SECTION A – Multiple-choice questions

Instructions for Section A
Answer all questions in pencil on the answer sheet provided for multiple-choice questions. Choose the response that is correct for the question. A correct answer scores 1; an incorrect answer scores 0. Marks will not be deducted for incorrect answers. No marks will be given if more than one answer is completed for any question. Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Question 1
The linear function \( f: D \to \mathbb{R}, \ f(x) = 5 - x \) has range \([-4, 5)\).

The domain \( D \) is
A. \((0, 9]\)
B. \((0, 1]\)
C. \([5, -4)\)
D. \([-9, 0)\)
E. \([1, 9)\)

Question 2
Let \( f: \mathbb{R} \to \mathbb{R}, \ f(x) = 1 - 2\cos \left( \frac{\pi x}{2} \right) \).

The period and range of this function are respectively
A. 4 and \([-2, 2]\)
B. 4 and \([-1, 3]\)
C. 1 and \([-1, 3]\)
D. \(4\pi\) and \([-1, 3]\)
E. \(4\pi\) and \([-2, 2]\)
Question 3
Part of the graph \( y = f(x) \) of the polynomial function \( f \) is shown below.

\[
\begin{align*}
&\quad (\frac{1}{3}, 100) \\
&\quad (-2, -9) \\
&\quad O
\end{align*}
\]

\( f'(x) < 0 \) for

A. \( x \in (\frac{1}{3}, \infty) \) \\
B. \( x \in \left( -9, \frac{100}{27} \right) \) \\
C. \( x \in (-\infty, -2) \cup \left( \frac{1}{3}, \infty \right) \) \\
D. \( x \in (-2, \frac{1}{3}) \) \\
E. \( x \in (-\infty, -2] \cup (1, \infty) \)

Question 4
The average rate of change of the function \( f \) with rule \( f(x) = 3x^2 - 2x + 1 \), between \( x = 0 \) and \( x = 3 \), is

A. 8 \\
B. 25 \\
C. \( \frac{53}{9} \) \\
D. \( \frac{25}{3} \) \\
E. \( \frac{13}{9} \)
Question 5
Which one of the following is the inverse function of \( g: [3, \infty) \to R, \ g(x) = \sqrt{2x-6} \)?

A. \( g^{-1}: [3, \infty) \to R, \ g^{-1}(x) = \frac{x^2 + 6}{2} \)

B. \( g^{-1}: [0, \infty) \to R, \ g^{-1}(x) = (2x - 6)^2 \)

C. \( g^{-1}: [0, \infty) \to R, \ g^{-1}(x) = \sqrt{\frac{x^2 + 6}{2}} \)

D. \( g^{-1}: [0, \infty) \to R, \ g^{-1}(x) = \frac{x^2 + 6}{2} \)

E. \( g^{-1}: R \to R, \ g^{-1}(x) = \frac{x^2 + 6}{2} \)

Question 6
Consider the graph of the function defined by \( f: [0, 2\pi] \to R, \ f(x) = \sin(2x) \).

The square of the length of the line segment joining the points on the graph for which \( x = \frac{\pi}{4} \) and \( x = \frac{3\pi}{4} \) is

A. \( \frac{\pi^2 + 16}{4} \)

B. \( \pi + 4 \)

C. 4

D. \( \frac{3\pi^2 + 16\pi}{4} \)

E. \( \frac{10\pi^2}{16} \)
Question 8
The UV index, $y$, for a summer day in Melbourne is illustrated in the graph below, where $t$ is the number of hours after 6 am.

The graph is most likely to be the graph of

A. $y = 5 + 5 \cos\left(\frac{\pi t}{7}\right)$
B. $y = 5 - 5 \cos\left(\frac{\pi t}{7}\right)$
C. $y = 5 + 5 \cos\left(\frac{\pi t}{14}\right)$
D. $y = 5 - 5 \cos\left(\frac{\pi t}{14}\right)$
E. $y = 5 + 5 \sin\left(\frac{\pi t}{14}\right)$

Question 9
Given that $\frac{d(xe^{kx})}{dx} = (kx + 1)e^{kx}$, then $\int xe^{kx} dx$ is equal to

A. $\frac{xe^{kx}}{kx + 1} + c$
B. $\left(\frac{kx + 1}{k}\right)e^{kx} + c$
C. $\frac{1}{k} \int e^{kx} dx$
D. $\frac{1}{k} \left(xe^{kx} - \int e^{kx} dx\right) + c$
E. $\frac{1}{k^2} \left(xe^{kx} - e^{kx}\right) + c$
Question 10
For the curve \( y = x^2 - 5 \), the tangent to the curve will be parallel to the line connecting the positive x-intercept and the y-intercept when \( x \) is equal to

A. \( \sqrt{5} \)
B. 5
C. \( -5 \)
D. \( \frac{\sqrt{5}}{2} \)
E. \( \frac{1}{\sqrt{5}} \)

Question 11
The function \( f \) has the property \( f(x) = f(y) = (y - x) f(xy) \) for all non-zero real numbers \( x \) and \( y \). Which one of the following is a possible rule for the function?

A. \( f(x) = x^2 \)
B. \( f(x) = x^2 + x^4 \)
C. \( f(x) = x \log_e(x) \)
D. \( f(x) = \frac{1}{x} \)
E. \( f(x) = \frac{1}{x^2} \)

Question 12
The graph of a function \( f \) is obtained from the graph of the function \( g \) with rule \( g(x) = \sqrt{2x - 5} \) by a reflection in the x-axis followed by a dilation from the y-axis by a factor of \( \frac{1}{2} \). Which one of the following is the rule for the function \( f \)?

A. \( f(x) = \sqrt{5 - 4x} \)
B. \( f(x) = -\sqrt{x - 5} \)
C. \( f(x) = \sqrt{x + 5} \)
D. \( f(x) = -\sqrt{4x - 5} \)
E. \( f(x) = -\sqrt{4x - 10} \)
Question 13

Consider the graphs of the functions $f$ and $g$ shown below.

The area of the shaded region could be represented by

A. $\int_{a}^{d} (f(x) - g(x)) \, dx$

B. $\int_{0}^{d} (f(x) - g(x)) \, dx$

C. $\int_{0}^{b} (f(x) - g(x)) \, dx + \int_{b}^{c} (f(x) - g(x)) \, dx$

D. $\int_{a}^{d} f(x) \, dx + \int_{a}^{c} (f(x) - g(x)) \, dx + \int_{b}^{d} f(x) \, dx$

E. $\int_{0}^{d} f(x) \, dx - \int_{a}^{c} g(x) \, dx$
**Question 14**
A rectangle is formed by using part of the coordinate axes and a point \((u, v)\), where \(u > 0\) on the parabola \(y = 4 - x^2\).

Which one of the following is the maximum area of the rectangle?

A. 4  
B. \(\frac{2\sqrt{3}}{3}\)  
C. \(\frac{8\sqrt{3} - 4}{3}\)  
D. \(\frac{8}{3}\)  
E. \(\frac{16\sqrt{3}}{9}\)
Question 20
Consider the transformation $T$, defined as

$$T : R^2 \rightarrow R^2, T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

The transformation $T$ maps the graph of $y = f(x)$ onto the graph of $y = g(x)$. If $\int_{0}^{3} f(x) \, dx = 5$, then $\int_{-3}^{0} g(x) \, dx$ is equal to

A. 0  
B. 15  
C. 20  
D. 25  
E. 30
SECTION 1

Instructions for Section 1

Answer all questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is correct for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1
Let \( f : \mathbb{R} \to \mathbb{R}, f(x) = 2\sin(3x) - 3. \)
The period and range of this function are respectively
A. period = \( \frac{2\pi}{3} \) and range = \([-5, -1]\]
B. period = \( \frac{2\pi}{3} \) and range = \([-2, 2]\]
C. period = \( \frac{\pi}{3} \) and range = \([-1, 5]\]
D. period = \( 3\pi \) and range = \([-1, 5]\]
E. period = \( 3\pi \) and range = \([-2, 2]\]

Question 2
The inverse function of \( f : (-2, \infty) \to \mathbb{R}, f(x) = \frac{1}{\sqrt{x + 2}} \) is
A. \( f^{-1} : \mathbb{R}^+ \to \mathbb{R} \quad f^{-1}(x) = \frac{1}{x^2} - 2 \)
B. \( f^{-1} : \mathbb{R}\setminus\{0\} \to \mathbb{R} \quad f^{-1}(x) = \frac{1}{x^2} - 2 \)
C. \( f^{-1} : \mathbb{R}^+ \to \mathbb{R} \quad f^{-1}(x) = \frac{1}{x^2} + 2 \)
D. \( f^{-1} : (-2, \infty) \to \mathbb{R} \quad f^{-1}(x) = x^2 + 2 \)
E. \( f^{-1} : (2, \infty) \to \mathbb{R} \quad f^{-1}(x) = \frac{1}{x^2} - 2 \)
Question 3

The rule for a function with the graph above could be
A. \( y = -2(x + b)(x - c)^2(x - d) \)
B. \( y = 2(x + b)(x - c)^2(x - d) \)
C. \( y = -2(x - b)(x - c)^2(x - d) \)
D. \( y = 2(x - b)(x - c)(x - d) \)
E. \( y = -2(x - b)(x + c)^2(x + d) \)

Question 4
Consider the tangent to the graph of \( y = x^2 \) at the point (2, 4).
Which of the following points lies on this tangent?
A. (1, -4)
B. (3, 8)
C. (-2, 6)
D. (1, 8)
E. (4, -4)
Question 5
Part of the graph of \( y = f(x) \) is shown below.

The corresponding part of the graph of the inverse function \( y = f^{-1}(x) \) is best represented by

A.  
\[ 
\begin{array}{c}
\text{A.} \\
\begin{array}{c}
\text{y} \\
\text{x}
\end{array}
\end{array}
\]

B.  
\[ 
\begin{array}{c}
\text{B.} \\
\begin{array}{c}
\text{y} \\
\text{x}
\end{array}
\end{array}
\]

C.  
\[ 
\begin{array}{c}
\text{C.} \\
\begin{array}{c}
\text{y} \\
\text{x}
\end{array}
\end{array}
\]

D.  
\[ 
\begin{array}{c}
\text{D.} \\
\begin{array}{c}
\text{y} \\
\text{x}
\end{array}
\end{array}
\]

E.  
\[ 
\begin{array}{c}
\text{E.} \\
\begin{array}{c}
\text{y} \\
\text{x}
\end{array}
\end{array}
\]
Question 6
For the polynomial \( P(x) = x^3 - ax^2 - 4x + 4 \), \( P(3) = 10 \), the value of \( a \) is
A. \(-3\)  
B. \(-1\)  
C. 1  
D. 3  
E. 10

Question 7
The range of the function \( f: (–1, 2] \rightarrow \mathbb{R}, \ f(x) = –x^2 + 2x – 3 \) is
A. \( \mathbb{R} \)  
B. \((-6, -3]\)  
C. \((-6, -2]\)  
D. \([-6, -3]\)  
E. \([-6, -2]\)

Question 8
The graph of a function \( f: [-2, p] \rightarrow \mathbb{R} \) is shown below.

\[
\begin{align*}
\text{(This means that the value of the definite integral from -2 to p of the function } f(x) \text{ is equal to zero)}
\end{align*}
\]

The average value of \( f \) over the interval \([-2, p]\) is zero.  
The area of the shaded region is \( \frac{25}{8} \).  
If the graph is a straight line, for \( 0 \leq x \leq p \), then the value of \( p \) is
A. 2  
B. 5  
C. \( \frac{5}{4} \)  
D. \( \frac{5}{2} \)  
E. \( \frac{25}{4} \)
Question 11
The transformation that maps the graph of \( y = \sqrt[3]{8x^3 + 1} \) onto the graph of \( y = \sqrt[3]{x^3 + 1} \) is a

A. dilation by a factor of 2 from the \( y \)-axis.
B. dilation by a factor of 2 from the \( x \)-axis.
C. dilation by a factor of \( \frac{1}{2} \) from the \( x \)-axis.
D. dilation by a factor of 8 from the \( y \)-axis.
E. dilation by a factor of \( \frac{1}{2} \) from the \( y \)-axis.
Question 15
If \( \int_0^5 g(x)\,dx = 20 \) and \( \int_0^5 (2g(x) + ax)\,dx = 90 \), then the value of \( a \) is
A. 0
B. 4
C. 2
D. –3
E. 1

Question 16
Let \( f(x) = ax^m \) and \( g(x) = bx^n \), where \( a, b, m \) and \( n \) are positive integers. The domain of \( f = \) domain of \( g = \mathbb{R} \). If \( f'(x) \) is an antiderivative of \( g(x) \), then which one of the following must be true?
A. \( \frac{m}{n} \) is an integer
B. \( \frac{n}{m} \) is an integer
C. \( \frac{a}{b} \) is an integer
D. \( \frac{b}{a} \) is an integer
E. \( n - m = 2 \)

Question 17
A graph with rule \( f(x) = x^3 - 3x^2 + c \), where \( c \) is a real number, has three distinct \( x \)-intercepts. The set of all possible values of \( c \) is
A. \( \mathbb{R} \)
B. \( \mathbb{R}^+ \)
C. \{0, 4\}
D. (0, 4)
E. (–\( \infty \), 4)
Question 19
If \( f(x) = \int_0^x (\sqrt{t^2 + 4}) \, dt \), then \( f'(-2) \) is equal to

A. \( \sqrt{2} \)
B. \( -\sqrt{2} \)
C. \( 2\sqrt{2} \)
D. \( -2\sqrt{2} \)
E. \( 4\sqrt{2} \)

Question 20
If \( f(x - 1) = x^2 - 2x + 3 \), then \( f(x) \) is equal to

A. \( x^2 - 2 \)
B. \( x^2 + 2 \)
C. \( x^2 - 2x + 2 \)
D. \( x^2 - 2x + 4 \)
E. \( x^2 - 4x + 6 \)

Question 21
The graphs of \( y = mx + c \) and \( y = ax^2 \) will have no points of intersection for all values of \( m, c \) and \( a \) such that

A. \( a > 0 \) and \( c > 0 \)
B. \( a > 0 \) and \( c < 0 \)
C. \( a > 0 \) and \( c > -\frac{m^2}{4a} \)
D. \( a < 0 \) and \( c > -\frac{m^2}{4a} \)
E. \( m > 0 \) and \( c > 0 \)
SECTION 1

Instructions for Section 1

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Question 1
The point $P (4, -3)$ lies on the graph of a function $f$. The graph of $f$ is translated four units vertically up and then reflected in the $y$-axis. The coordinates of the final image of $P$ are

A. $(-4, 1)$
B. $(-4, 3)$
C. $(0, -3)$
D. $(4, -6)$
E. $(-4, -1)$

Question 2
The linear function $f : D \rightarrow R$, $f(x) = 4 - x$ has range $[-2, 6)$. The domain $D$ of the function is

A. $[-2, 6)$
B. $(-2, 2]$  
C. $R$
D. $(-2, 6)$
E. $[-6, 2]$

Question 3
The area of the region enclosed by the graph of $y = x(x + 2)(x - 4)$ and the $x$-axis is

A. $\frac{128}{3}$
B. $\frac{20}{3}$
C. $\frac{236}{3}$
D. $\frac{148}{3}$
E. $36$
Question 4
Let $f$ be a function with domain $\mathbb{R}$ such that $f'(5) = 0$ and $f'(x) < 0$ when $x \neq 5$.
At $x = 5$, the graph of $f$ has a
A. local minimum.
B. local maximum.
C. gradient of 5.
D. gradient of $-5$.
E. stationary point of inflection.

Question 5
The random variable $X$ has a normal distribution with mean 12 and standard deviation 0.5.
If $Z$ has the standard normal distribution, then the probability that $X$ is less than 11.5 is equal to
A. $\Pr(Z > -1)$
B. $\Pr(Z < -0.5)$
C. $\Pr(Z > 1)$
D. $\Pr(Z \geq 0.5)$
E. $\Pr(Z < 1)$

Question 6
The function $f : D \rightarrow \mathbb{R}$ with rule $f(x) = 2x^3 - 9x^2 - 168x$ will have an inverse function for
A. $D = \mathbb{R}$
B. $D = (7, \infty)$
C. $D = (-4, 8)$
D. $D = (-\infty, 0)$
E. $D = \left[-\frac{1}{2}, \infty\right)$
Question 8

If \( \int_{1}^{4} f(x) \, dx = 6 \), then \( \int_{1}^{4} (5 - 2f(x)) \, dx \) is equal to

A. 3 
B. 4 
C. 5 
D. 6 
E. 16
Question 9
The inverse of the function \( f : \mathbb{R}^+ \rightarrow \mathbb{R}, \; f(x) = \frac{1}{\sqrt{x}} + 4 \) is

A. \( f^{-1} : (4, \infty) \rightarrow \mathbb{R} \; \quad f^{-1}(x) = \frac{1}{(x - 4)^2} \)

B. \( f^{-1} : \mathbb{R}^+ \rightarrow \mathbb{R} \; \quad f^{-1}(x) = \frac{1}{x^2} + 4 \)

C. \( f^{-1} : \mathbb{R}^+ \rightarrow \mathbb{R} \; \quad f^{-1}(x) = (x + 4)^2 \)

D. \( f^{-1} : (-4, \infty) \rightarrow \mathbb{R} \; \quad f^{-1}(x) = \frac{1}{(x + 4)^2} \)

E. \( f^{-1} : (-\infty, 4) \rightarrow \mathbb{R} \; \quad f^{-1}(x) = \frac{1}{(x - 4)^2} \)

Question 10
Which one of the following functions satisfies the functional equation \( f(f(x)) = x \) for every real number \( x \)?

A. \( f(x) = 2x \)

B. \( f(x) = x^2 \)

C. \( f(x) = 2\sqrt{x} \)

D. \( f(x) = x - 2 \)

E. \( f(x) = 2 - x \)
**Question 12**

The transformation $T: R^2 \rightarrow R^2$ with rule

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

maps the line with equation $x - 2y = 3$ onto the line with equation

A. $x + y = 0$
B. $x + 4y = 0$
C. $-x - y = 4$
D. $x + 4y = -6$
E. $x - 2y = 1$

**Question 13**

The domain of the function $h$, where $h(x) = \cos(\log_a(x))$ and $a$ is a real number greater than 1, is chosen so that $h$ is a one-to-one function.

Which one of the following could be the domain?

A. $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$
B. $(0, \pi)$
C. $\left[1, \frac{\pi}{2}\right]$  
D. $\left[\frac{\pi}{2}, \frac{\pi}{2}\right]$  
E. $\left[\frac{\pi}{2}, \frac{\pi}{2}\right]$
Question 15
Zoe has a rectangular piece of cardboard that is 8 cm long and 6 cm wide. Zoe cuts squares of side length \( x \) centimetres from each of the corners of the cardboard, as shown in the diagram below.

![Diagram of cardboard with squares cut from corners]

Zoe turns up the sides to form an open box.

The value of \( x \) for which the volume of the box is a maximum is closest to
A. 0.8  
B. 1.1  
C. 1.6  
D. 2.0  
E. 3.6

Question 16
The continuous random variable \( X \), with probability density function \( p(x) \), has mean 2 and variance 5.

The value of \( \int_{-\infty}^{\infty} x p(x) \, dx \) is
A. 1  
B. 7  
C. 9  
D. 21  
E. 29

Question 17
The simultaneous linear equations \( ax - 3y = 5 \) and \( 3x - ay = 8 - a \) have no solution for
A. \( a = 3 \)  
B. \( a = -3 \)  
C. both \( a = 3 \) and \( a = -3 \)  
D. \( a \in \mathbb{R}\{3\} \)  
E. \( a \in \mathbb{R}[{-3, 3}] \)
Question 18
The graph of \( y = kx - 4 \) intersects the graph of \( y = x^2 + 2x \) at two distinct points for
A. \( k = 6 \)
B. \( k > 6 \) or \( k < -2 \)
C. \( -2 \leq k \leq 6 \)
D. \( 6 - 2\sqrt{3} \leq k \leq 6 + 2\sqrt{3} \)
E. \( k = -2 \)

Question 19
Jake and Anita are calculating the area between the graph of \( y = x \) and the y-axis between \( y = 0 \) and \( y = 4 \). Jake uses a partitioning, shown in the diagram below, while Anita uses a definite integral to find the exact area.

The difference between the results obtained by Jake and Anita is
A. 0
B. \( \frac{22}{3} \)
C. \( \frac{26}{3} \)
D. 14
E. 35
Question 21
The trapezium $ABCD$ is shown below. The sides $AB$, $BC$ and $DA$ are of equal length, $p$. The size of the acute angle $BCD$ is $x$ radians.

The area of the trapezium is a maximum when the value of $x$ is

A. $\frac{\pi}{12}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

E. $\frac{5\pi}{12}$