Section A: Multiple Choice

Circle the correct alternative.

Question 1

The discriminant of $x^2 + 5x - 10$ is

A 5

B −5

C −15

D 65

E $\sqrt{65}$

$a = 1,\ b = 5,\ c = -10$

$\Delta = b^2 - 4ac$

$= 5^2 - 4 \times 1 \times -10$

$= 65$

Question 2

The equation of the axis of symmetry of the graph of $y = 2x^2 - 12x + 17$ is

A. $x = 6$
B. $x = -6$
C. $x = 3$
D. $x = -3$
E. $x = 0$

$x = \frac{-b}{2a}$

$= \frac{12}{4} = 3$

Question 3

The equation $4x^2 + mx + 1 = 0$ will have two distinct real roots when the value(s) of m is/are

A. $m > \pm 4$

B. $m\ \{m: m > 4\} \cup \{m: m < -4\}$

C. $-4 < m < 4$

D. $m > 16$

E. $m = \pm 4$

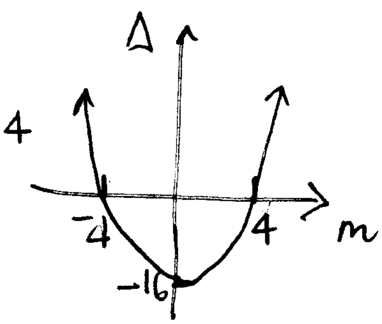
$\Delta = (m)^2 - 4 \times 4 \times 1$

$= m^2 - 16$

For 2 solutions,

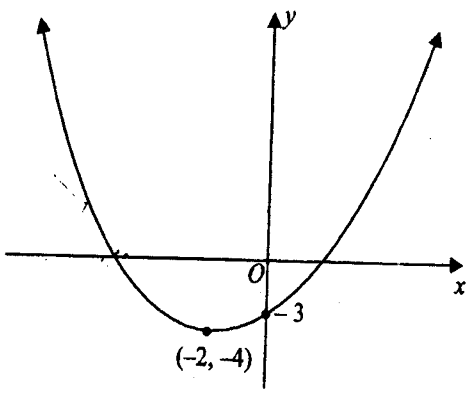
$\Delta > 0$

$\therefore m < -4$ or $m > 4$



Question 4

The following shows part of the graph of the curve with equation: $y = A(x+B)^2 - 4$



$y = a(x+2)^2 - 4$

$-3 = a(0+2)^2 - 4$

$-3 = 4a - 4$

$\therefore a = \frac{1}{4}$

The values of A and B are respectively:

$\therefore A = \frac{1}{4},\ B = 2$

	A	B
A.	$\frac{1}{4}$	-4
B.	4	2
C.	-2	4
D.	$-\frac{1}{4}$	2
E.	$\frac{1}{4}$	2

Question 5

The co-ordinates of the turning point of the parabola with equation: $y = -2x^2 - 28x - 1$ are:

A. (-7, 97)
B. (-7, -295)
~~C.~~ (7, -785)
~~D.~~ -7
~~E.~~ 7

$x = \frac{-b}{2a} = \frac{28}{-4}$

$= -7$

when $x = -7$,

$y = -2 \times 49 + 28 \times 7 - 1$

$= -98 + 196 - 1$

$= 97$

Questions 6 and 7 refer to the information below:

The line with equation $y = 3x + c$ touches the parabola $y = x^2 - 5x + 11$ in exactly one point.

Question 6

The value of c is

A. 7
B. 4
C. – 5
D. 5
E. 16

$3x + c = x^2 - 5x + 11$

$x^2 - 8x + 11 - c = 0$

$\Delta = 0$

$\therefore (-8)^2 - 4(11-c) = 0$

$64 - 44 + 4c = 0$

$20 + 4c = 0$

$\therefore c = -5$

Question 7

The point of contact between the line and the parabola is:

A. (-4, 47)
B. (7,4)
C. (4,0)
D. (4,7)
E. (5,11)

$x^2 - 8x + 16 = 0$

$(x-4)^2 = 0$

$\therefore x = 4$

Line: $y = 3x - 5$

If $x = 4$,

$y = 7$

$\therefore (4,7)$

Question 8

The equation of the parabola that passes through the point (0, 11) and has its vertex at (3, –7) is

A $y = 2(x + 3)^2 + 7$

B $y = (x + 3)^2 + 7$

C $y = (x + 3)^2 – 7$

D $y = 2(x – 3)^2 – 7$

E $y = 2(x – 3)^2 + 7$

$y = a(x-3)^2 - 7$

$11 = a(0-3)^2 - 7$

$18 = 9a$

$\therefore a = 2$

$\therefore y = 2(x-3)^2 - 7$

Question 9

The solution of the inequality $3x^2 \leq 5x$ is

A. $0 \leq x \leq \frac{3}{5}$

B. $x \geq \frac{5}{3}$

C. $x \geq 0$

D. $x \leq \frac{5}{3}$

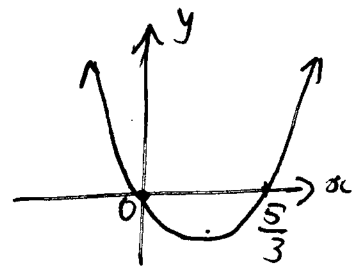
E. $0 \leq x \leq \frac{5}{3}$

$3x^2 - 5x \leq 0$

Critical points:

$x(3x - 5) = 0$

$x = 0, \frac{5}{3}$



$\therefore 3x^2 - 5x \leq 0$

for $0 \leq x \leq \frac{5}{3}$

Question 10

The parabola $y = x^2$ is dilated by a factor of 3 vertically, reflected in the x-axis and is then translated by 7 units in the negative y direction. The equation of the new parabola is:

A. $y = -3(x-7)^2$

B. $y = -3(x+7)^2$

C. $y = -3x^2 + 7$

D. $y = -3x^2 - 7$

E. $y = -7x^2 - 3$

$y = x^2$

$y = 3x^2$

$y = -3x^2$

$y = -3x^2 - 7$

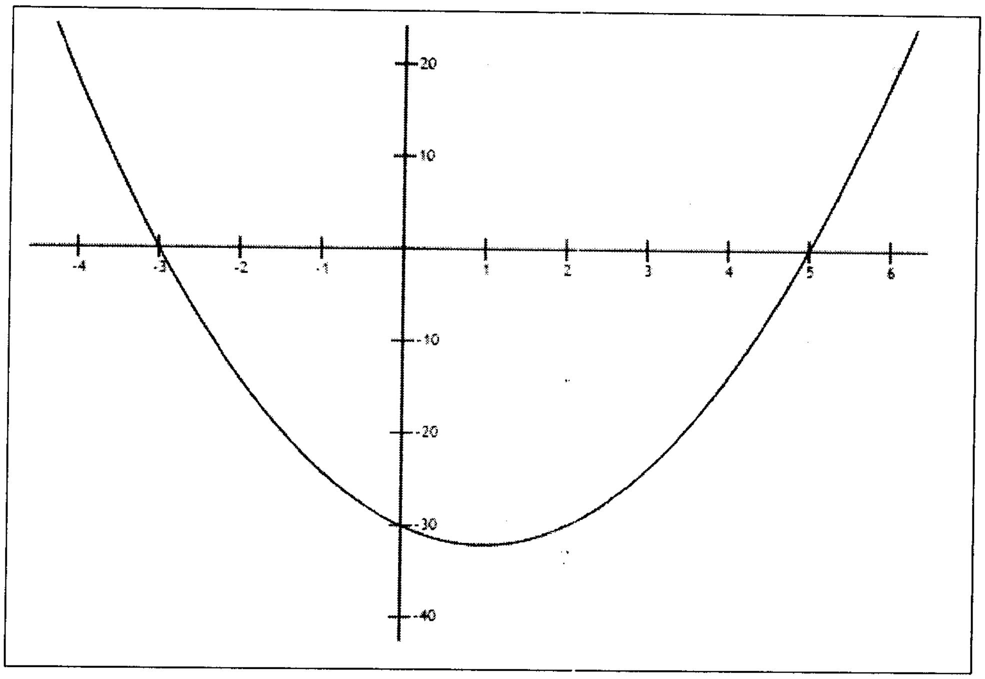
Section B

Short Answer Questions

To gain all the available marks, full working must be shown.

Question 1

Consider the parabola shown in the diagram below:



It has *x*-intercepts at (-3,0) and at (5,0), and a *y*-intercept at (0, -30).

a. Determine the equation of this parabola.

$$y = a(x+3)(x-5)$$

$$-30 = a \times 3 \times -5$$

$$a = 2$$

$$y = 2(x+3)(x-5)$$

b. Determine the co-ordinates of its turning point.

$$\frac{-3+5}{2} = 1 \qquad \therefore x = 1 \text{ is axis of symmetry}$$

When $x = 1$, $y = 2 \times 4 \times -4 = -32$

$\therefore$ T/p $= (1, -32)$

(2 + 1 = 3 marks)

Question 2

a. Use the Quadratic formula to find exact solutions of the equation:

$$3x^2 - 6x + 1 = 0$$

$a = 3$

$b = -6$

$c = 1$

$$x = \frac{6 \pm \sqrt{36 - 4 \times 3 \times 1}}{6}$$

$$x = \frac{6 \pm \sqrt{24}}{6}$$

$$x = \frac{6 \pm 2\sqrt{6}}{6}$$

$$x = \frac{3 \pm \sqrt{6}}{3}$$

3 marks

b. Sketch the graph of the curve: $y = 3x^2 - 6x + 1$, labelling the exact co-ordinates of all key points.

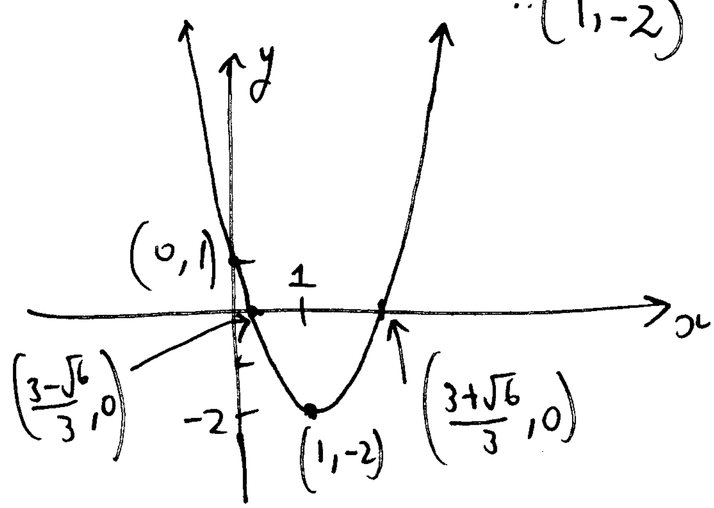
Let $x = 0$, $y = 1$ $\therefore (0,1)$ is y-int.

T/p : occurs at $x = 1$

When $x = 1$, $y = 3 \times 1^2 - 6 \times 1 + 1$

$= -2$

$\therefore (1,-2)$



2 marks

Question 3

The quadratic function: $y = ax^2 + bx + c$ passes through the point with co-ordinates (0, - 1).

a. Find the value of c.

$c = -1$

1 mark

b. It also passes through the two points with co-ordinates (1, 1) and (-2, -23). Determine two equations linking a and b from this information.

$y = ax^2 + bx - 1$

$1 = a + b - 1$

$\therefore \ 2 = a + b \quad (1)$

$-23 = a(-2)^2 + b(-2) - 1$

$\therefore -22 = 4a - 2b$

2 marks

$\therefore -11 = 2a - b \quad (2)$

c. Hence, find the values of a and b.

$2 = a + b$

$-11 = 2a - b$

$\therefore -9 = 3a \qquad \therefore a = -3$

$\therefore b = 5$

3 marks

Question 4

a. Complete the square for the quadratic function $y = -x^2 + 8x + 4$

$y = -(x^2 - 8x) + 4$

$= -(x^2 - 8x + 16 - 16) + 4$

$= -[(x-4)^2 - 16] + 4$

$= -(x-4)^2 + 16 + 4$

$= -(x-4)^2 + 20$

b. State the maximum value of y.

$y_{max} = 20.$

(3 + 1 = 4 marks)

Question 5

Consider the quadratic equation $x^2 + px + 2p = 0$.

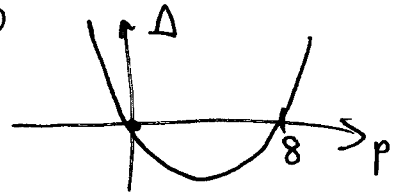
a Find the discriminant.

$$\Delta = p^2 - 4 \times 1 \times 2p$$
$$= p^2 - 8p$$

1 mark

b. i. Find the values of p for which there are 2 solutions.

$\Delta > 0$



$\therefore \Delta > 0$ if

$p > 8$ or $p < 0$

2 marks

ii. Find any value of p for which there are two rational solutions.

Δ must be a perfect square

Try: $p^2 - 8p = 9$

$p^2 - 8p - 9 = 0$

$(p-9)(p+1) = 0 \quad \therefore p = 9$

(or $p = -1$)

2 marks

c. Find the values of p for which there are no solutions.

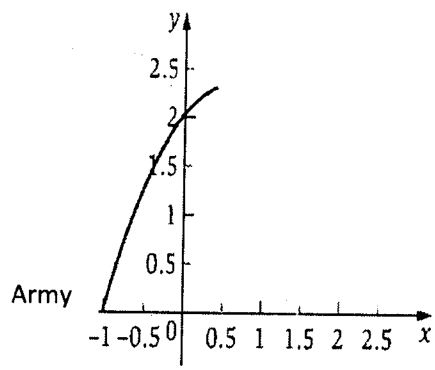
$\Delta < 0 \quad \therefore 0 < p < 8$

1 mark

Section C

Extended Response question

Two countries have a common border and are at war with each other. Take horizontal ground level to be the x-axis, and let the y-axis measure vertical height above the ground. The origin marks the position of the border between the two countries. One army is stationed 1 km to the left of the border at the point (-1, 0). This army fires a missile towards the enemy. The equation of the missile's path is : $y = -x^2 + bx + c$.
The missile passes 2 km vertically above the border. See the diagram below:



a. Find the values of b and c.

$y = -x^2 + bx + c$

$(0,2)$: $2 = c \quad \therefore c = 2$

$(-1,0)$: $0 = -1 - b + 2$

$\therefore 0 = 1 - b$

$\therefore b = 1$

3 marks

b. How far into enemy territory will the missile land?

$y = -x^2 + x + 2$

$= -(x^2 - x - 2)$

$y = -(x-2)(x+1)$

$y = 0$ if $x = 2, -1$

$\therefore$ Lands 2km into enemy territory.

1 mark

c. What is the greatest height (in kilometers) reached by the missile?

Axis of symmetry : $x = \frac{-b}{2a}$

$= \frac{-1}{-2} = \frac{1}{2}$

1 mark

When $x = \frac{1}{2}$,

$y = -\frac{1}{4} + \frac{1}{2} + 2 = 2\frac{1}{4}$ $\quad \therefore$ Max height $= \frac{9}{4}$ km

Moments after the first missile is fired, peace was declared between the two countries. As a result, just after the first missile was fired, a second missile was fired to destroy the first one. The trajectory of the second missile is a parabola $y = ax^2 + bx + c$ which passes through the points with co-ordinates: $(1, \frac{6}{5})$, $\left(\frac{3}{2}, \frac{5}{4}\right)$ and $\left(3, \frac{4}{5}\right)$.

d. Determine the values of a, b and c.

$$\frac{6}{5} = a + b + c$$
$$\frac{5}{4} = \frac{9a}{4} + \frac{3b}{2} + c$$
$$\frac{4}{5} = 9a + 3b + c$$

Solving: $a = -\frac{1}{5}$, $b = \frac{3}{5}$, $c = \frac{4}{5}$

4 marks

e. The second missile was launched from a point $(k,0)$ on the x-axis where $k > 0$. Determine the value of k.

$$y = -\frac{1}{5}x^2 + \frac{3}{5}x + \frac{4}{5}$$
$$y = -\frac{1}{5}(x^2 - 3x - 4)$$
$$= -\frac{1}{5}(x+1)(x-4)$$
$$= 0 \text{ if } x = -1, 4$$

Since $k > 0$,
$k = 4$

1 mark

f. Find the co-ordinates of the point where the two missiles collide.

$$y = -x^2 + x + 2$$
$$y = -\frac{1}{5}(x+1)(x-4)$$

Solving: $-x^2 + x + 2 = -\frac{1}{5}(x+1)(x-4)$

gives: $x = -1, \frac{3}{2}$

Must intersect In air. $\therefore x = \frac{3}{2}$

$\therefore$ [illegible] $\left(\frac{3}{\ }, \frac{5}{\ }\right)$

1 mark