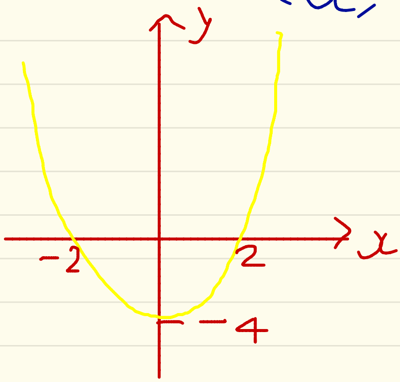
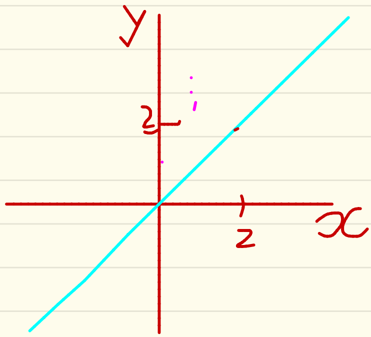
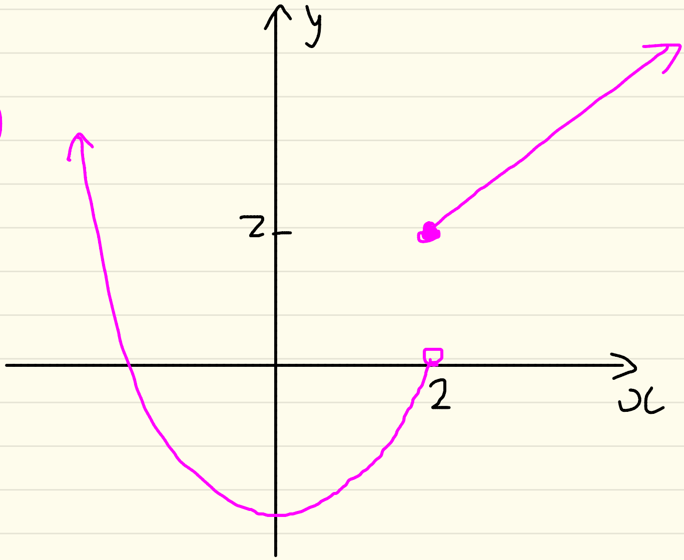
SOLUTION for Q8

(a) $f(x) = \begin{cases} x^2 - 4, & x \in (-\infty, 2) \\ x, & x \in [2, \infty) \end{cases}$





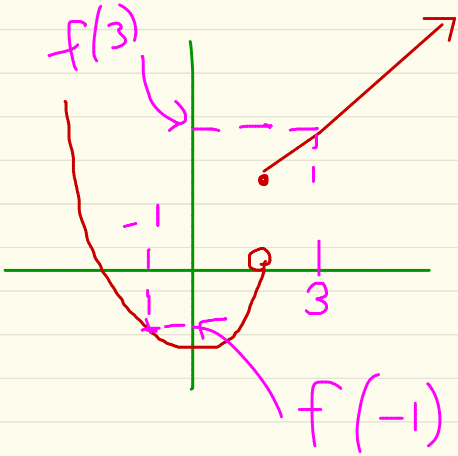
$y = f(x)$



(b) (i) $f(-1) = (-1)^2 - 4$

$= -3$

(ii) $f(3) = 3$



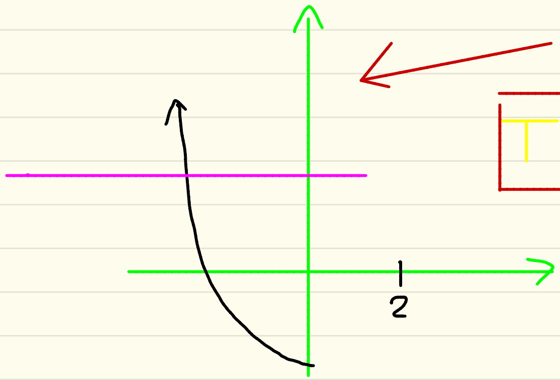
(c) $g : S \to R$, $g(x) = \begin{cases} x^2 - 4 \\ x \end{cases}$

find S such that $-1 \in S$ and g^{-1} exists.

We must define S so that g is ONE-to-ONE.

(i) [Cont]

This means that a horizontal line cuts in ONE point.



This is a ONE-to ONE function

The set S is $x \in (-\infty, 0]$

(d) $h(x) = 2x$

$2x$

$$f(h(x)) = \begin{cases} (h(x))^2 - 4, & h(x) < 2 \\ h(x), & h(x) \geq 2 \end{cases}$$

$2x$

If $h(x) < 2$

then $2x < 2$

$x < 1$

If $h(x) \geq 2$

then $2x \geq 2$

$x > 1$

So $$f(h(x)) = \begin{cases} 4x^2 - 4, & x < 1 \\ 2x, & x \geq 1 \end{cases}$$

$$h(f(x)) = \begin{cases} 2(x^2 - 4), & x < 2 \\ 2x, & x \geq 2 \end{cases}$$

$$h(f(x)) = \begin{cases} 2x^2 - 8, & x < 2 \\ 2x, & x \geq 2 \end{cases}$$