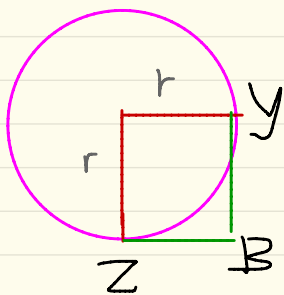
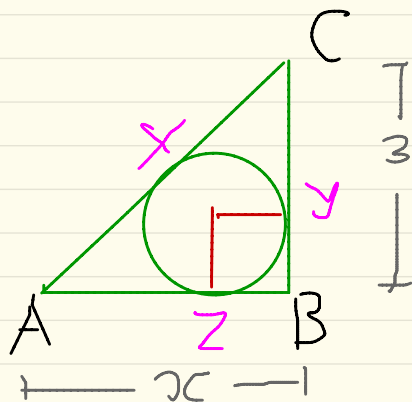


Ch 1 Review Solutions

Q 11.



From this "zoom"
we can see
that $YB = r$

$$ZB = r$$

(i) $YB = r$

(ii) $ZB = r$

(iii) $AZ = AB - r = x - r$

$$(iv) \quad CY = CB - r = 3 - r$$

$$(b) \quad CX = CX$$

$$\therefore CX = 3 - r$$

$$AX = AZ$$

$$\therefore AX = x - r$$

$$\text{But } AX + XC = AC$$

$$\therefore 3 - r + x - r = AC$$

$$x + 3 - 2r = AC$$

Using Pythagoras:

$$AC^2 = x^2 + 3^2$$

$$AC^2 = x^2 + 9$$

$$AC = \sqrt{x^2 + 9}$$

Equating the two expressions for AC we get:

$$\sqrt{x^2 + 9} = x + 3 - 2r$$

$$2r = x + 3 - \sqrt{x^2 + 9}$$

$$r = \frac{1}{2} (x + 3 - \sqrt{x^2 + 9})$$

$$\text{If } x = 4,$$

$$r = \frac{1}{2} \left(4 + 3 - \sqrt{4^2 + 3^2} \right)$$

$$r = \frac{1}{2} \left(7 - \sqrt{25} \right)$$

$$r = \frac{1}{2} \times (7 - 5)$$

$$r = 1$$

$$\text{If } r = 0.5,$$

$$0.5 = \frac{1}{2} \left(x + 3 - \sqrt{x^2 + 9} \right)$$

$$1 = x + 3 - \sqrt{x^2 + 9}$$

$$-2 - x = -\sqrt{x^2 + 9}$$

$$x + 2 = \sqrt{x^2 + 9}$$

$$(x + 2)^2 = x^2 + 9$$

$$x^2 + 4x + 4 = x^2 + 9$$

$$4x = 5$$

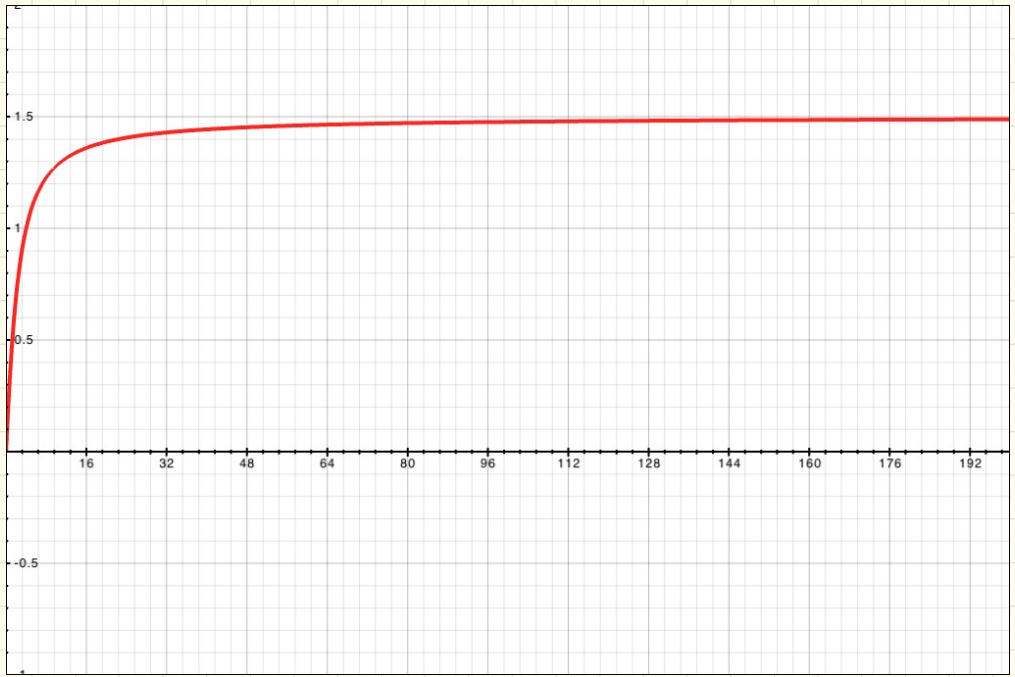
$$x = \frac{5}{4}$$

d.

Let

$$f(x) = \frac{1}{2} \left(x + 3 - \sqrt{x^2 + 9} \right)$$

Graph this function
on CAS for $x \geq 0$.



You will see that as x gets larger and larger, the value of the function $f(x)$ gets closer and closer to $y = 1.5$.

We can confirm this graphical observation by direct calculation as follows:

On CAS, define

$$f(x) = \frac{1}{2} (x + 3 - \sqrt{x^2 + 9})$$

Now evaluate $f(x)$ for large values of x :

$$f(100) = 1.47751$$

$$f(1000) = 1.49775$$

$$f(100000) = 1.49998$$

From these calculations
and from the graph
we conclude:

As $x \rightarrow \infty$

(as x increases without limit)

the value of

$$r = 0.5(x + 3 - \sqrt{x^2 + 9})$$

$\rightarrow 1.5$