SOLUTIONS for Q12

(a) $$f(x) = \frac{px+q}{x+r}$$

$$f(-x) = \frac{p(-x)+q}{-x+r}$$

$$= \frac{-px+q}{r-x}$$

We are asked to find under what circumstances:

$$f(x) = f(-x)$$

$$\therefore \frac{px+q}{x+r} = \frac{q-px}{r-x}$$

$$(px+q)(r-x) = (x+r)(q-px)$$

$$pxr - px^2 + qr - qx = qx - px^2 + rq - prx$$

$$2prx - 2qx = 0$$

$$2x(pr-q) = 0$$

This equation can only be true for ALL values of x if $pr - q = 0$

$$\therefore pr = q$$

But $f(x) = \dfrac{px+q}{x+r}$

Substitute $q = pr$

$$f(x) = \frac{px + pr}{x+r}$$

$$= \frac{p(x+r)}{x+r}$$

$= p$, as stated.

(b)

We are now asked to find the circumstances under which

$$f(-x) = -f(x)$$

$$\frac{q - px}{r - x} = \frac{-px - q}{x + r}$$

$$(x+r)(q-px) = (r-x)(-px-q)$$

$$qx - px^2 + rq - prx = -rpx - rq + px^2 + qx$$

$$0 = 2px^2 - 2qr$$

For this to be true INDEPENDENTLY of the value of x, we must have: $p = 0$

This gives $-2qr = 0$

Since we are required to find $f(x)$ in terms of q,

$-2qr = 0$ if $r = 0$

Thus, $f(x) = \dfrac{0.x + q}{x + 0}$

$\therefore f(x) = \dfrac{q}{x}$

(c) $f(x) = \dfrac{3x + 8}{x - 3}$

$f(x) = \dfrac{3(x-3)}{(x-3)} + \dfrac{9 + 8}{x - 3}$

$= 3 + \dfrac{17}{x-3}$

DOMAIN/RANGE TABLE

$\mathrm{dom}(f)$ $\mathbb{R}\setminus\{3\}$	$\mathrm{ran}(f)$ $\mathbb{R}\setminus\{3\}$
$\mathrm{dom}(f^{-1})$ $\mathbb{R}\setminus\{3\}$	$\mathrm{ran}(f^{-1})$ $\mathbb{R}\setminus\{3\}$

$$y = 3 + \frac{17}{x-3}$$

Interchange x and y:

$$x = 3 + \frac{17}{y-3}$$

$$x - 3 = \frac{17}{y-3}$$

$$y = 3 + \frac{17}{x-3}$$

The inverse function is:

$$f^{-1} : R \backslash \{3\} \to \mathbb{R},\ f^{-1}(x) = 3 + \frac{17}{x-3}$$

$$\therefore f^{-1}(x) = \frac{3x+8}{x-3}$$

$$= f(x)$$