## SECTION A – Multiple-choice questions

### Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1**

The linear function $f: D \to R,\ f(x) = 5 - x$ has range [–4, 5).
The domain $D$ is

**A.** (0, 9]
**B.** (0, 1]
**C.** [5, –4)
**D.** [–9, 0)
**E.** [1, 9)

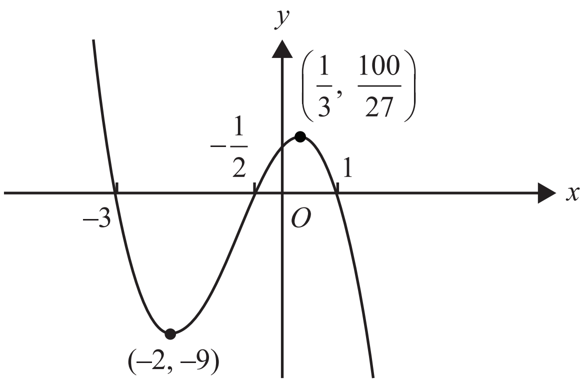
**Question 2**

Let $f: R \to R,\ f(x) = 1 - 2\cos\left(\frac{\pi x}{2}\right)$.
The period and range of this function are respectively

**A.** 4 and [–2, 2]
**B.** 4 and [–1, 3]
**C.** 1 and [–1, 3]
**D.** $4\pi$ and [–1, 3]
**E.** $4\pi$ and [–2, 2]

**Question 3**

Part of the graph $y = f(x)$ of the polynomial function $f$ is shown below.




$f'(x) < 0$ for

**A.** $x \in (-2, 0) \cup \left(\frac{1}{3}, \infty\right)$

**B.** $x \in \left(-9, \frac{100}{27}\right)$

**C.** $x \in (-\infty, -2) \cup \left(\frac{1}{3}, \infty\right)$

**D.** $x \in \left(-2, \frac{1}{3}\right)$

**E.** $x \in (-\infty, -2] \cup (1, \infty)$

**Question 4**

The average rate of change of the function $f$ with rule $f(x) = 3x^2 - 2\sqrt{x+1}$, between $x = 0$ and $x = 3$, is

**A.** 8

**B.** 25

**C.** $\frac{53}{9}$

**D.** $\frac{25}{3}$

**E.** $\frac{13}{9}$

**Question 5**

Which one of the following is the inverse function of $g: [3, \infty) \to R,\ g(x) = \sqrt{2x - 6}$ ?

**A.** $g^{-1}: [3, \infty) \to R,\ g^{-1}(x) = \dfrac{x^2 + 6}{2}$

**B.** $g^{-1}: [0, \infty) \to R,\ g^{-1}(x) = (2x - 6)^2$

**C.** $g^{-1}: [0, \infty) \to R,\ g^{-1}(x) = \sqrt{\dfrac{x}{2} + 6}$

**D.** $g^{-1}: [0, \infty) \to R,\ g^{-1}(x) = \dfrac{x^2 + 6}{2}$

**E.** $g^{-1}: R \to R,\ g^{-1}(x) = \dfrac{x^2 + 6}{2}$

**Question 6**

Consider the graph of the function defined by $f: [0, 2\pi] \to R,\ f(x) = \sin(2x)$.

The square of the length of the line segment joining the points on the graph for which $x = \dfrac{\pi}{4}$ and $x = \dfrac{3\pi}{4}$ is

**A.** $\dfrac{\pi^2 + 16}{4}$

**B.** $\pi + 4$

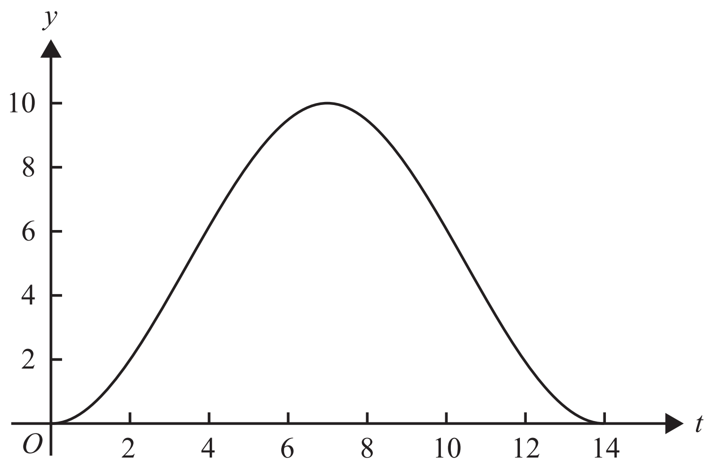
**C.** 4

**D.** $\dfrac{3\pi^2 + 16\pi}{4}$

**E.** $\dfrac{10\pi^2}{16}$

**Question 8**

The UV index, $y$, for a summer day in Melbourne is illustrated in the graph below, where $t$ is the number of hours after 6 am.




The graph is most likely to be the graph of

**A.** $y = 5 + 5\cos\left(\frac{\pi t}{7}\right)$

**B.** $y = 5 - 5\cos\left(\frac{\pi t}{7}\right)$

**C.** $y = 5 + 5\cos\left(\frac{\pi t}{14}\right)$

**D.** $y = 5 - 5\cos\left(\frac{\pi t}{14}\right)$

**E.** $y = 5 + 5\sin\left(\frac{\pi t}{14}\right)$

**Question 9**

Given that $\frac{d(xe^{kx})}{dx} = (kx+1)e^{kx}$, then $\int xe^{kx}dx$ is equal to

**A.** $\frac{xe^{kx}}{kx+1} + c$

**B.** $\left(\frac{kx+1}{k}\right)e^{kx} + c$

**C.** $\frac{1}{k}\int e^{kx}dx$

**D.** $\frac{1}{k}\left(xe^{kx} - \int e^{kx}dx\right) + c$

**E.** $\frac{1}{k^2}\left(xe^{kx} - e^{kx}\right) + c$

**Question 10**

For the curve $y = x^2 - 5$, the tangent to the curve will be parallel to the line connecting the positive $x$-intercept and the $y$-intercept when $x$ is equal to

**A.** $\sqrt{5}$

**B.** 5

**C.** –5

**D.** $\frac{\sqrt{5}}{2}$

**E.** $\frac{1}{\sqrt{5}}$

**Question 11**

The function $f$ has the property $f(x) - f(y) = (y - x)f(xy)$ for all non-zero real numbers $x$ and $y$.
Which one of the following is a possible rule for the function?

**A.** $f(x) = x^2$

**B.** $f(x) = x^2 + x^4$

**C.** $f(x) = x\log_e(x)$

**D.** $f(x) = \frac{1}{x}$

**E.** $f(x) = \frac{1}{x^2}$

**Question 12**

The graph of a function $f$ is obtained from the graph of the function $g$ with rule $g(x) = \sqrt{2x - 5}$ by a reflection in the $x$-axis followed by a dilation from the $y$-axis by a factor of $\frac{1}{2}$.
Which one of the following is the rule for the function $f$?

**A.** $f(x) = \sqrt{5 - 4x}$

**B.** $f(x) = -\sqrt{x - 5}$

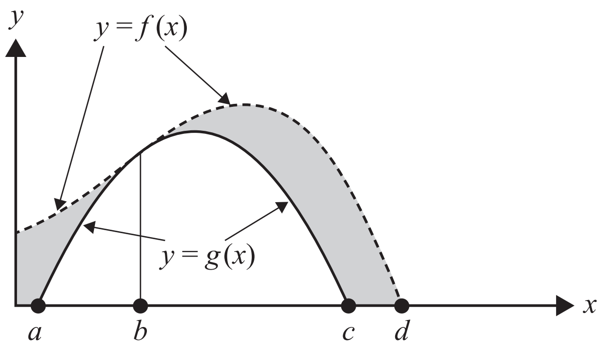
**C.** $f(x) = \sqrt{x + 5}$

**D.** $f(x) = -\sqrt{4x - 5}$

**E.** $f(x) = -\sqrt{4x - 10}$

**Question 13**

Consider the graphs of the functions $f$ and $g$ shown below.




The area of the shaded region could be represented by

**A.** $\int_a^d \left(f(x)-g(x)\right)dx$

**B.** $\int_0^d \left(f(x)-g(x)\right)dx$

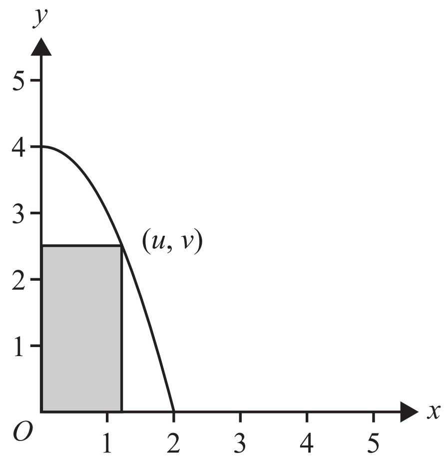
**C.** $\int_0^b \left(f(x)-g(x)\right)dx+\int_b^c \left(f(x)-g(x)\right)dx$

**D.** $\int_0^a f(x)dx+\int_a^c \left(f(x)-g(x)\right)dx+\int_b^d f(x)dx$

**E.** $\int_0^d f(x)dx-\int_a^c g(x)dx$

**Question 14**

A rectangle is formed by using part of the coordinate axes and a point $(u, v)$, where $u > 0$ on the parabola $y = 4 - x^2$.




Which one of the following is the maximum area of the rectangle?

**A.** 4

**B.** $\frac{2\sqrt{3}}{3}$

**C.** $\frac{8\sqrt{3}-4}{3}$

**D.** $\frac{8}{3}$

**E.** $\frac{16\sqrt{3}}{9}$

**Question 20**

Consider the transformation *T*, defined as

$$T : R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

The transformation *T* maps the graph of $y = f(x)$ onto the graph of $y = g(x)$.

If $\int_0^3 f(x)dx = 5$, then $\int_{-3}^0 g(x)dx$ is equal to

**A.** 0
**B.** 15
**C.** 20
**D.** 25
**E.** 30

**END OF SECTION A**

## SECTION 1

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**Question 1**

Let $f: R \to R, f(x) = 2\sin(3x) - 3$.

The period and range of this function are respectively

**A.** period = $\frac{2\pi}{3}$ and range = $[-5, -1]$

**B.** period = $\frac{2\pi}{3}$ and range = $[-2, 2]$

**C.** period = $\frac{\pi}{3}$ and range = $[-1, 5]$

**D.** period = $3\pi$ and range = $[-1, 5]$

**E.** period = $3\pi$ and range = $[-2, 2]$

**Question 2**

The inverse function of $f: (-2, \infty) \to R,\ f(x) = \frac{1}{\sqrt{x+2}}$ is

**A.** $f^{-1}: R^+ \to R$ $\quad f^{-1}(x) = \frac{1}{x^2} - 2$

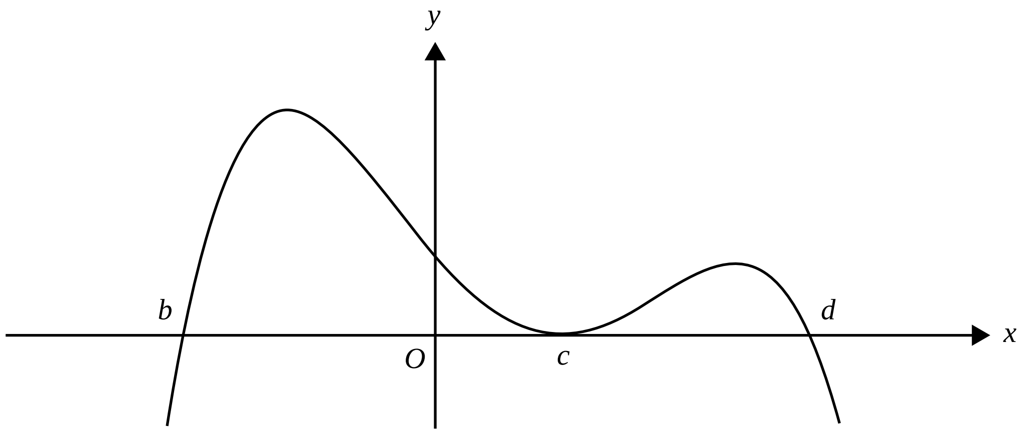
**B.** $f^{-1}: R\backslash\{0\} \to R$ $\quad f^{-1}(x) = \frac{1}{x^2} - 2$

**C.** $f^{-1}: R^+ \to R$ $\quad f^{-1}(x) = \frac{1}{x^2} + 2$

**D.** $f^{-1}: (-2, \infty) \to R$ $\quad f^{-1}(x) = x^2 + 2$

**E.** $f^{-1}: (2, \infty) \to R$ $\quad f^{-1}(x) = \frac{1}{x^2 - 2}$

**Question 3**




The rule for a function with the graph above could be

**A.** $y = -2(x + b)(x - c)^2(x - d)$

**B.** $y = 2(x + b)(x - c)^2(x - d)$

**C.** $y = -2(x - b)(x - c)^2(x - d)$

**D.** $y = 2(x - b)(x - c)(x - d)$

**E.** $y = -2(x - b)(x + c)^2(x + d)$

**Question 4**

Consider the tangent to the graph of $y = x^2$ at the point (2, 4).

Which of the following points lies on this tangent?

**A.** (1, –4)

**B.** (3, 8)

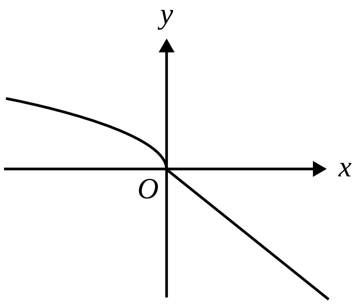
**C.** (–2, 6)

**D.** (1, 8)

**E.** (4, –4)

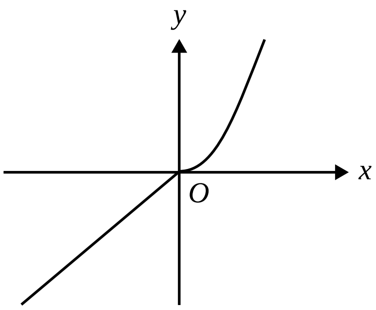
**Question 5**

Part of the graph of $y = f(x)$ is shown below.

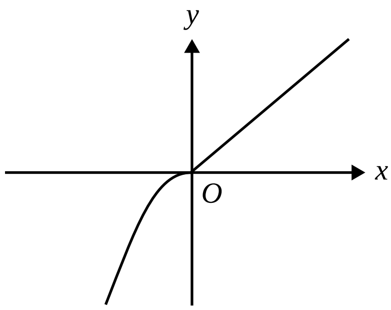



The corresponding part of the graph of the inverse function $y = f^{-1}(x)$ is best represented by

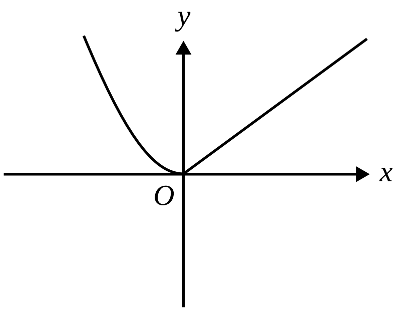
**A.**



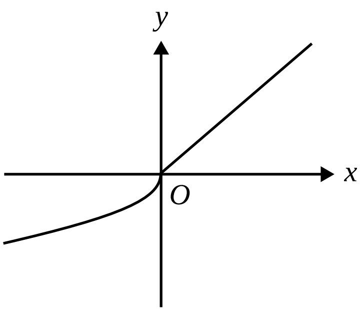

**B.**



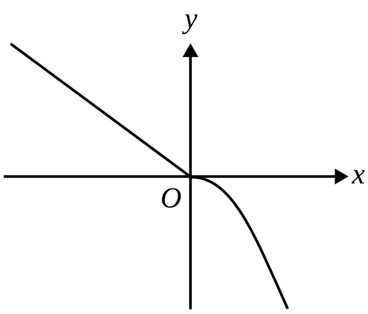

**C.**




**D.**




**E.**



**Question 6**

For the polynomial $P(x) = x^3 - ax^2 - 4x + 4$, $P(3) = 10$, the value of $a$ is

**A.** $-3$

**B.** $-1$

**C.** $1$

**D.** $3$

**E.** $10$

**Question 7**

The range of the function $f: (-1, 2] \rightarrow R$, $f(x) = -x^2 + 2x - 3$ is

**A.** $R$

**B.** $(-6, -3]$

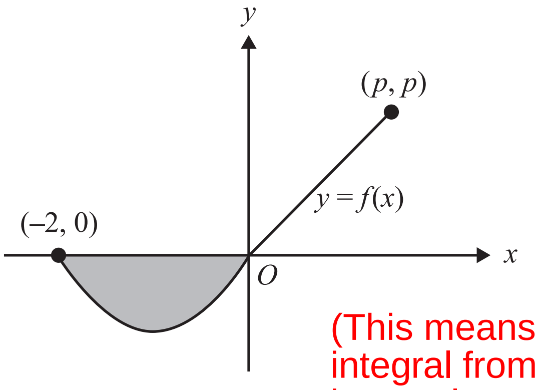
**C.** $(-6, -2]$

**D.** $[-6, -3]$

**E.** $[-6, -2]$

**Question 8**

The graph of a function $f: [-2, p] \rightarrow R$ is shown below.




(This means that the value of the definite integral from -2 to p of the function f(x) is equal to zero)

The average value of $f$ over the interval $[-2, p]$ is zero.

The area of the shaded region is $\frac{25}{8}$.

If the graph is a straight line, for $0 \le x \le p$, then the value of $p$ is

**A.** $2$

**B.** $5$

**C.** $\frac{5}{4}$

**D.** $\frac{5}{2}$

**E.** $\frac{25}{4}$

**Question 11**

The transformation that maps the graph of $y = \sqrt{8x^3 + 1}$ onto the graph of $y = \sqrt{x^3 + 1}$ is a

**A.** dilation by a factor of 2 from the *y*-axis.

**B.** dilation by a factor of 2 from the *x*-axis.

**C.** dilation by a factor of $\frac{1}{2}$ from the *x*-axis.

**D.** dilation by a factor of 8 from the *y*-axis.

**E.** dilation by a factor of $\frac{1}{2}$ from the *y*-axis.

**Question 15**

If $\int_0^5 g(x)dx = 20$ and $\int_0^5 \left(2g(x) + ax\right)dx = 90$, then the value of $a$ is

**A.** 0

**B.** 4

**C.** 2

**D.** –3

**E.** 1

**Question 16**

Let $f(x) = ax^m$ and $g(x) = bx^n$, where $a$, $b$, $m$ and $n$ are positive integers. The domain of $f$ = domain of $g = R$.
If $f'(x)$ is an antiderivative of $g(x)$, then which one of the following must be true?

**A.** $\frac{m}{n}$ is an integer

**B.** $\frac{n}{m}$ is an integer

**C.** $\frac{a}{b}$ is an integer

**D.** $\frac{b}{a}$ is an integer

**E.** $n - m = 2$

**Question 17**

A graph with rule $f(x) = x^3 - 3x^2 + c$, where $c$ is a real number, has three distinct $x$-intercepts.
The set of all possible values of $c$ is

**A.** $R$

**B.** $R^+$

**C.** $\{0, 4\}$

**D.** $(0, 4)$

**E.** $(-\infty, 4)$

**Question 19**

If $f(x) = \int_0^x \left(\sqrt{t^2 + 4}\right) dt$, then $f'(-2)$ is equal to

**A.** $\sqrt{2}$

**B.** $-\sqrt{2}$

**C.** $2\sqrt{2}$

**D.** $-2\sqrt{2}$

**E.** $4\sqrt{2}$

**Question 20**

If $f(x-1) = x^2 - 2x + 3$, then $f(x)$ is equal to

**A.** $x^2 - 2$

**B.** $x^2 + 2$

**C.** $x^2 - 2x + 2$

**D.** $x^2 - 2x + 4$

**E.** $x^2 - 4x + 6$

**Question 21**

The graphs of $y = mx + c$ and $y = ax^2$ will have no points of intersection for all values of $m$, $c$ and $a$ such that

**A.** $a > 0$ and $c > 0$

**B.** $a > 0$ and $c < 0$

**C.** $a > 0$ and $c > -\dfrac{m^2}{4a}$

**D.** $a < 0$ and $c > -\dfrac{m^2}{4a}$

**E.** $m > 0$ and $c > 0$

## SECTION 1

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**Question 1**

The point $P$ (4, –3) lies on the graph of a function $f$. The graph of $f$ is translated four units vertically up and then reflected in the $y$-axis.

The coordinates of the final image of $P$ are

**A.** (–4, 1)
**B.** (–4, 3)
**C.** (0, –3)
**D.** (4, –6)
**E.** (–4, –1)

**Question 2**

The linear function $f: D \to R, f(x) = 4 - x$ has range [–2, 6).

The domain $D$ of the function is

**A.** [–2, 6)
**B.** (–2, 2]
**C.** $R$
**D.** (–2, 6]
**E.** [–6, 2]

**Question 3**

The area of the region enclosed by the graph of $y = x(x+2)(x-4)$ and the $x$-axis is

**A.** $\frac{128}{3}$

**B.** $\frac{20}{3}$

**C.** $\frac{236}{3}$

**D.** $\frac{148}{3}$

**E.** 36

**Question 4**

Let $f$ be a function with domain $R$ such that $f'(5) = 0$ and $f'(x) < 0$ when $x \neq 5$.

At $x = 5$, the graph of $f$ has a

**A.** local minimum.

**B.** local maximum.

**C.** gradient of 5.

**D.** gradient of –5.

**E.** stationary point of inflection.

**Question 6**

The function $f : D \to R$ with rule $f(x) = 2x^3 - 9x^2 - 168x$ will have an inverse function for

**A.** $D = R$

**B.** $D = (7, \infty)$

**C.** $D = (-4, 8)$

**D.** $D = (-\infty, 0)$

**E.** $D = \left[-\frac{1}{2}, \infty\right)$

**Question 8**

If $\int_1^4 f(x)\,dx = 6$ , then $\int_1^4 \left(5 - 2f(x)\right)dx$ is equal to

**A.** 3
**B.** 4
**C.** 5
**D.** 6
**E.** 16

**Question 9**

The inverse of the function $f: R^+ \to R,\ f(x) = \dfrac{1}{\sqrt{x}} + 4$ is

**A.** $f^{-1}: (4, \infty) \to R \qquad f^{-1}(x) = \dfrac{1}{(x-4)^2}$

**B.** $f^{-1}: R^+ \to R \qquad f^{-1}(x) = \dfrac{1}{x^2} + 4$

**C.** $f^{-1}: R^+ \to R \qquad f^{-1}(x) = (x+4)^2$

**D.** $f^{-1}: (-4, \infty) \to R \qquad f^{-1}(x) = \dfrac{1}{(x+4)^2}$

**E.** $f^{-1}: (-\infty, 4) \to R \qquad f^{-1}(x) = \dfrac{1}{(x-4)^2}$

**Question 10**

Which one of the following functions satisfies the functional equation $f(f(x)) = x$ for every real number $x$?

**A.** $f(x) = 2x$

**B.** $f(x) = x^2$

**C.** $f(x) = 2\sqrt{x}$

**D.** $f(x) = x - 2$

**E.** $f(x) = 2 - x$

**Question 12**

The transformation $T: R^2 \rightarrow R^2$ with rule

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

maps the line with equation $x - 2y = 3$ onto the line with equation

**A.** $x + y = 0$

**B.** $x + 4y = 0$

**C.** $-x - y = 4$

**D.** $x + 4y = -6$

**E.** $x - 2y = 1$

**Question 13**

The domain of the function $h$, where $h(x) = \cos(\log_a(x))$ and $a$ is a real number greater than 1, is chosen so that $h$ is a one-to-one function.

Which one of the following could be the domain?

**A.** $\left(a^{-\frac{\pi}{2}}, a^{\frac{\pi}{2}}\right)$

**B.** $(0, \pi)$

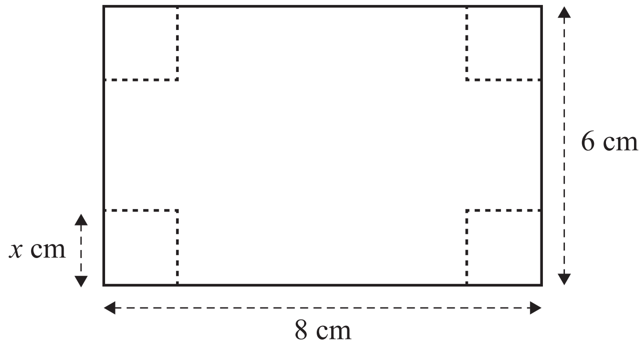
**C.** $\left[1, a^{\frac{\pi}{2}}\right]$

**D.** $\left[a^{-\frac{\pi}{2}}, a^{\frac{\pi}{2}}\right)$

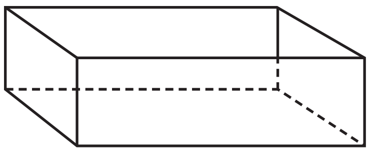
**E.** $\left[a^{-\frac{\pi}{2}}, a^{\frac{\pi}{2}}\right]$

**Question 15**

Zoe has a rectangular piece of cardboard that is 8 cm long and 6 cm wide. Zoe cuts squares of side length $x$ centimetres from each of the corners of the cardboard, as shown in the diagram below.




Zoe turns up the sides to form an open box.



The value of $x$ for which the volume of the box is a maximum is closest to

**A.** 0.8

**B.** 1.1

**C.** 1.6

**D.** 2.0

**E.** 3.6

**Question 17**

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 8 - a$ have **no solution** for

**A.** $a = 3$

**B.** $a = -3$

**C.** both $a = 3$ and $a = -3$

**D.** $a \in R\backslash\{3\}$

**E.** $a \in R\backslash[-3, 3]$

**Question 18**

The graph of $y = kx - 4$ intersects the graph of $y = x^2 + 2x$ at two distinct points for

**A.** $k = 6$

**B.** $k > 6$ or $k < -2$

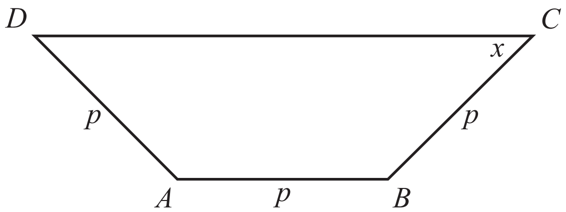
**C.** $-2 \leq k \leq 6$

**D.** $6 - 2\sqrt{3} \leq k \leq 6 + 2\sqrt{3}$

**E.** $k = -2$

**Question 21**

The trapezium *ABCD* is shown below. The sides *AB*, *BC* and *DA* are of equal length, *p*. The size of the acute angle *BCD* is *x* radians.




The area of the trapezium is a maximum when the value of *x* is

**A.** $\frac{\pi}{12}$

**B.** $\frac{\pi}{6}$

**C.** $\frac{\pi}{4}$

**D.** $\frac{\pi}{3}$

**E.** $\frac{5\pi}{12}$