SHORT ANSWER, Ch. 9 Review.

Q4. $y = \left(4x + \frac{9}{x}\right)^2$

$$\therefore \frac{dy}{dx} = 2\left(4x + \frac{9}{x}\right)^1 \times \left(4 - \frac{9}{x^2}\right)$$

$$= 2\left(4 - \frac{9}{x^2}\right)\left(4x + \frac{9}{x}\right)$$

$\frac{dy}{dx} = 0$ ~~at~~ if:

$$2\left(4 - \frac{9}{x^2}\right)\left(4x + \frac{9}{x}\right) = 0$$

$\therefore 4 - \frac{9}{x^2} = 0$ or $4x + \frac{9}{x} = 0$

$\therefore 4 = \frac{9}{x^2}$ $\quad$ $4x + \frac{9}{x} = 0$

$4x^2 + 9 = 0$

$x^2 = \frac{9}{4}$ $\quad$ This has no solution

$\therefore x = \pm \frac{3}{2}$

$\therefore \frac{dy}{dx} = 0$ at $x = \pm \frac{3}{2}$

Q5 (a) $y = \dfrac{2x-3}{x^2+4}$

$u = 2x-3 \qquad v = x^2+4$

$u' = 2 \qquad v' = 2x$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

$$= \frac{2(x^2+4) - 2x(2x-3)}{(x^2+4)^2}$$

$$= \frac{2x^2+8-4x^2+6x}{(x^2+4)^2}$$

$$= \frac{-2x^2+6x+8}{(x^2+4)^2}$$

(b) We require: $y>0$ and $\dfrac{dy}{dx}>0$

$y>0$ if $\dfrac{2x-3}{x^2+4}>0 \quad \therefore 2x-3>0$

$\therefore x > \frac{3}{2}$ ①

$\dfrac{dy}{dx}>0$ if: $\dfrac{-2x^2+6x+8}{(x^2+4)^2} > 0$

$\therefore -2x^2+6x+8>0$

$\therefore -x^2+3x+4>0$

$\therefore x^2-3x-4<0$

$\therefore (x-4)(x+1)<0$

$\therefore x: -1<x<4$ ②

Combining ① and ②,

the required values of x are: $\frac{3}{2}<x<4$.

Q 6.

(a) Let $y = x f'(x)$ $u = x$ $v = f(x)$

$u' = 1$ $v' = f'(x)$

$$\frac{dy}{dx} = uv' + vu'$$
$$= x f'(x) + f(x)$$

(b) $|f(x)| = f(x)$ if $f(x) \geq 0$ $\therefore x \in [0, 4]$

But at $x = 0, 4$, the function $y = |f(x)|$ will not be differentiable.

$|f(x)| = -f(x)$ if $x < 0 \cup x > 4$

$\therefore$ The derivative of $|f(x)|$ is:

$$\frac{d}{dx}\left(|f(x)|\right) = f'(x), \quad x \in (0, 4)$$
$$= -f'(x), \quad x \in (-\infty, 0) \cup (4, \infty)$$

(c) Let $y = \dfrac{x^2}{[f(x)]^2}$ $u = x^2$ $v = [f(x)]^2$

$u' = 2x$ $v' = 2f(x) \cdot f'(x)$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

$$= \frac{2x(f(x))^2 - 2x^2 f(x) \cdot f'(x)}{[f(x)]^4}$$

$$= \frac{2x}{[f(x)]^2} - \frac{2x^2 f'(x)}{[f(x)]^3}$$