overall, 30% of the fish are over 25 cm in length. Furthermore, 42% of the fish are female and 25 cm or under in length.

(a) Use this information to complete the tree diagram below by determining the probabilities P_1 to P_5 on the branches of the diagram, and the probabilities to the right of the tree diagram.

(5 marks)



$0.5\ P_5 = 0.42$

$\therefore P_5 = 0.84$

$\therefore P_4 = 1 - 0.84 = 0.16$

$0.5\ P_2 = 0.22$

$\therefore P_2 = 0.44$

$P_3 = 1 - 0.44 = 0.56$

(b) What is the probability that a randomly caught fish will be:

(i) 25 cm or under in length? (1 mark)

$P(\leq 25\text{cm}) = 0.28 + 0.42 = 0.7$

(ii) either female or over 25 cm in length? (1 mark)

$Pr(F \cup > 25\text{cm})$

$= Pr(F) + Pr(> 25\text{cm}) - Pr(F \cap > 25\text{cm})$

$= 0.5 + 0.3 - 0.08 = 0.72$

(iii) female, given that it is over 25 cm in length? (2 marks)

$$Pr(F \mid > 25\text{cm}) = \frac{Pr(F \cap > 25\text{cm})}{Pr(> 25\text{cm})}$$

$$= \frac{0.08}{0.3}$$

$$= \frac{8}{30}$$

$$= \frac{4}{15}$$

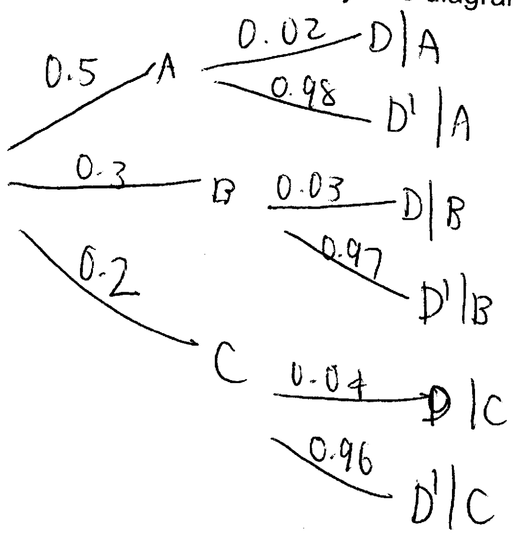
DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 16

(7 marks)

A toy manufacturer buys pre-assembled robotic arms from three different suppliers: 50% of the total order comes from Supplier A, 30% from Supplier B, and the remaining 20% from Supplier C. Past data shows that the quality control standards of the three suppliers are different. While 2% of the arms produced by Supplier A are defective, Suppliers B and C produce defective arms at rates of 3% and 4%, respectively.

(a) Construct a probability tree diagram for the above information. (4 marks)



$$Pr(D) = 0.5 \times 0.02 + 0.3 \times 0.03 + 0.2 \times 0.04$$
$$= 0.01 + 0.009 + 0.008$$
$$= 0.027$$

(b) Given that a robotic arm is defective, determine the probability that the arm did not come from Supplier A. (3 marks)

$$Pr(A' \mid D) = \frac{Pr(A' \cap D)}{Pr(D)}$$

$$Pr(A' \cap D) = Pr(B \cap D) + Pr(C \cap D)$$
$$= 0.009 + 0.008$$
$$= 0.017$$

$$\therefore Pr(A' \mid D) = \frac{0.017}{0.027} = \frac{17}{27}$$