SOLUTIONS FOR Q13

(a) $f(x) = \dfrac{x+1}{x-1}$

(i) $f(2) = \dfrac{2+1}{2-1} = \dfrac{3}{1} = \boxed{3}$

$f(f(2)) = f(3) = \dfrac{3+1}{3-1} = \dfrac{4}{2} = \boxed{2}$

$f(f(f(2)) = f(2) = 3$

(ii) $f(f(x)) = \dfrac{f(x)+1}{f(x)-1}$

$= \dfrac{\dfrac{x+1}{x-1} + 1}{\dfrac{x+1}{x-1} - 1}$

$$= \frac{\frac{x+1}{x-1} + \frac{x-1}{x-1}}{\frac{x+1}{x-1} - \frac{(x-1)}{x-1}}$$

$$= \frac{\frac{2x}{x-1}}{\frac{2}{x-1}}$$

$$= \frac{2x}{x-1} \times \frac{x-1}{2x}$$

$$= x$$

(b) Let $f(x) = \dfrac{x-3}{x+1}$

$$f(f(x)) = \frac{f(x)-3}{f(x)+1}$$

$$= \frac{\dfrac{x-3}{x+1} - 3}{\dfrac{x-3}{x+1} + 1}$$

$$= \frac{\dfrac{x-3}{x+1} \quad \dfrac{-3(x+1)}{x+1}}{\dfrac{x-3}{x+1} + \dfrac{x+1}{x+1}}$$

$$= \frac{x-3-3x-3}{x-3+x+1}$$

$$= \frac{-2x-6}{2x-2}$$

$$= \frac{-x-3}{x-1}$$

$$\therefore f(f(x)) = \frac{-x-3}{x-1}$$

$$f\Big(f\big(f(x)\big)\Big)$$

$$= f\left(\frac{-x-3}{x-1}\right)$$

$$= \frac{\frac{-x-3}{x-1} - 3}{\frac{-x-3}{x-1} + 1}$$

$$= \frac{\frac{-x-3}{x-1} - \frac{3(x-1)}{x-1}}{\frac{-x-3}{x-1} + \frac{x-1}{x-1}}$$

$$= \frac{-x-3-3x+3}{-x-3+x-1}$$

$$= \frac{-4x}{-4}$$

$$= x$$