EXTENDED RESPONSE – Ch. 17

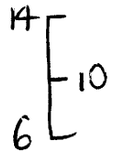
Q1.

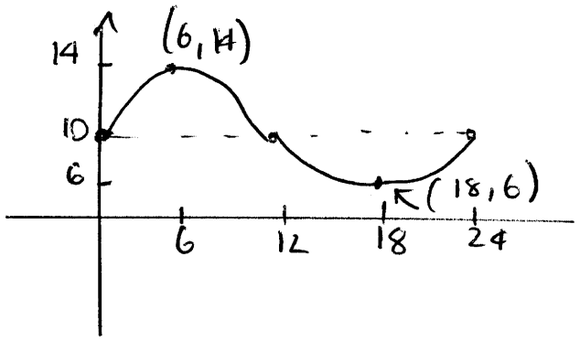
$$h(t) = 10 + 4\sin(15t)^\circ, \quad 0 \le t \le 24$$

(a) Period $= \frac{360}{15} = 24$

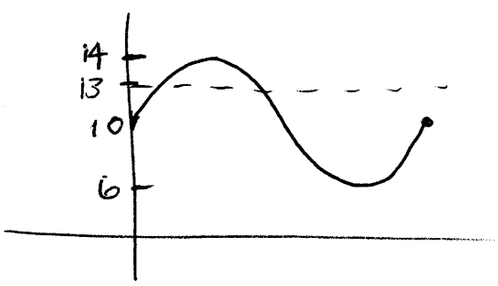
Quarter period $= 24 \div 4 = 6$

Range:





(b)



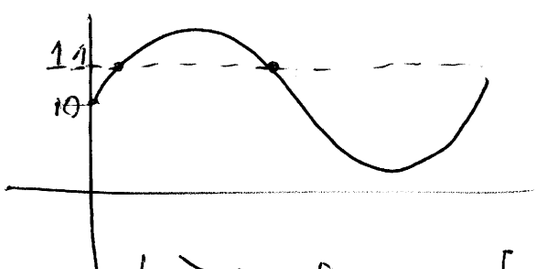
$13 = 10 + 4\sin(15t)^\circ$ (CAS must be in degree mode!!)

$\frac{3}{4} = \sin(15t)^\circ$

Solving for $0 \le t \le 24$ gives:

$t = 3.24, 8.76$

(c)



Solve: $11 = 10 + 4\sin(15t)$

$t = 0.97, 11.03$

$\therefore h \ge 11$ for $t \in [0.97, 11.03]$

Q2.

(a) $N(0) = 40 \times 2^0 = 40$

(b) (i) $N(2) = 40 \times 2^{1.5 \times 2}$

$= 40 \times 2^3$

$= 40 \times 8$

$= 320$

(ii) $N(4) = 40 \times 2^{1.5 \times 4}$

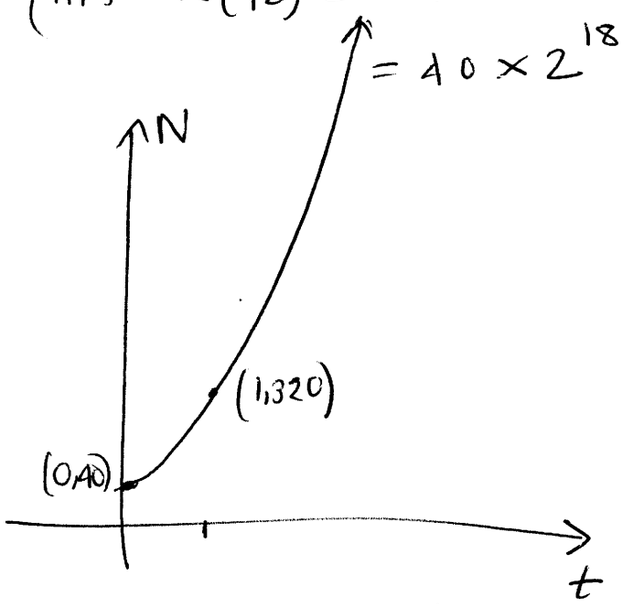
$= 40 \times 2^6$

$= 2560$

(iii) $N(12) = 40 \times 2^{1.5 \times 12}$

$= 40 \times 2^{18}$

(c)



(d) $80 = 40 \times 2^{1.5t}$

$\therefore \frac{80}{40} = 2^{1.5t}$

$2^1 = 2^{1.5t}$

$\therefore 1 = 1.5t$

$\therefore t = \frac{2}{3}$ $\therefore$ It takes $\frac{2}{3}$ hour

Q3. [Cont]

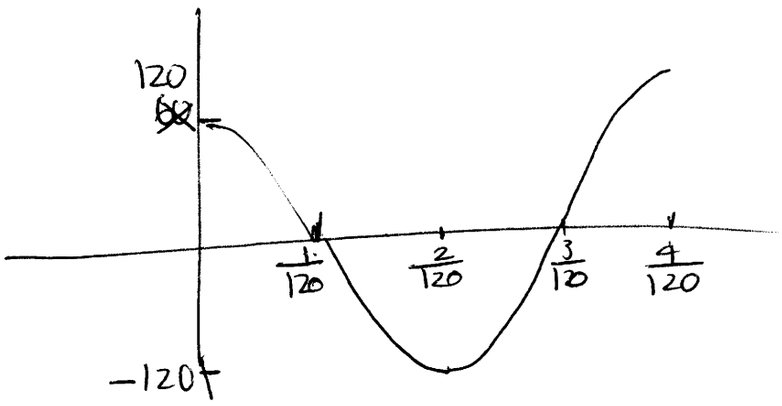
height of person is 15.5 m at $t = 0, 20, 60$.

Q4(a) $V = 120 \cos(60\pi t)$

Period $= \frac{2\pi}{60\pi} = \frac{1}{30}$ sec

Quarter period $= \frac{1}{120}$ sec

Amp $= 120$



(b) $60 = 120 \cos(60\pi t), \quad 0 \le t \le \frac{1}{30}$

$\frac{1}{2} = \cos(60\pi t)$

$\therefore 60\pi t = \frac{\pi}{3} \quad \therefore t = \frac{1}{180}$ sec

(c) At $t = 0, \frac{1}{30}, \frac{2}{30}, \frac{3}{30}, \ldots$

$\therefore$ Voltage is maximized if $t = \frac{k}{30}$, $k \in J$.

Q5.

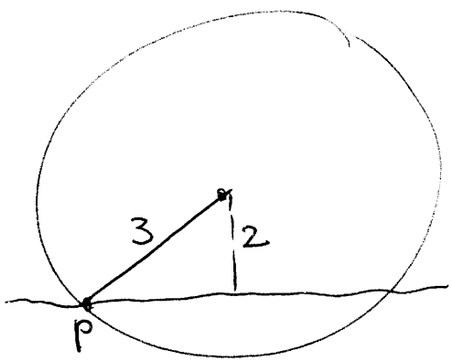
(a) (i) Since it makes 4 rev / minute, it makes 1 rev in 15 sec

$\therefore$ Period = 15 sec

(ii) $a = 3$

(iii) $15 = \frac{2\pi}{C} \quad \therefore C = \frac{2\pi}{15}$

(b)



When $t = 0, d = 0$

$$d = a + b\sin\left(\frac{2\pi}{15}(t-h)\right)$$

$a = 2, b = 3$

$$\therefore d = 2 + 3\sin\left(\frac{2\pi}{15}(t-h)\right)$$

When $t = 0, d = 0$

$$\therefore 0 = 2 + 3\sin\left(\frac{2\pi}{15}(0-h)\right)$$

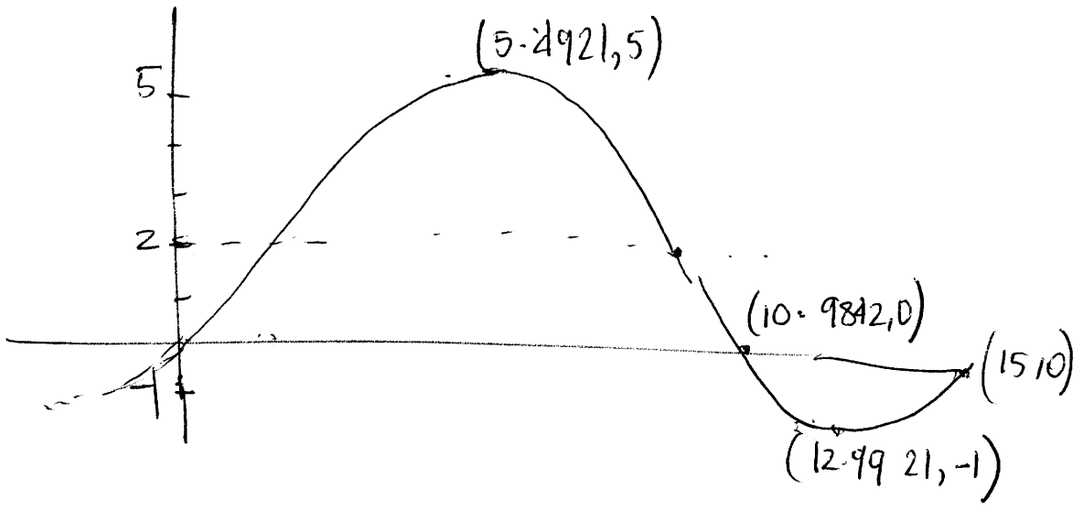
$$\therefore \frac{-2}{3} = \sin\left(\frac{2\pi}{15}(-h)\right)$$

Solving for $0 \leq h \leq 15$ gives:

$h = 1.7421, 5.7579.$

Taking the smaller value, $h = 1.7421$

Q5. (c)



Q6.

(a) (i) $h(0) = 30 \times 1.65^{0} = 30$

(ii) $h(1) = 30 \times 1.65 = 49.5$

(iii) $h(2) = 30 \times 1.65^{2} = 81.675$

(b) $h(N+1) = k\,h(N)$

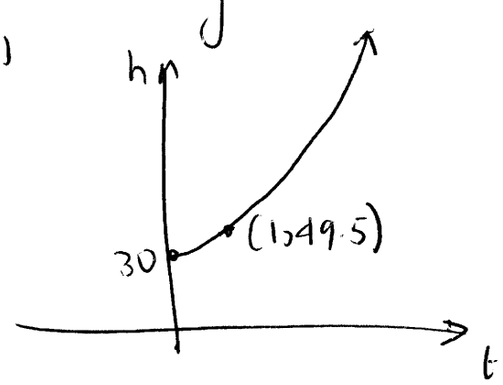
$\therefore 30 \times 1.65^{N+1} = k \times 30 \times 1.65^{N}$

$\therefore 1.65 = k$

(c) $900 = 30 \times 1.65^{x}$

Solving: $x = 6.792$

(d)



Q7.

(a) $\theta = 80 \times 2^{-t} + 20$

$\theta(0) = 80 \times 2^{0} + 20 = 80 + 20 = 100$

$\theta(1) = 60$

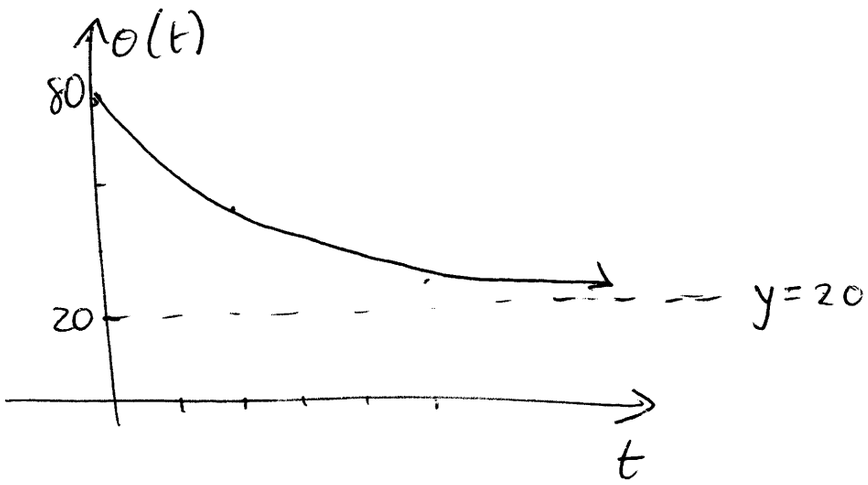
$\theta(2) = 40$

$\theta(3) = 30$

$\theta(4) = 25$

$\theta(5) = 22.5$

(b)



(c) $t = 1$

(d) $\theta(3.5) = 27.07$

Q8.(a) Let all functions be the values in millions:

For A: $A(t) = 70 + 30 \times \left(\frac{t}{10}\right)$

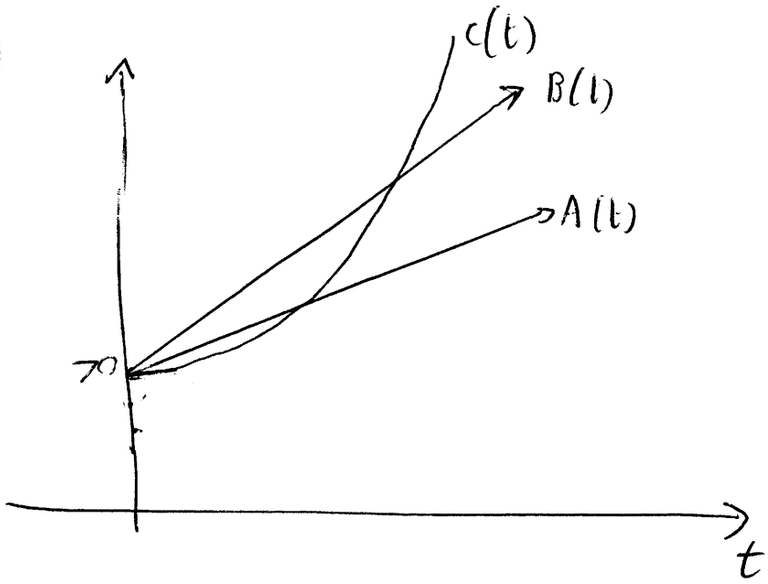
$A(t) = 70 + 3t$

For B: $B(t) = 70 + 50 \times \left(\frac{t}{10}\right)$

$\therefore B(t) = 70 + 5t$

For C: $C(t) = 70 \times 1.3^{\frac{t}{10}}$

(b)



(c) (1) $A(t) = c(t)$

$\therefore 70 + 3t = 70 \times 1.3^{t/10}$

Solving: $t = 34.78$ years

$B(t) = c(t)$

Solving: $70 + 5t = 70 \times 1.3^{t/10}$

$t = 66.82$ years

Q9.

$$P = 4 \times 2^{\frac{t-1975}{35}}$$

(a) (i) $P(1975) = 4 \times 2^0 = 4$ (population = 4 billion)

(ii) $P(1995) = 5.94$ (population = 5.94 billion)

(iii) $P(2005) = 7.25$ (population = 7.25 billion)

(b)

Defining $P(t) = 4 \times 2^{\frac{t-1975}{35}}$

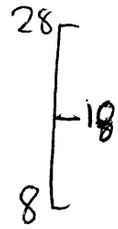
solve: $P(t) = 2 \times P(1997)$

$t = 2032$

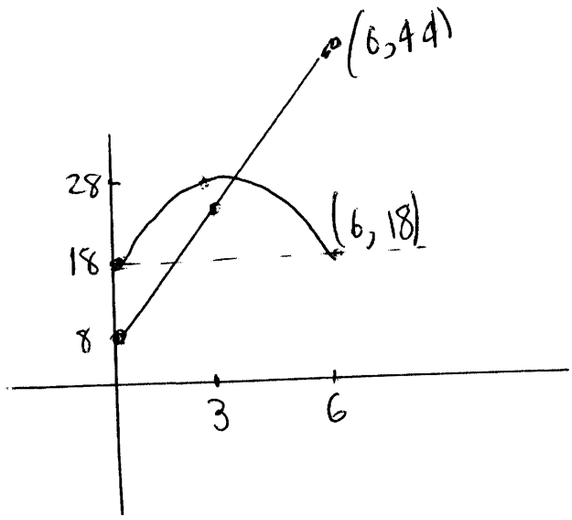
Q11. (a)

$$h_1(t) = 18 + 10\sin\left(\frac{\pi t}{6}\right)$$

Period $= \frac{2\pi}{\frac{\pi}{6}} = 12$



Range: $[8, 28]$

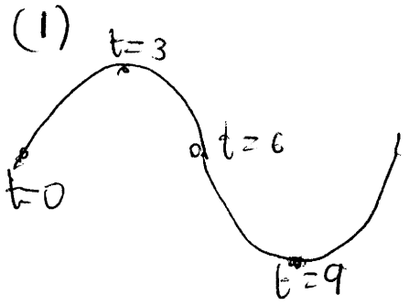


(b) $h_1(t) = h_2(t)$ Solving: $18 + 10\sin\left(\frac{\pi t}{6}\right) = 8 + 6t$,

where $0 \leq t \leq 6$.

$t = 3.31$

(c) (i)



Reaches a minimum at $t = 9$.

(ii) When the gate is closed, the height of the water is 2 m. On the non-tidal side water then flows in so that height increases by 6m every hour

$\therefore h = 2 + 6t$.