BINOMIAL PROBABILITY CAS QUESTIONS

Question 10

The binomial random variable, X, has $E(X) = 2$ and $\text{Var}(X) = \frac{4}{3}$.

$\Pr(X = 1)$ is equal to

A. $\left(\frac{1}{3}\right)^6$

B. $\left(\frac{2}{3}\right)^6$

C. $\frac{1}{3} \times \left(\frac{2}{3}\right)^2$

D. $6 \times \frac{1}{3} \times \left(\frac{2}{3}\right)^5$

E. $6 \times \frac{2}{3} \times \left(\frac{1}{3}\right)^5$

$np = 2$

$npq = \frac{4}{3}$

$\therefore 2q = \frac{4}{3}$

$\therefore q = \frac{2}{3}$

$\therefore p = \frac{1}{3}, n = 6$

$\Pr(X = 1) = \binom{6}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^5$

$= 6 \times \frac{1}{3} \times \left(\frac{2}{3}\right)^5$

Question 22

John and Rebecca are playing darts. The result of each of their throws is independent of the result of any other throw. The probability that John hits the bullseye with a single throw is $\frac{1}{4}$. The probability that Rebecca hits the bullseye with a single throw is $\frac{1}{2}$. John has four throws and Rebecca has two throws.

The ratio of the probability of Rebecca hitting the bullseye at least once to the probability of John hitting the bullseye at least once is

A. 1:1

B. 32:27

C. 64:85

D. 2:1

E. 192:175

John:

$X \overset{d}{=} Bi(n = 4, p = \frac{1}{4})$

$\Pr(X \geq 1) = 1 - \Pr(X = 0) = 1 - \binom{4}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^4$

$= 1 - \left(\frac{3}{4}\right)^4$

$= 1 - \frac{81}{256}$

$= \frac{175}{256}$

Rebecca

$Y \overset{d}{=} Bi(n = 2, p = \frac{1}{2})$

$\Pr(Y \geq 1) = 1 - \Pr(Y = 0)$

$= 1 - \binom{2}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^2$

$= 1 - \frac{1}{4} = \frac{3}{4}$

Ratio: $\frac{3}{4} : \frac{175}{256} = \frac{192}{256} : \frac{175}{256}$

Question 9

Harry is a soccer player who practises penalty kicks many times each day.

Each time Harry takes a penalty kick, the probability that he scores a goal is 0.7, independent of any other penalty kick.

One day Harry took 20 penalty kicks.

$X \stackrel{d}{=} Bi(n = 20, p = 0.7)$

Given that he scored at least 12 goals, the probability that Harry scored exactly 15 goals is closest to

A. 0.1789

B. 0.8867

C. 0.8

D. 0.6396

E. 0.2017

$$Pr(X = 15 \mid X \geq 12)$$
$$= \frac{Pr(X = 15 \cap X \geq 12)}{Pr(X \geq 12)}$$
$$= \frac{Pr(X = 15)}{Pr(X \geq 12)} = \frac{0.178863}{0.88667} \simeq 0.2017$$

"at least" = "greater than or equal to"

Question 12

A soccer player is practising her goal kicking. She has a probability of $\frac{3}{5}$ of scoring a goal with each attempt. She has 15 attempts.

The probability that the number of goals she scores is less than 7 is closest to

A. 0.0612

B. 0.0950

C. 0.1181

D. 0.2131

E. 0.7869

$X \stackrel{d}{=} Bi(n = 15, p = \frac{3}{5})$

$Pr(X < 7) \approx 0.0950$

Question 13

A fair coin is tossed twelve times.

The probability (correct to four decimal places) that at most 4 heads are obtained is

A. 0.0730

B. 0.1209

C. 0.1938

D. 0.8062

E. 0.9270

$X \stackrel{d}{=} Bi(n = 12, p = 0.5)$

$Pr(X \leq 4) \simeq 0.1938$

"at most" = "less than or equal to"

Question 13

According to a survey, 30% of employed women have never been married.

If 10 employed women are selected at random, the probability (correct to four decimal places) that at least 7 have never been married is

$$X \overset{d}{=} Bi(n=10, p=0.3)$$

$$Pr(X \geq 7) \approx 0.0106$$

A. 0.0016

B. 0.0090

C. 0.0106

D. 0.9894

E. 0.9984

Question 14

The minimum number of times that a fair coin can be tossed so that the probability of obtaining a head on each trial is less than 0.0005 is

$$(0.5)^n < 0.0005$$

A. 8

B. 9

C. 10

D. 11

E. 12

Question 12

Fifty-four percent of employees at a large corporation are parents.
A random sample of thirty of the corporation's employees is taken.
The probability that less than half of them are parents is closest to

$$X \overset{d}{=} Bi(n=30, p=0.54)$$

$$Pr(X < 15) \simeq 0.2661$$

A. 0.1048

B. 0.1312

C. 0.2661

D. 0.3129

E. 0.3973

Question 12

A die has been altered so that the probability of throwing a 6 is 0.3.
Ellen rolls the die ten times. The probability that she obtains a 6 more than twice is closest to

A. 0.2668

B. 0.2335

C. 0.3828

D. 0.8507

E. 0.6172

$X \stackrel{d}{=} Bi(n=10, p=0.3)$

$Pr(X>2) \simeq 0.6172$

Question 8

According to a survey, 20% of primary school students at a particular primary school like the colour red. If 10 students from this primary school are selected at random, the probability that at least 6 of them like the colour red is

A. 0.0064

B. 0.0009

C. 0.9936

D. 0.0055

E. 0.9945

$X \stackrel{d}{=} Bi(n=10, p=0.2)$

$Pr(X \geq 6) \simeq 0.0064$

Question 10

Let X be a discrete random variable with a binomial distribution. The mean of X is 2 and the variance is 1.6. The values of n (number of independent trials) and p (the probability of each trial) are

A. $p = 2$ and $n = 20$

B. $p = 0.8$ and $n = 10$

C. $p = 0.2$ and $n = 10$

D. $p = 0.02$ and $n = 100$

E. $p = 2$ and $n = 100$

$E(X) = 2$ $\therefore np = 2$

$var(X) = 1.6$ $\therefore npq = 1.6$

$\therefore 2q = 1.6$

$\therefore q = 0.8$

$\therefore p = 0.2$

$\therefore n = \frac{2}{0.2} = 10$

$\therefore n = 10,$
$p = 0.2,$
$q = 0.8$

Question 21

Ben has constructed a spinner that will randomly display an integer between 0 and 4 with the following probabilities.

Number	x	0	1	2	3	4
Probability	$Pr(X=x)$	0.2	0.3	0.15	0.25	0.1

Ben spins the spinner 5 times. The probability of obtaining at least 3 odd numbers is

A. 0.55

B. 0.55^3

C. $(0.55)^3 + (0.55)^4 + (0.55)^5$

D. $(0.45)^2(0.55)^3 + (0.45)(0.55)^4 + (0.55)^5$

E. $10(0.45)^2(0.55)^3 + 5(0.45)(0.55)^4 + (0.55)^5$

$Pr(\text{odd no}) = Pr(1) + Pr(3)$
$= 0.55$

Y = no. of odd numberr

$Y \overset{d}{=} Bi(n=5, p=0.55)$

$Pr(Y \geq 3) = \binom{5}{3}(0.55)^3(0.45)^2 + \binom{5}{4}(0.55)^4(0.45)^1 + \binom{5}{5}(0.55)^5$

$= 10 \times (0.55)^3(0.45)^2 + 5 \times (0.55)^4 \times 0.45 + (0.55)^5$

Question 17

In a particular population the probability a person has blue eyes is 0.36. A group of 8 people are selected from this population. It is known that less than 5 of the 8 have blue eyes. Correct to four decimal places, the probability that exactly 3 have blue eyes is

"it is known " indicates conditional probability

A. 0.2890
B. 0.3181
C. 0.4922
D. 0.5069
E. 0.5417

$X \overset{d}{=} Bi(n=8, p=0.36)$

$Pr(X=3 \mid X<5)$

$= \dfrac{Pr(X=3 \cap X<5)}{Pr(X<5)}$

$= \dfrac{Pr(X=3)}{Pr(X\leq 5)} = \dfrac{0.2805}{0.8820} \simeq 0.318$

Question

Ms Marshall is playing against a chess computer. The probability that she wins any individual game against the computer (which is playing in Grandmaster mode) is 0.27. What is the minimum number of games that she will need to play in order to have a probability of more than 0.9 of winning at least one game?

Let $X \stackrel{\star}{=}$ no. of games won

$X \stackrel{d}{=} Bi(n, p = 0.27)$

$Pr(X \geq 1) \geq 0.9$

$\therefore 1 - Pr(X = 0) \geq 0.9$

$\therefore Pr(X = 0) \leq 0.1$

$\therefore \binom{n}{0}(0.27)^0(0.73)^n \leq 0.1$

$\therefore (0.73)^n \leq 0.1$

$(0.73)^7 = 0.1104$

$(0.73)^8 = 0.0806$

$\therefore n = 8$ is the least number.

Question

Rex is shooting at a target. His probability of hitting the target is 0.6. What is the minimum number of shots needed for the probability of Rex hitting the target exactly 5 times to be more than 25% ?

Let X = no of hits

$X \stackrel{d}{=} Bi(n, p = 0.6)$

$Pr(X = 5) \geq 0.25$

If $n = 10$, $Pr(X = 5) = 0.2007$

If $n = 9$, $Pr(X = 5) = 0.2508$

If $n = 8$, $Pr(X = 5) = 0.2787$

If $n = 7$, $Pr(X = 5) = 0.2613$

If $n = 6$, $Pr(X = 5) = 0.1866$

$\therefore$ Minimum no. of shots is 7.

Question 5

Thomas Lindsay is a star full forward for the local football team. He has found from past experience that his chance of scoring a goal is 0.7 and is independent of any attempt he makes. At training he is practising going for goal and makes ten attempts.

a. **i.** What is the probability, correct to 4 decimal places, that Thomas scores goals with his first four attempts?

1 mark

$\underline{0.7}\,G \quad \underline{0.7}\,G \quad \underline{0.7}\,G \quad \underline{0.7}\,G$

$(0.7)^4 \approx 0.2401$

ii. What is the probability that Thomas scores exactly four goals out of his ten attempts, correct to 4 decimal places?

1 mark

$X = $ no. of goals $\qquad X \overset{d}{=} Bi(n=10, p=0.7)$

$\Pr(X=4) = 0.1327$

iii. What is the probability that the last six of Thomas' ten attempts miss, given that exactly four of his ten attempts are goals? Give your answer correct to 4 decimal places.

2 marks

$\Pr(????MMMMMM \mid X=4)$

$= \dfrac{\Pr(GGGGMMMMMM)}{\Pr(X=4)}$

The intersection event is the sequence: GGGGMMMMMM, since only for this sequence does X=4 and the last six shots are misses.

$= \dfrac{(0.7)^4(0.3)^6}{\binom{10}{4}(0.7)^4(0.3)^6} = \dfrac{1}{210} \approx 0.0048$

Question 3

At a supermarket, store data indicates that the probability that a customer has one or more deli items in their purchase is 0.4.

a. Sue is an employee at the supermarket and during her shift at a checkout, she serves ninety customers. Let the random variable X represent the number of customers Sue serves who have one or more deli items in their purchases.

$$X \overset{d}{=} Bi(n = 90, p = 0.4)$$

i. Find the probability that the first four customers that Sue serves **don't** have any deli items in their purchase.

(1 mark)

$$(0.6)^4 = 0.1296$$

ii. Find the mean number of customers Sue serves in her shift who have deli items in their purchase.

(1 mark)

$$E(X) = np$$
$$= 90 \times 0.4$$
$$= 36$$

iii. Find the probability, correct to 4 decimal places, that at least thirty of the customers Sue serves have deli items in their purchase.

(1 mark)

$$Pr(X \geq 30) \approx 0.9203$$

Data collected at the store indicates that when an employee at a checkout serves n customers, there is a probability of $\frac{96n}{15\,625}$ that all but one of those customers will have deli items in their purchase.

b. Show that n equals 6.

$X \overset{d}{=} Bi(n, p = 0.4)$

(3 marks)

$$\Pr(X = n-1) = \frac{96n}{15625}$$

$$\therefore \binom{n}{n-1}(0.4)^{n-1}(0.6) = \frac{96n}{15625}$$

$$\frac{n!}{1!(n-1)!} \times (0.4)^{n-1} \times 0.6 = \frac{96n}{15625}$$

$$\therefore n \times (0.4)^{n-1} \times 0.6 = \frac{96n}{15625}$$

$$\therefore 0.6 \times (0.4)^{n-1} = \frac{96}{15625}$$

$$\therefore \frac{3}{5} \times \left(\frac{2}{5}\right)^{n-1} = \frac{96}{15625}$$

$$\therefore \frac{3 \times 2^{n-1}}{5^n} = \frac{96}{15625}$$

$$\therefore 5^n = 15625 \quad \therefore n = 6$$

$$3 \times 2^{6-1} = 3 \times 2^5 = 96$$

Question

A cadet fires shots at a target at distances ranging from 25 m to 90 m. The probability of hitting the target with a single shot is p. When firing from a distance d m, $p = \frac{3}{200}(90 - d)$.

Each shot is fired independently.

The cadet fires 10 shots from a distance of 40 m.

(a) (i) Find the probability that exactly 6 shots hit the target.

(ii) Find the probability that at least 8 shots hit the target.

The cadet fires 20 shots from a distance of x m.

(b) Find, to the nearest integer, the value of x if the cadet has an 80% chance of hitting the target at least once.

(a) (i) When $d = 40$, $p = \frac{3}{200} \times (90 - 40) = \frac{150}{200} = \frac{3}{4}$

$X \stackrel{d}{=} Bi(n = 10, p = 0.75)$ $x = 0, 1, 2, \ldots, 10$

$Pr(X = 6) = 0.1460$

(ii) $Pr(X \geq 8) \simeq 0.5256$

(b) $Y \stackrel{d}{=} Bi(n = 20, p = ?)$

$Pr(Y \geq 1) = 0.8$

$\therefore 1 - Pr(Y = 0) = 0.8$ $\therefore Pr(Y = 0) = 0.2$

$\therefore \binom{20}{0} p^0 (1-p)^{20} = 0.2$

$\therefore (1-p)^{20} = 0.2$ $\therefore 1 - p = (0.2)^{\frac{1}{20}}$

$\therefore 1 - p = 0.922681$

$\therefore p = 0.0773$

$\therefore 0.0773 = \frac{3}{200}(90 - d)$

$\therefore d = -\frac{200}{3} \times 0.0773 + 90 \simeq 84.84 \simeq 85$ m

Question

The probability of a telesales representative making a sale on a customer call is 0.15

Find the probability that

(a) no sales are made in 10 calls,

(b) more than 3 sales are made in 20 calls.

Representatives are required to achieve a mean of at least 5 sales each day.

(c) Find the least number of calls each day a representative should make to achieve this requirement.

(d) Calculate the least number of calls that need to be made by a representative for the probability of at least 1 sale to exceed 0.95

(a) X = no. of sales $\quad X = 0, 1, 2, \ldots, 10$

$X \overset{d}{=} Bi(n = 10, p = 0.15)$

$Pr(X = 0) \simeq 0.1969$

(b) Y = no. of sales, $\quad Y = 0, 1, 2, \ldots, 20$

$Y \overset{d}{=} Bi(n = 20, p = 0.15)$

$Pr(Y > 3) \simeq 0.3522$

(c) $E(X) = np \quad = 0.15n$

$0.15n \geq 5$

$n \geq 5/0.15 \quad n \geq 33.33$

$\therefore n \geq 34$.

(d) $X \overset{d}{=} Bi(n = ?, p = 0.15) \quad Pr(X \geq 1) > 0.95$

$\therefore 1 - Pr(X = 0) > 0.95$

$\therefore 0.05 > \binom{n}{0}(0.15)^0(0.85)^n$

If $n = 18$, $(0.85)^{18} \approx 0.0536$

If $n = 19$ $(0.85)^{19} \simeq 0.0456$

$\therefore n = 19$

Question

In a large restaurant an average of 3 out of every 5 customers ask for water with their meal.

A random sample of 10 customers is selected.

(a) Find the probability that

(i) exactly 6 ask for water with their meal.

(ii) less than 9 ask for water with their meal.

(5)

A second random sample of 50 customers is selected.

(b) Find the smallest value of m such that

$$P(X < m) \geqslant 0.9$$

where the random variable X represents the number of these customers who ask for water.

(3)

(a) Let X = no. of customers who ask for water

$X \overset{d}{=} Bi(n = 10, p = 0.6)$

(i) $Pr(X = 6) = 0.2508$

(ii) $Pr(X < 9) = 0.9536$

(b) $X \overset{d}{=} Bi(n=50, p = 0.6)$

Find m: $Pr(X < m) \geq 0.9$ where m is the smallest possible value.

Trial and Error: $Pr(X < 34) \approx 0.843$

$Pr(X < 35) \approx 0.904$

$\therefore m = 35$