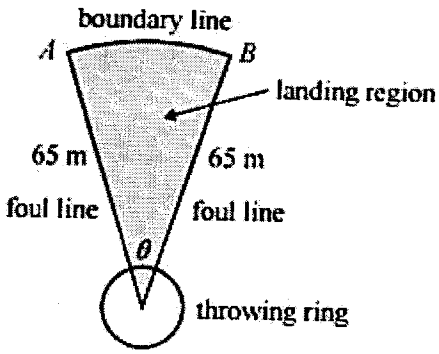
One of the field events at athletics competitions is the discus.

The field markings for the discus event consist of a circular throwing ring, foul lines and the boundary line of the field, as shown in the diagram below. The shaded area on the diagram is the landing region for a discus throw.



The foul lines meet the boundary line at points *A* and *B*, 65 m from the centre of the throwing ring.
The angle θ is 34.92°.

a. What is the length of the boundary line from point *A* to point *B*?
Write your answer in metres, rounded to two decimal places. **1 mark**

$$S = \frac{\pi r \theta}{180}$$

$$S = \frac{\pi \times 65 \times 34.92}{180}$$

$$S \simeq 39.62 \text{ m}$$

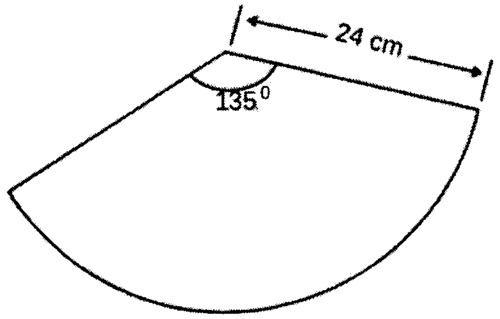
b. Calculate the area of the landing region.
Round your answer to the nearest square metre. **2 marks**

$$A = \frac{\pi r^2 \theta}{360}$$

$$\therefore A = \frac{\pi \times 65^2 \times 34.92}{360}$$

$$A \simeq 1288 \text{ m}^2$$

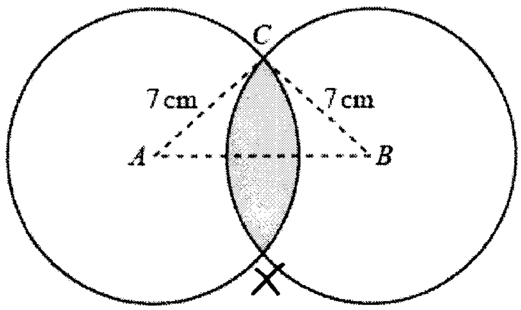
What is the perimeter of this shape (2 d.p.)?



$$P = 24 + 24 + \frac{\pi r \theta}{180}$$

$$= 48 + \frac{\pi \times 24 \times 135}{180}$$

$$= 104{\cdot}55\text{ cm}$$



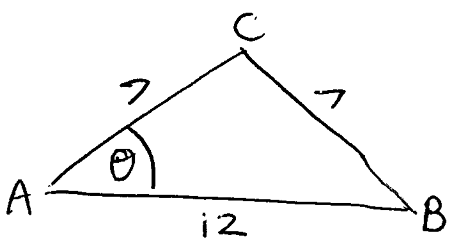
The diagram shows two circles of radius 7 cm with centres A and B. The distance AB is 12 cm and the point C lies on both circles. The region common to both circles is shaded.

(i) Show that angle CAB is 31.00° correct to 4 significant figures [2]

(ii) Find the perimeter of the shaded region. [2]

(iii) Find the area of the shaded region. [5]

(i)

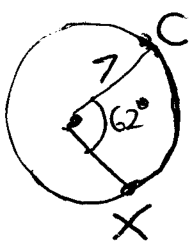


$$\cos\theta = \frac{7^2 + 12^2 - 7^2}{2 \times 7 \times 12}$$

$$\therefore \theta = \cos^{-1}\left(\frac{7^2 + 12^2 - 7^2}{2 \times 7 \times 12}\right)$$

$$\theta \simeq 31{\cdot}00^{\circ}$$

(ii)



$$S = \frac{\pi r \theta}{180}$$

$$\therefore S = \frac{\pi \times 7 \times 62}{180}$$

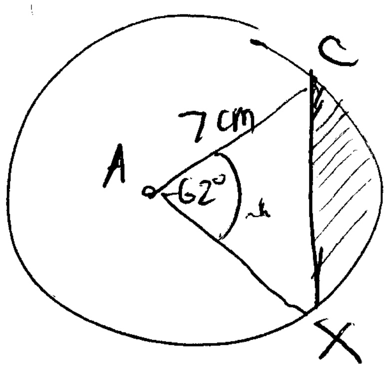
$$S \simeq 7{\cdot}5754$$

$\therefore$ Perimeter

$$= 2 \times 7{\cdot}5754$$

$$= 15{\cdot}15\text{ cm}$$

(iii)



Required area

= 2 × Segment shaded

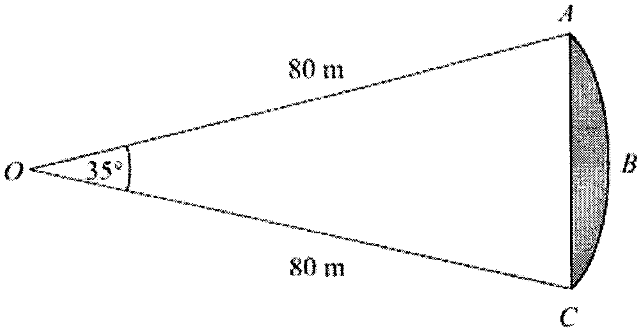
$$A_{segment} = \frac{\pi r^2 \theta}{360} - \frac{1}{2} r^2 \sin\theta$$

$$= \frac{\pi \times 7^2 \times 62}{360} - \frac{1}{2} \times 7^2 \sin(62°)$$

$= 4.8806 \text{ cm}^2$

$\therefore$ Shaded area $= 2 \times 4.8806 \text{ cm}^2$

$\approx 9.76 \text{ cm}^2$



ABC is an arc of a circle centre *O* with radius 80 m.
AC is a chord of the circle.
Angle *AOC* = 35°.

Calculate the area of the shaded region.
Give your answer correct to 3 significant figures.

$$A_{segment} = \frac{\pi r^2 \theta}{360} - \frac{1}{2} r^2 \sin\theta$$

$$= \frac{\pi \times 80^2 \times 35}{360} - \frac{1}{2} \times 80^2 \sin(35^\circ)$$

$$\approx 119.3 \text{ m}^2$$

$$\simeq 119 \text{ m}^2 \quad (3 \text{ sig figures})$$

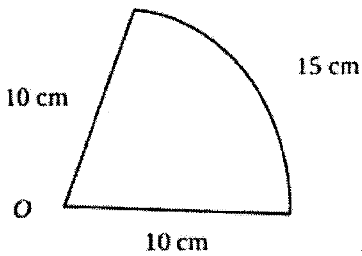


Diagram **NOT** accurately drawn

The diagram shows a sector of a circle, centre *O*, radius 10 cm.
The arc length of the sector is 15 cm.

Calculate the area of the sector.

$$S = \frac{\pi r \theta}{180}$$

$$\therefore 15 = \frac{\pi \times 10 \theta}{180}$$

$$\theta = \frac{180 \times 15}{\pi \times 10}$$

$$\theta \simeq 85.944^\circ$$

$$A = \frac{\pi r^2 \theta}{360}$$

$$\therefore A = \frac{\pi \times 10^2 \times 85.944}{360}$$

$$= 75 \text{ cm}^2$$

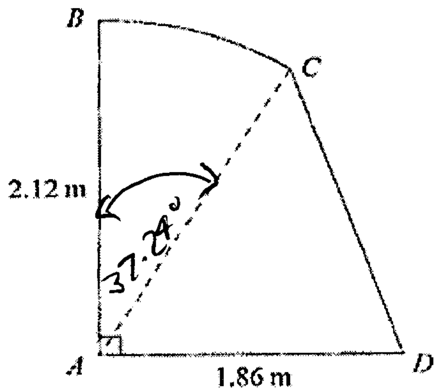


Figure 2 shows the cross section $ABCD$ of a small shed.
The straight line AB is vertical and has length 2.12 m.
The straight line AD is horizontal and has length 1.86 m.
The curve BC is an arc of a circle with centre A, and CD is a straight line.
Given that the size of $\angle BAC$ is $37.24°$, find:

(a) the length of the arc BC, in m, to 2 decimal places,

(b) the area of the sector BAC, in m², to 2 decimal places

(c) the size of $\angle CAD$, in degrees, to 4 significant figures

(d) the total area of the cross section to the nearest square metre.

(a) $$S = \frac{\pi r \theta}{180}$$

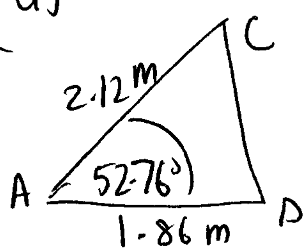
$$\therefore S = \frac{\pi \times 2.12 \times 37.24}{180} = 1.38\,m$$

(b) $$A = \frac{\pi r^2 \theta}{360} = \frac{\pi \times 2.12^2 \times 37.24}{360}$$

$$= 1.46\ m^2$$

(c) $$\angle CAD = 90° - 37.24° = 52.76°$$

(d)



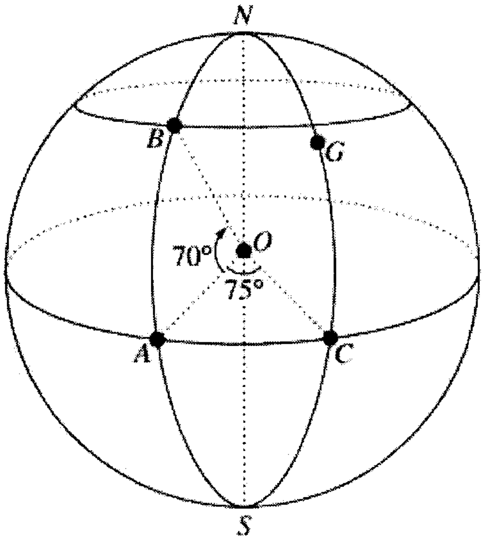
$$A_{triangle} = \frac{1}{2} bc \sin(A)$$

$$= 0.5 \times 2.12 \times 1.86 \sin(52.76°)$$

$$= 1.5696\ m^2$$

$$\therefore A_{total} = 1.3779 + 1.5696$$

$$\simeq 2.95\ m^2 \simeq 3\,m^2$$



NOT TO SCALE

In this diagram of the Earth, O represents the centre and G represents Greenwich. The point A lies on the equator.

(i) What is the time difference between Greenwich and point A? (Ignore time zones.) **1**

(ii) What is the latitude of point B? **1**

(iii) Calculate, to the nearest kilometre, the great circle distance from point A to point B. (You may assume that the radius of the Earth is 6400 km, **2**

(i) $\frac{75^\circ}{15^\circ} = 5$ hours (A is 5 hours behind Greenwich)

(ii) 70° N

(iii) $70^\circ - 0^\circ = 70^\circ$ $\quad S = \frac{\pi R \theta}{180} = \frac{\pi \times 6400 \times 70}{180}$

$= 7,819$ km

(b) The table shows the approximate coordinates for two cities.

City	Latitude	Longitude
Buenos Aires	35° S	60° W
Adelaide	35° S	140° E

(i) What is the time difference between Adelaide and Buenos Aires? (Ignore time zones.) **1**

(ii) Roy lives in Adelaide and his cousin Juan lives in Buenos Aires. Roy wants to telephone Juan at 7 pm on a Friday night, Buenos Aires time. At what time, and on what day, should Roy make the call? **2**

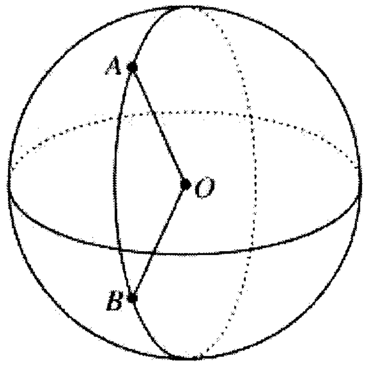
(i) $60 + 140 = 200^\circ$ $\quad \frac{200}{15} = 13.3\dot{3} \approx 13$ hours

(ii) Buenos Aires Time — 7 pm Friday

Adelaide Time — 7pm + 13 hours = 8:00AM Sat

Needs to phone 8:00 AM Saturday.

This diagram represents Earth. O is at the centre, and A and B are points on the surface. 2



A: 35°N 20°E

B: 8°S 20°E

Calculate the distance from A to B along the great circle through A and B. ~~Give your answer in nautical miles.~~

(Radius of Earth is 6400 km.

$$35^\circ + 8^\circ = 43^\circ$$

$$S = \frac{\pi R \theta}{180}$$

$$S = \frac{\pi \times 6400 \times 43}{180}$$

$$= 4803 \text{ km}$$

Cassie flew from London (52°N, 0°E) to Manila (15°N, 120°E). 3

Her plane left London at 9.30 am Monday (London time), stopped for 5 hours in Singapore and arrived in Manila at 4.00 pm Tuesday (Manila time).

What was the total flying time?

$120 - 0 = 120$

$\frac{120}{15} = 8$ hours

	London Time	Manila Time
plane leaves	9:30 AM	9:30 AM + 8 hours = 5:30 PM (Monday)
Plane lands		4:00 PM (Tuesday)

Time difference between departure and landing = 4:00PM Tuesday − 5:30PM Monday

= 24 − 1.5 = 22.5 hours

Since there was a 5 hour stop, total flying time = 17.5 hours

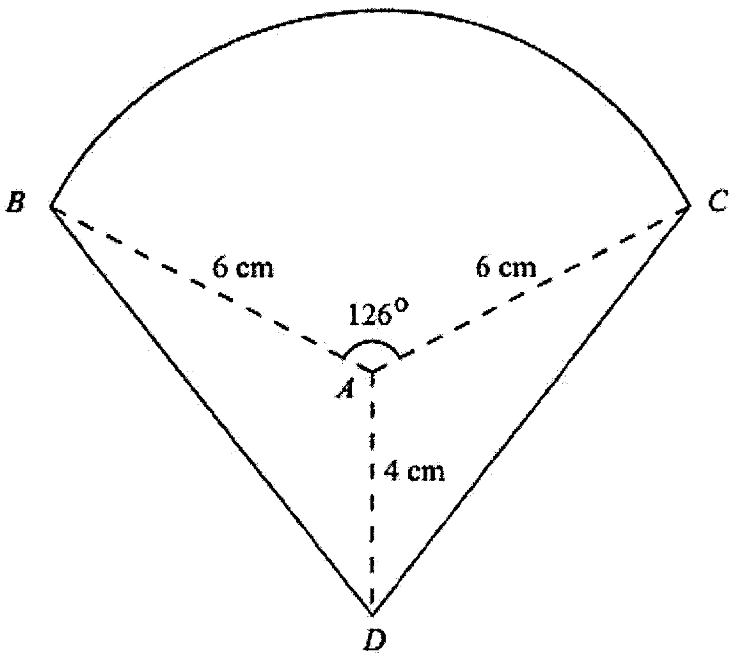


Figure 3

The shape *BCD* shown in Figure 3 is a design for a logo.

The straight lines *DB* and *DC* are equal in length. The curve *BC* is an arc of a circle with centre *A* and radius 6 cm. The size of $\angle BAC$ is $126°$ and $AD = 4$ cm.

Find

(a) the area of the sector *BAC*, in cm^2.

(2)

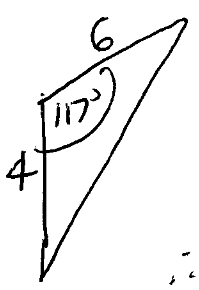
(b) the size of $\angle DAC$, in degrees to 3 significant figures,

(c) the area of the entire logo, to the nearest square cm

(a) $A = \frac{\pi r^2 \theta}{360} = \frac{\pi \times 6^2 \times 126}{360} = 39.58 \text{ cm}^2$

(b) $360° - 126° = 234°$ $\therefore \angle DAC = \frac{234}{2} = 117°$

(c)



$A_{triangle} = 0.5 \times 4 \times 6 \sin(117°) = 10.692 \text{ cm}^2$

$\therefore$ Area of logo

$= 39.58 + 2 \times 10.692 \simeq 61 \text{ cm}^2$

Singapore is located at 1°N 104°E and Sydney is located at 34°S 151°E. 2

What is the time difference between Singapore and Sydney? (Ignore daylight saving.)

$151° - 104° = 47°$

$47 \div 15 = 3.1\dot{3}$

≈ 3 hours

(a) An aircraft travels at an average speed of 913 km/h. It departs from a town in Kenya (0°, 38°E) on Tuesday at 10 pm and flies east to a town in Borneo (0°, 113°E).

(i) What is the distance, to the nearest kilometre, between the two towns? (Assume the radius of Earth is 6400 km.) 2

(ii) How long will the flight take? (Answer to the nearest hour.) 1

(iii) What will be the local time in Borneo when the aircraft arrives? 2

(i) $113° - 38° = 75°$

$S = \frac{\pi R \theta}{180}$

$S = \frac{\pi \times 6400 \times 75}{180}$

$= 8378$ km

(ii) $\frac{8378}{913} \approx 9$ hours

(iii) $75 \div 15 = 5$ hours

How many kilometres is it along the 146°E meridian of longitude from Cairns in Queensland (17°S, 146°E) to Burnie in Tasmania (41°S, 146°E)?

	Kenyan Time	time
Departs	10 pm Tuesday	3:00 AM Wednesday
Lands		3:00 AM + 9 hours

$\therefore$ Lands at 12:00PM Wednesday in Borneo ~~Burnie~~

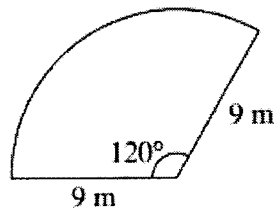
$41° - 17° = 24°$

$S = \frac{\pi R \theta}{180} = \frac{\pi \times 24 \times 6400}{180}$

$\simeq 2681$ km

Question 1

This is a sketch of a sector of a circle.



Calculate the area of this sector (correct to one decimal place).

(A) 9.4 m^2

(B) 18.8 m^2

(C) 36.8 m^2

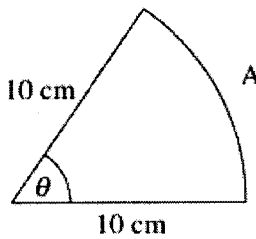
(D) 84.8 m^2

$A = \frac{\pi \times 9^2 \times 120}{360}$

$\simeq 84 \cdot 82 \text{ m}^2$

Question 2

This is a sketch of a sector of a circle.



NOT TO SCALE

Find the value of θ to the nearest degree.

(A) 47°

(B) 48°

(C) 68°

(D) 69°

$S = \frac{\pi r \theta}{180}$

$\therefore 12 = \frac{\pi \times 10\theta}{180}$

$\theta = \frac{180 \times 12}{\pi \times 10} \simeq 69°$

Question 3

The location of Town A is 25°N 45°E. The location of Town B is 10°N 105°E.

Which of the following is true?

(A) Town A is four hours behind Town B.

(B) Town A is four hours ahead of Town B.

(C) Town A is one hour behind Town B.

(D) Town A is one hour ahead of Town B.

$105° - 45°$

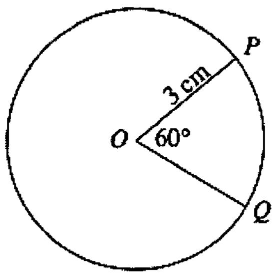
$= 60°$

$\frac{60}{15} = 4$ hours

$\therefore$ A is 4 hours behind B.

Question 4

P and Q are points on the circumference of a circle with centre O and radius 3 cm.



NOT
TO
SCALE

What is the length of the arc PQ, in centimetres, correct to three significant figures?

(A) 1.57
(B) 3.14
(C) 4.71
(D) 18.8

$$s = \frac{\pi r\theta}{180}$$

$$= \frac{\pi \times 3 \times 60}{180}$$

$$\simeq 3.14\text{ cm}$$

Question 5

Kim lives in Perth (32°S, 115°E). He wants to watch an ice hockey game being played in Toronto (44°N, 80°W) starting at 10.00 pm on Wednesday.

What is the time in Perth when the game starts?

(A) 9.00 am on Wednesday
(B) 7.40 pm on Wednesday
(C) 12.20 am on Thursday
(D) 11.00 am on Thursday

$115° + 80° = 195°$

$\frac{195}{15} = 13$ hours

Toronto time
10:00PM Wed

Perth time
10:00PM + 13 hrs
= 11:00AM Thursday

Question 6

Makoua and Macapá are two towns on the equator.

The longitude of Makoua is 16°E and the longitude of Macapá is 52°W.

How far apart are these two towns if the radius of Earth is approximately 6400 km?

(A) 4000 km
(B) 7600 km
(C) 1 447 600 km
(D) 2 734 400 km

$16° + 52° = 68°$

$$S = \frac{\pi R\theta}{180} = \frac{\pi \times 6400 \times 68}{180}$$

$$\simeq 7596\text{ km}$$

Question 7

Which expression will give the shortest distance, in kilometres, between Mount Isa (20°S 140°E) and Tokyo (35°N 140°E)?

(A) $\frac{15}{360} \times 2 \times \pi \times 6400$

(B) $\frac{55}{360} \times 2 \times \pi \times 6400$

(C) $\frac{140}{360} \times 2 \times \pi \times 6400$

(D) $\frac{305}{360} \times 2 \times \pi \times 6400$

$35° + 20° = 55°$

$S = \frac{\pi R \theta}{180}$

$= \frac{55}{180} \times 6400 \times \pi$

$= \frac{55}{360} \times 2 \times \pi \times 6400$

Question 8

Perth in Western Australia is 8 hours ahead of Greenwich in England. Cape Town in South Africa is 2 hours ahead of Greenwich.

What is the time in Cape Town when it is 1 pm in Perth?

(A) 3 am

(B) 7 am

(C) 7 pm

(D) 11 pm

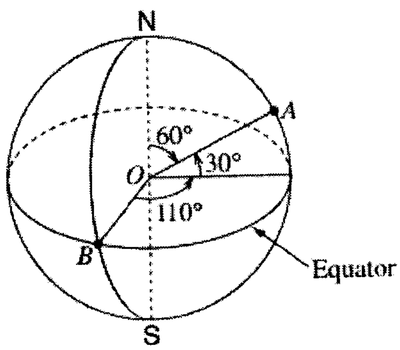
Greenwich	Cape Town	Perth
0	+2	+8

1pm − 6 hours ← 1pm

= 7:00AM

Question 9

In this diagram of the Earth, *O* represents the centre and *B* lies on both the Equator and the Greenwich Meridian.



NOT TO SCALE

What is the latitude and longitude of point *A*?

30°N, 110°E

(A) 30°N 110°E

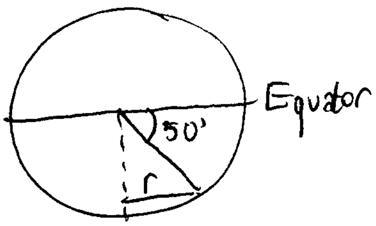
(B) 30°N 110°W

(C) 60°N 110°E

(D) 60°N 110°W

Question 1

A plane flies **due east** from position A $50°S, 60°E$ to position B $50°S, 80°E$. Calculate the distance it has flown, to the nearest km.



$r = 6400\cos(50°)$

$r = 4113.84$ km

$80° - 60° = 20°$

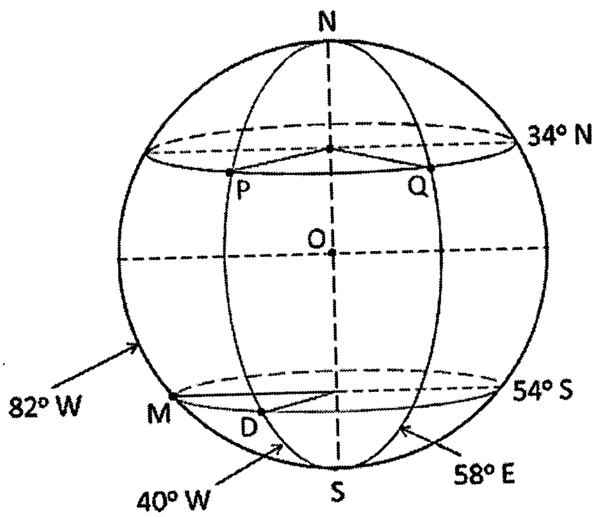
$\therefore S = \frac{\pi r\theta}{180}$

$\therefore S = \frac{\pi \times 4113.84 \times 20}{180}$

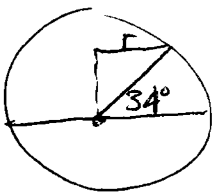
≈ 718 km

Question 2

Consider the diagram shown below:



a. Calculate the distance PQ, along the parallel of latitude at 34° N.



$r = 6400\cos(34°)$

$= 5305.84$ km

$58° + 40° = 98°$

$S = \frac{\pi r\theta}{180}$

$S = \frac{\pi \times 98 \times 5305.84}{180}$

$= 9075$ km

b. Calculate the distance DM, measured along the parallel of latitude at 54°S.

$r = 6400\cos(54°)$

≈ 3761.83 km

$\theta = 82° - 40°$

$= 42°$

$S = \frac{\pi \times 3761.83 \times 42}{180}$

≈ 2758 km

Question 3

Trigville is located at 37°N, 72°W and Mathsville is located at 37°N, 17°E. Calculate the distance between Trigville and Mathsville travelling due east from Trigville along the parallel of latitude, to the nearest km.

$$72^\circ + 17^\circ = 89^\circ$$

$$r = 6400\cos(37^\circ)$$
$$\simeq 5111.27 \text{ km}$$

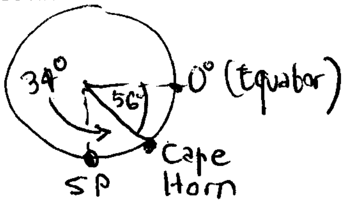
$$S = \frac{\pi r \theta}{180}$$

$$\therefore S = \frac{\pi \times 5111.27 \times 89}{180}$$
$$= \boxed{7940 \text{ km}}$$

Question 4

The southern most tip of mainland Chile is Cape Horn, located at $56°S, 67°W$

a. Calculate the shortest distance between Cape Horn and the South Pole, to the nearest km.



$$S = \frac{\pi R \theta}{180}$$

$$S = \frac{\pi \times 6400 \times 34}{180} \simeq 3798 \text{ km}$$

b. Calculate, to the nearest km, the total length of the $56°S$ parallel of latitude, to the nearest km.

$$r = 6400\cos(56^\circ)$$
$$= 3578.83 \text{ km}$$

$$C = 2\pi r$$

$$\therefore C = 2\pi \times 3578.83$$
$$\simeq 22{,}486 \text{ km}$$

c. Melbourne is located at $38°S, 145°E$.

i. What is the time difference between Cape Horn and Melbourne, to the nearest hour?

$$67^\circ + 145^\circ = 212^\circ \qquad \frac{212}{15} = 14.1\dot{3}$$
$$\simeq \boxed{14 \text{ hours}}$$

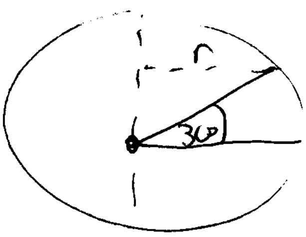
ii. If it is 1:00 pm on Wednesday in Melbourne, what day and time is it at Cape Horn?

1:00pm Wednesday − 14 hours

= 11:00pm Tuesday

Question 5

A ship starts from position Latitude 30° north, 15° west and travels **due east** for 2,400 km. Calculate its new co-ordinates.



$$r = 6400\cos(30^\circ)$$
$$\simeq 5542.56 \text{ km}$$

$$s = 2400 \text{ km}$$

$$\therefore 2400 = \frac{\pi \times 5542.56 \times \theta}{180}$$

$$\theta = \frac{2400 \times 180}{\pi \times 5542.56} = 24.8^\circ$$

New longitude co-ordinate

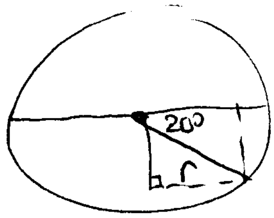
$$= 15^\circ W + 24.8^\circ$$
$$\simeq 9.8^\circ E$$

$\therefore$ New co-ordinates:

30°N, 9.8°E

Question

From 20° S, 15° E a submarine travels 4,500 km due west along the $20°S$ latitude to a naval base. Calculate the co-ordinates of the naval base.



$S = 4500$

$\therefore 4500 = \dfrac{\pi \times 6014.03 \times \theta}{180}$

$\dfrac{4500 \times 180}{\pi \times 6014.03} = \theta$

$\theta \simeq 42.87°$

$r = 6400 \cos(20°)$

$\simeq 6014.03$ km

New longitude co-ordinate

$= 15°E - 42.87°$

$= 27.87°W$

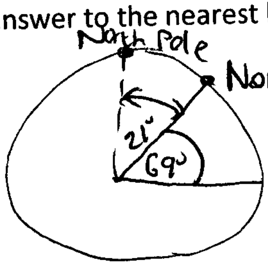
Naval base:

20°S, 27.87°W

Question

Norilsk is the northern most city in the world. It is located in Siberia and has co-ordinates: $69°N, 88°E$.

a. Calculate the shortest distance between Norilsk and the North Pole. Give your answer to the nearest km.



$S = \dfrac{\pi R \theta}{180}$

$= \dfrac{\pi \times 6400 \times 21}{180}$ $\simeq 2346$ km

b. The Arctic Circle is located at a latitude of $66°N$. How much shorter than the Arctic Circle is the parallel of latitude on which Norilsk lies? Give your answer to the nearest km.

Arctic Circle:

$r = 6400 \cos(66) = 2603.1145$ km

Norilsk latitude:

$r = 6400 \cos(69°) \simeq 2293.5549$ km

Length of Arctic Circle:

$C = 2\pi r$

$C = 2\pi \times 2603.1145$

$\simeq 16355.851$ km

Norilsk latitude

$C = 2\pi \times 2293.55 \simeq 14410.83$ km

Difference

$16355.851 - 14410.83$

$\simeq 1945$ km

c. A plane takes off from Norilsk at 9:00 AM local time and flies to Moscow $56°N, 38°E$. If the plane flight takes 4 hours 40 minutes, at what time does the plane arrive in Moscow? Give your answer to the nearest minute.

Difference in longitude

$= 88° - 38° = 50°$

Time Difference $= \dfrac{50}{15} = 3.\dot{3}$

≈ 3 hours

	Moscow	Norilsk
Plane takes off	6:00AM	9:00AM
Lands	10:40AM	

$\therefore$ Lands at 10:40AM in Moscow.