SOLUTIONS to Q7.

(a) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x+1$

$g : \mathbb{R} \to \mathbb{R}$, $g(x) = 2 + x^3$

$\text{dom}(f) = \mathbb{R}$ $\quad$ $\text{ran}(f) = \mathbb{R}$

$\text{dom}(g) = \mathbb{R}$ $\quad$ $\text{ran}(g) = \mathbb{R}$

$g \circ f = g(f(x))$

is defined because

$$\text{ran}(f) \subseteq \text{dom}(g)$$

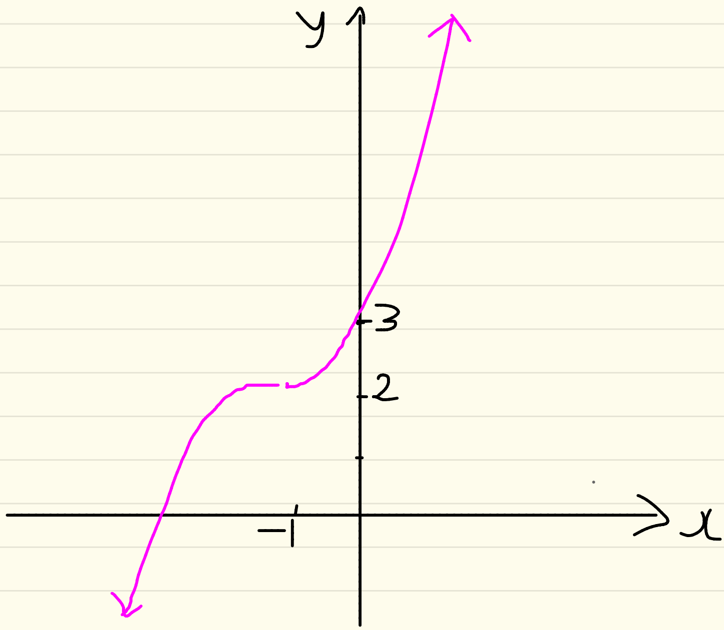
(In fact, $\text{ran}(f) = \text{dom}(g) = \mathbb{R}$)

$$\begin{aligned} g \circ f(x) &= g(f(x)) \\ &= 2 + (f(x))^3 \\ &= 2 + (x+1)^3, \quad x \in \mathbb{R} \end{aligned}$$

(b) $g \circ f(x) = 2 + (1+x)^3$

gof is a function because for each y value in the range there is exactly one corresponding x value in the domain.

gof is also a one-to-one function:



Therefore, an INVERSE function exists.

(b)

Let $g \circ f^{-1}(10) = p$

Then, $(10, p)$ lies on the graph of $g \circ f^{-1}$.

This means that $(p, 10)$ lies on the graph of $g \circ f$.

$$\therefore 10 = 2 + (1+p)^3$$

$$8 = (1+p)^3$$

$$\sqrt[3]{8} = 1+p$$

$$2 = 1+p.$$

$$p = 1$$

Therefore, $g \circ f^{-1}(10) = 1$