Section A: Multiple Choice

Question 1

The gradient of the tangent to the curve $y = g(x)$ at the point where $x = 7$ is given by:

A. $\frac{g(7+h)-g(7)}{h}$
B. $\lim_{h \to 0} g(7) - h$
C. $\lim_{h \to 0} \frac{g(7+h)}{h}$
D. $\lim_{h \to 0} \frac{g(7+h)-g(7)}{h}$
E. $\lim_{h \to 0} \frac{g(7+h)-g(7)}{7}$

Question 2

If $y = x^5 - 8x^3 + 12x - 10$ then $\frac{dy}{dx}$ is equal to:

A. $x^5 - 8x^3 + 12$
B. $5x^4 - 24x^2 + 12$
C. $5x^4 - 24x^2 + 12x - 10$
D. $5x^5 - 24x^2 + 12x$
E. $4x^4 - 23x^2 + 11$

$$\frac{dy}{dx} = 5x^4 - 24x^2 + 12$$

Question 3

If $f(x) = x^3(x+5)$ then $f'(1)$ is equal to:

A. 3
B. 6
C. 10
D. 15
E. 19

$$f(x) = x^4 + 5x^3$$
$$f'(x) = 4x^3 + 15x^2$$
$$\therefore f'(1) = 4 + 15 = 19$$

Question 4

If $y = \frac{8x^2+12x}{3x}$ then $\frac{dy}{dx}$ is equal to:

A. $\frac{16x+12}{3}$
B. $\frac{16x+12}{-3x^2}$
C. $\frac{8}{3}$
D. $\frac{8x}{3} + 4$
E. $\frac{8x^3+6x^2}{x^2}$

$$y = \frac{8x^2 + 12x}{3x}$$
$$\therefore y = \frac{x(8x+12)}{3x}$$
$$\therefore y = \frac{8x}{3} + 4 \qquad \therefore \frac{dy}{dx} = \frac{8}{3}$$

Question 5

The points on the curve with equation: $y = x^3 - 4x^2 + 8x + 2$, where the tangent is parallel to the line with equation $y = 4x$ have x co-ordinates equal to:

A. 2 and -2
B. 2 and $\frac{2}{3}$
C. 3 and $-\frac{1}{2}$
D. $-\frac{3}{2}$ and 1
E. $-\frac{2}{3}$ and -3

$$\frac{dy}{dx} = 4$$

$$\therefore 3x^2 - 8x + 8 = 4$$

$$3x^2 - 8x + 4 = 0$$

$$(3x-2)(x-2) = 0$$

$$\therefore x = \frac{2}{3}, 2$$

Question 6

Let $f(x) = \frac{2x^3+3x}{6x^4}$. The derivative function, $f'(x)$ is equal to:

A. $\frac{6x^2+3}{24x^3}$
B. $\frac{x^{-1}}{3} + \frac{x^{-3}}{2}$
C. $-\frac{1}{3x^2} - \frac{9}{2x^4}$
D. $\frac{5(x^4+x^2)}{12x^5}$
E. $-\frac{1}{3x^2} - \frac{3}{2x^4}$

$$f(x) = \frac{2x^3}{6x^4} + \frac{3x}{6x^4}$$

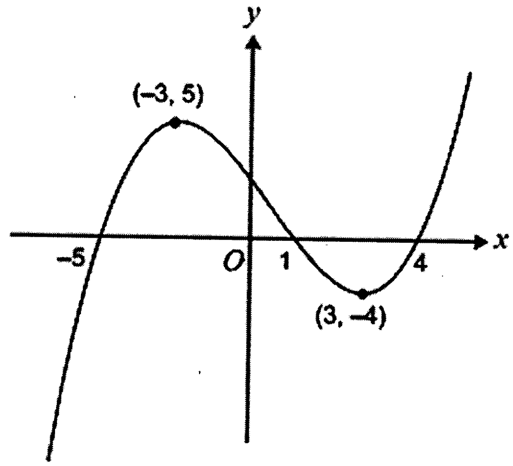
$$= \frac{1}{3x} + \frac{1}{2x^3}$$

$$= \frac{x^{-1}}{3} + \frac{x^{-3}}{2}$$

$$\therefore f'(x) = \frac{-x^{-2}}{3} - \frac{3x^{-4}}{2}$$

$$= -\frac{1}{3x^2} - \frac{3}{2x^4}$$

Question 7



For the graph of $y = f(x)$ shown above, $f'(x)$ is negative when

A. $-3 < x < 3$
B. $-3 \leq x \leq 3$
C. $x < -3$ or $x > 3$
D. $x \leq -3$ or $x \geq 3$
E. $-5 < x < 1$ or $x > 4$

Question 8

$\int 12x^3 - 1dx$ is equal to:

A. $3x^4 - x + c$
B. $3x^4 + c$
C. $36x^2$
D. $4x^3 - x + c$
E. $3x^4 + cx$

$$\int 12x^3 - 1\,dx$$

$$= \frac{12x^4}{4} - x + C$$

$$= 3x^4 - x + C$$

Question 9

The average rate of change of the function with rule $f(x) = x^3 - \sqrt{x+1}$ between $x = 0$ and $x = 3$ is

A. 0

B. 12

C. $\frac{26}{3}$

D. $\frac{25}{3}$

E. 8

$$\frac{f(3) - f(0)}{3 - 0}$$

$f(3) = 27 - \sqrt{4}$

$= 25$

$f(0) = 0 - \sqrt{1}$

$= -1$

$\therefore$ Av. rate of change $= \frac{25 - -1}{3} = \frac{26}{3}$

Question10

A function g with domain R has the following properties.

- $g'(x) = x^2 - 2x$
- the graph of $g(x)$ passes through the point $(1, 0)$

$g(x)$ is equal to

A. $2x - 2$

B. $\frac{x^3}{3} - x^2$

C. $\frac{x^3}{3} - x^2 + \frac{2}{3}$

D. $x^2 - 2x + 2$

E. $3x^3 - x^2 - 1$

$g(x) = \int x^2 - 2x \, dx$

$= \frac{x^3}{3} - x^2 + c$

$g(1) = 0$

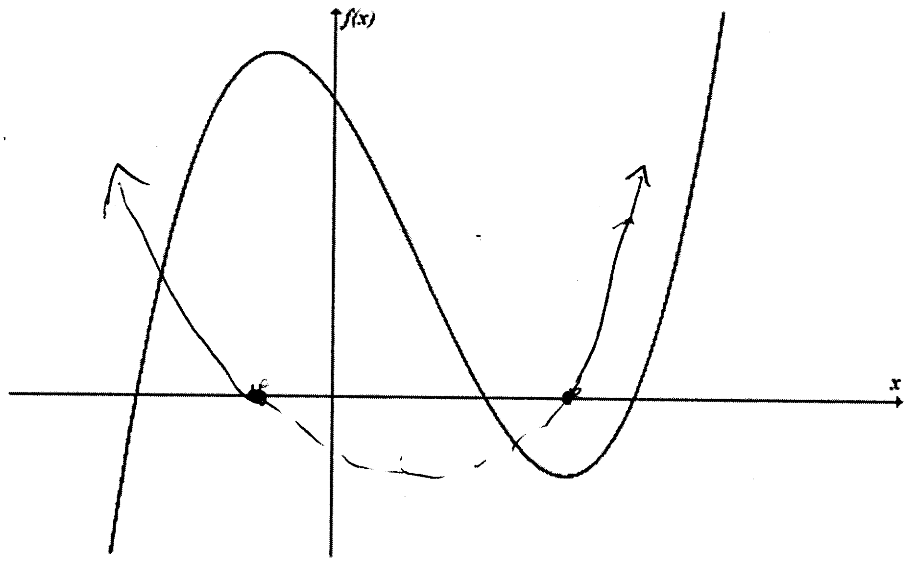
$\therefore 0 = \frac{1}{3} - 1 + c$

$0 = -\frac{2}{3} + c \quad \therefore c = \frac{2}{3}$

$\therefore g(x) = \frac{x^3}{3} - x^2 + \frac{2}{3}$

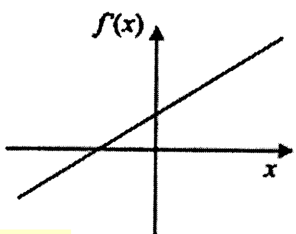
Question 11

The graph of $y = f(x)$ is shown.



The graph of the gradient function, $f'(x)$, could be

A



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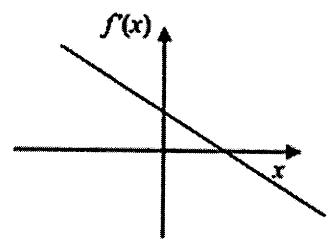

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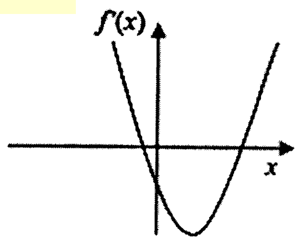

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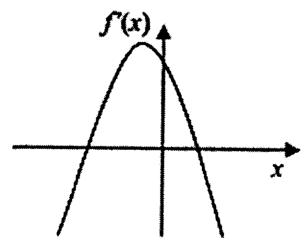

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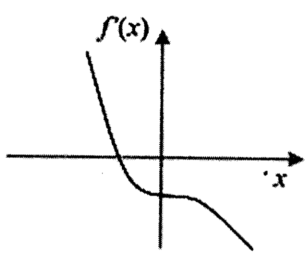
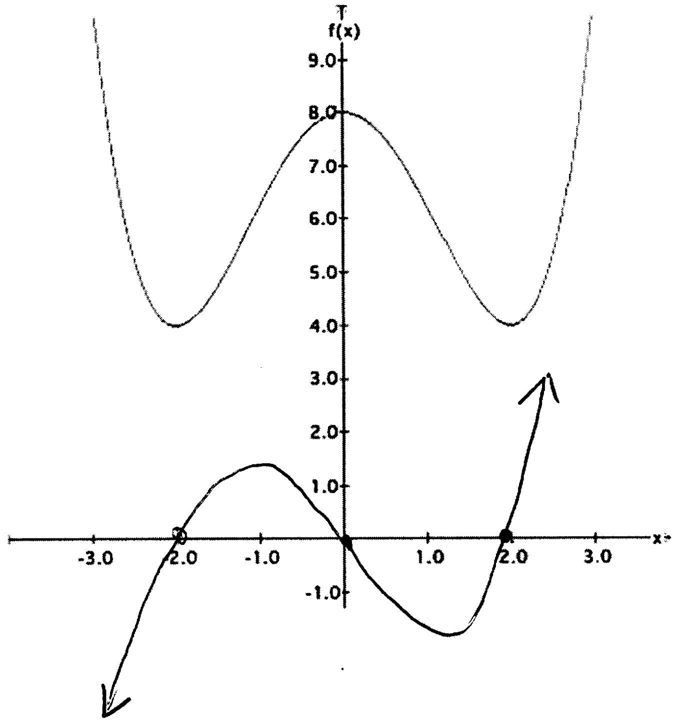


Section B: Short Answer Section

Question 1

Given the graph of the function $f(x)$ below. Sketch a graph to indicate the derivative of $f(x)$.

(2 marks)



Question 2

Use First Principles to differentiate the function $g(x) = x^2 - 8x$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 8(x+h) - (x^2 - 8x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 8x - 8h - x^2 + 8x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 8h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h - 8)}{h}$$

3 marks

$$= \lim_{h \to 0} (2x - 8 + h) = 2x - 8$$

Question 3

Given that: $f'(x) = (3x - 6)(2x + 6)$ and $f(-1) = 40$

a. Determine the function $f(x)$

$$f'(x) = 6x^2 + 6x - 36$$

$$\therefore f(x) = \int 6x^2 + 6x - 36 \, dx$$

$$= 2x^3 - 3x^2 - 36x + C$$

Since $f(-1) = 40$

$$40 = 2\times -1 - 3 + 36 + C$$

$$\therefore 40 = -2 - 3 + 36 + C$$

$$\therefore C = 9$$

$$\therefore f(x) = 2x^3 - 3x^2 - 36x + 9$$

b. Evaluate $f(1)$

$$f(1) = 2\times 1^3 - 3\times 1^2 - 36\times 1 + 9$$

$$= 2 - 3 - 36 + 9 = -28$$

4 marks

Question 4

Find the point(s) on the curve $y = x^3 - 3x$ where the gradient is equal to 24.

$$\frac{dy}{dx} = 24$$

$$\therefore 3x^2 - 3 = 24$$

$$3x^2 = 27$$

$$x^2 = 9$$

$$\therefore x = \pm 3$$

If $x = 3$, $y = 3^3 - 3\times 3 = 18$

If $x = -3$, $y = -27 + 9 = -18$

$\therefore (3, 18)$ and $(-3, -18)$

Question 5

The curve with equation $y = ax^2 + bx + 1$ has a gradient of 2 at the point (1, 0). Find the values of a and b

$$\left.\frac{dy}{dx}\right|_{x=1} = 2 \qquad \frac{dy}{dx} = 2ax + b$$

$$y = 0 \text{ at } x = 1$$

$$\therefore \quad 0 = a + b + 1 \quad (1)$$

$$2 = 2a + b \quad (2)$$

From (2), $b = 2 - 2a$

Sub. into (1):

$$0 = a + 2 - 2a + 1$$

$$\therefore 0 = -a + 3$$

$$\boxed{\begin{gathered}\therefore a = 3 \\ \therefore b = -4\end{gathered}}$$

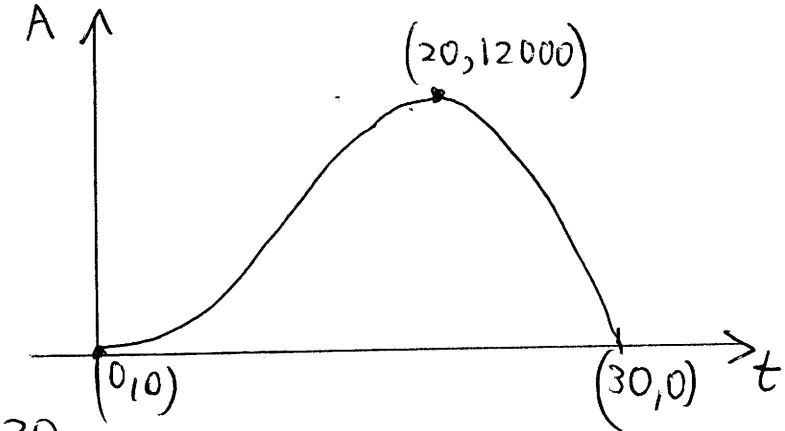
3 marks

Section C: Analysis Section

Question 1

The area of a bushfire, A hectares, t hours after the fire has started is modelled by the rule $A = 90t^2 - 3t^3$, $t > 0$.

a. Sketch a graph of the model over an appropriate domain, labelling all significant points.



$A(t) = 0$ at $t = 0, 30$

3 marks

b. Find $\frac{dA}{dt}$ for all values of t in the domain.

$$\frac{dA}{dt} = 180t - 9t^2$$

1 mark

c. Find the rate of change of the area of the bushfire at t equals:

i. 4

$$A'(4) = 180 \times 4 - 9 \times 4^2$$
$$= 576 \text{ ha/hour}$$

ii. 20

$$A'(20) = 180 \times 20 - 9 \times 20^2$$
$$= 0$$

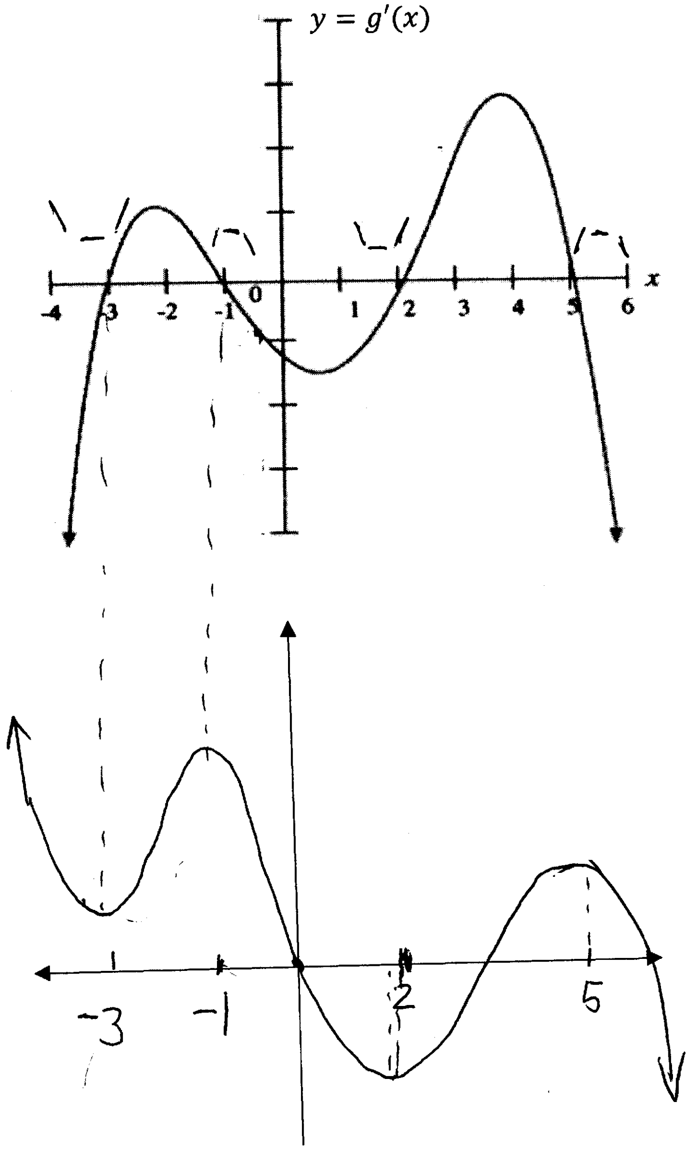
2 marks

d. Explain what happens after 20 hours

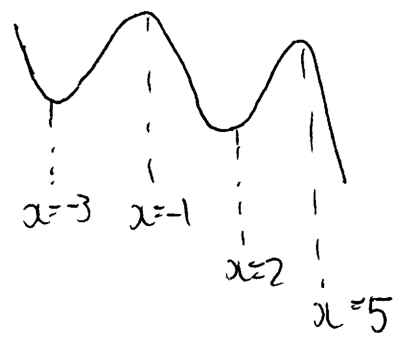
At $t=20$, the fire instantaneously stops increasing in area and just after $t=20$ begins to decrease in area, until $t=30$ when it is finally extinguished.

Question 2

Consider the function shown in the graph below. Given that this function is the derivative function $g'(x)$ of a function $g(x)$ and that $g(0) = 0$, sketch the graph of the function $g(x)$ on the set of axes underneath.



General shape:



The original graph has stationary points at $x = -3, -1, 2, 5$

3 marks