EXTENDED RESPONSE, Ch.9 Review

Q1 (a) To calculate $(f \circ g)'(1)$ we must first find an expression for $(f \circ g)'(x)$.

By chain rule, $(f \circ g)'(x) = \frac{d}{dx}(f(g(x)))$

$$= f'(g(x)) \times g'(x)$$

$$\therefore (f \circ g)'(1) = f'(g(1)) \times g'(1)$$

$$= f'(-1) \times -2$$

$$= 2 \times -2$$

$$= -4$$

(ii) $(g \circ f)'(1)$

By Chain Rule: $\frac{d}{dx}(g \circ f) = \frac{d}{dx}(g(f(x))$

$$= g'(f(x)) \times f'(x)$$

$$\therefore (g \circ f)'(1) = g'(f(1)) \times f'(1)$$

$$= g'(6) \times 6$$

$$= -1 \times 6 = -6$$

(iii) $(fg)'(1)$

By Product Rule: $\frac{d}{dx}(fg) = f'g + g'f$

$$\therefore (fg)'(1) = f'(1) \times g(1) + g'(1) \times f(1)$$

$$= 6 \times -1 + -2 \times 6$$

$$= -18$$

(iv) $(gf)'(1) = (fg)'(1) = -18$

(v) $\left(\frac{f}{g}\right)'(1)$

By Quotient Rule: $\left(\frac{f}{g}\right)'(x) = \frac{gf' - fg'}{g^2}$

$$\therefore \left(\frac{f}{g}\right)'(1) = \frac{g(1) \times f'(1) - f(1) \times g'(1)}{[g(1)]^2}$$

$$= \frac{-1 \times 6 - 6 \times -2}{(-1)^2}$$

$$= \frac{-6 - -12}{1} = 6$$

(vi) $\left(\frac{g}{f}\right)'(1)$ By Quotient Rule:

$$\left(\frac{g}{f}\right)'(x) = \frac{fg' - gf'}{f^2}$$

$$\therefore \left(\frac{g}{f}\right)'(1) = \frac{f(1) \times g'(1) - g(1) \times f'(1)}{[f(1)]^2}$$

$$= \frac{-6}{(6)^2}$$

$$= \frac{-1}{6}$$

Q 1 (b)

$$f(x) = ax^3 + bx^2 + cx + d$$

Given: $f(1) = 6$, $f(-1) = 8$, $f'(1) = 6$, $f'(-1) = 2$

$$f'(x) = 3ax^2 + 2bx + c$$

$\therefore$ $6 = a + b + c + d$ (1)

$8 = -a + b - c + d$ (2)

$6 = 3a + 2b + c$ (3)

$2 = 3a - 2b + c$ (4)

Solving on CAS gives:

$a = \frac{5}{2}$, $b = 1$, $c = \frac{-7}{2}$, $d = 6$