

**CAS FREE QUESTIONS****Question 6**

The probability density function of a continuous random variable  $X$  is given by

$$f(x) = \begin{cases} \frac{x}{12} & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

- a. Find  $\Pr(X < 3)$ .

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2 marks

- b. If  $\Pr(X \geq a) = \frac{5}{8}$ , find the value of  $a$ .

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2 marks



3) June 2005 no.3

The lifetime, in years, of an electrical appliance may be modelled by the random variable  $T$  with probability density function

$$f(t) = \begin{cases} \frac{k}{t^2} & 1 \leq t \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

- (i) Show that  $k = \frac{4}{3}$ . [2]
- (ii) Find the value of the mean of  $T$ , giving your answer in the form  $a \ln b$ . [3]
- (iii) Find the time  $t_0$  for which  $P(T > t_0) = 0.1$ . [3]



**Question 8** (4 marks)

A continuous random variable,  $X$ , has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{5}e^{-\frac{x}{5}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The median of  $X$  is  $m$ .

**a.** Determine the value of  $m$ .

2 marks

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**b.** The value of  $m$  is a number greater than 1.

Find  $\Pr(X < 1 | X \leq m)$ .

2 marks

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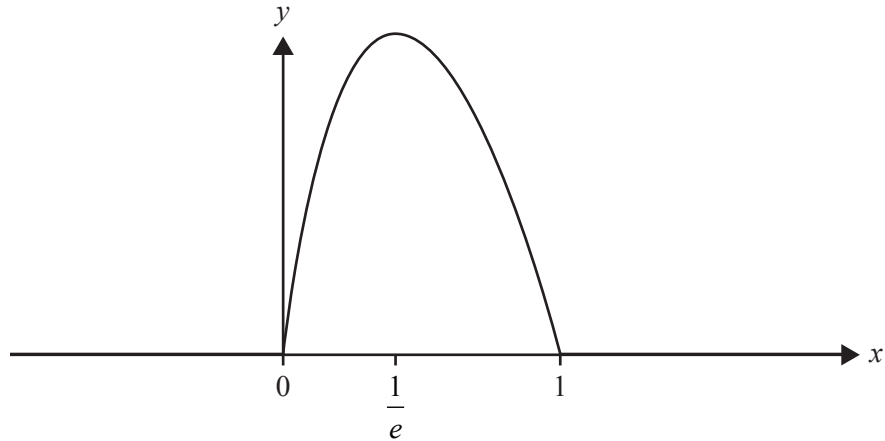


**Question 8** (6 marks)

Let  $X$  be a continuous random variable with probability density function

$$f(x) = \begin{cases} -4x \log_e(x) & 0 < x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Part of the graph of  $f$  is shown below. The graph has a turning point at  $x = \frac{1}{e}$ .



a. Show by differentiation that

$$\frac{x^k}{k^2} (k \log_e(x) - 1)$$

is an antiderivative of  $x^{k-1} \log_e(x)$ , where  $k$  is a positive real number.

2 marks

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**b. i.** Calculate  $\Pr\left(X > \frac{1}{e}\right)$ .

2 marks

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**ii.** Hence, explain whether the median of  $X$  is greater than or less than  $\frac{1}{e}$ , given that  $e > \frac{5}{2}$ . 2 marks

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