CAS FREE QUESTIONS

Binomial Distribution

Question 7

A biased coin is tossed three times. The probability of a head from a toss of this coin is p.

a. Find, in terms of p, the probability of obtaining

i. three heads from the three tosses

$X \overset{d}{=} Bi(n = 3, p)$ X = no. of heads

$X = 0, 1, 2, 3$

$Pr(X = 3) = \binom{3}{3} p^3 (1-p)^0 = p^3$

ii. two heads and a tail from the three tosses.

$Pr(X = 2) = \binom{3}{2} p^2 (1-p)$

$= 3p^2(1-p)$

1 + 1 = 2 marks

b. If the probability of obtaining three heads equals the probability of obtaining two heads and a tail, find p.

$p^3 = 3p^2(1-p)$

$\therefore p = 3(1-p), \quad p \neq 0$

$p = 3 - 3p$

$4p = 3$

$p = \frac{3}{4}$

2 marks

$\therefore p = 0, \frac{3}{4}$

Note that the possibility of p = 0 must also be included.

Question 4 (3 marks)

A paddock contains 10 tagged sheep and 20 untagged sheep. Four times each day, one sheep is selected at random from the paddock, placed in an observation area and studied, and then returned to the paddock.

a. What is the probability that the number of tagged sheep selected on a given day is zero? 1 mark

X = no. of tagged sheep selected $\quad X = 0, 1, 2, 3, 4$

$X \stackrel{d}{=} Bi(n=4, p=\frac{1}{3})$

$$Pr(X=0) = \binom{4}{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{4} = \frac{16}{81}$$

b. What is the probability that at least one tagged sheep is selected on a given day? 1 mark

$$Pr(X \geq 1) = 1 - Pr(X=0)$$
$$= 1 - \frac{16}{81}$$
$$= \frac{65}{81}$$

c. What is the probability that no tagged sheep are selected on each of six consecutive days?

Express your answer in the form $\left(\frac{a}{b}\right)^c$, where a, b and c are positive integers. 1 mark

Let Y = no. of days in which no tagged sheep is selected

$Y = 0, 1, 2, \ldots, 6$

$Y \stackrel{d}{=} Bi\left(n=6, p=\frac{16}{81}\right)$

$$Pr(Y=6) = \binom{6}{6}\left(\frac{16}{81}\right)^{6}\left(\frac{65}{81}\right)^{0}$$

$$= \left(\frac{16}{81}\right)^{6}$$

Sam plays basketball. The probability that Sam scores a goal every time she has a shot is 0.2

b. Given that she scores no more than one goal in four shots, what is the probability that the first two shots were **not** goals?

Let X = no. of goals $X \overset{d}{=} Bi(n=4, p=0.2)$ $X = 0,1,2,3,4$

We require: $Pr(G'G' \mid X \leq 1)$ where G' = not goal, G = goal

$$= \frac{Pr(G'G' \cap X \leq 1)}{Pr(X \leq 1)} = \frac{Pr(G'G' \cap X \leq 1)}{Pr(X=0) + Pr(X=1)}$$

$$= \frac{Pr(G'G'G'G') + Pr(G'G'GG') + Pr(G'G'G'G)}{\binom{4}{0}(0.2)^0(0.8)^4 + \binom{4}{1}(0.2)^1(0.8)^3}$$

$$= \frac{(0.8)^4 + (0.8)^3 \times 0.2 + (0.8)^3 \times 0.2}{(0.8)^4 + 4 \times 0.2 \times (0.8)^3}$$

$$= \frac{(0.8)^3(0.8 + 2 \times 0.2)}{(0.8)^3(0.8 + 0.8)}$$

$$= \frac{1.2}{1.6} = \frac{12}{16} = \frac{3}{4}$$

c. What is the least number of shots she needs to make to ensure that the probability that she gets at least one goal is 0.9, given that $log_{10}(8) \approx 0.903$

$Pr(X \geq 1) \geq 0.9$

$\therefore 1 - Pr(X=0) \geq 0.9$

$\therefore 0.1 \geq Pr(X=0)$

$\therefore 0.1 \geq (0.8)^n$

Solving: $0.1 = (0.8)^n$

gives $\log_{10}(0.1) = n\log_{10}(0.8)$

$$n = \frac{\log_{10}(0.1)}{\log_{10}(0.8)} = \frac{-1}{\log_{10}(0.8)}$$

But $\log_{10}(0.8) = \log_{10}\left(\frac{8}{10}\right)$

$= \log_{10}(8) - \log_{10}(10)$

$= \log_{10}(8) - \log_{10}(10)$

$= 0.903 - 1$

$= -0.097$

$\therefore n \geq \frac{-1}{-0.097} = \frac{1}{0.097}$

$= \frac{1000}{97}$

97 X 10 = 970

so 1000/97 is slightly more than 10 but less than 11.

Therefore, n = 11 is the least number required.