Ch. 9 Review EXTENDED RESPONSE

Q4 (a)

$$f(x) = \frac{x^n}{x^n+1}$$

Now, $1 - \frac{1}{x^n+1} = \frac{x^n+1}{x^n+1} - \frac{1}{x^n+1}$

$$= \frac{x^n}{x^n+1}$$

$$= f(x)$$

(b)

* Let $y = 1 - \frac{1}{x^n+1}$

Let $u = x^n + 1$ $\quad \therefore y = 1 - \frac{1}{u}$

$\frac{du}{dx} = nx^{n-1}$ $\quad \frac{dy}{du} = \frac{1}{u^2}$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= nx^{n-1} \times \frac{1}{u^2}$$

$$= \frac{nx^{n-1}}{(x^n+1)^2}$$

(c)

$f(x) = 1 - \frac{1}{x^n+1}$ where n is an even positive integer

$\therefore x^n + 1 \geq 1$ for all $x \in \mathbb{R}$

$\therefore \frac{1}{x^n+1} \leq 1$ for all $x \in \mathbb{R}$

But $\frac{1}{x^n+1}$ is always positive

$\therefore 0 < \frac{1}{x^n+1} \leq 1$

$\therefore 0 > -\frac{1}{x^n+1} \geq -1$

$$\therefore 1 > 1 - \frac{1}{x^n+1} \geq 0.$$

$$\therefore 1 > f(x) \geq 0 \text{ , as stated.}$$

(d) $f'(x) = 0$ if:

$$\frac{n x^{n-1}}{1+x^n} = 0$$

$$\therefore n x^{n-1} = 0$$

$$\therefore x^{n-1} = 0$$

$$\therefore x = 0.$$

(e) If n is even, $n-1$ is odd

$$f'(x) > 0 \text{ if } x^{n-1} > 0$$

$$\therefore x > 0$$

~~$f'(x)$~~

(f) $$f(x) = 1 - \frac{1}{x^n + 1}$$

$$f(-x) = 1 - \frac{1}{(-x)^n + 1}$$

But since n is even, $(-x)^n = x^n$

$$\therefore f(-x) = 1 - \frac{1}{x^n + 1}$$

$$= f(x)$$

$\therefore f(x)$ is an even function.