Question 3

Regular customers at a hairdressing salon can choose to have their hair cut by Shirley, Jen or Narj.

The salon has 600 regular customers who get their hair cut each month.

In June, 200 customers chose Shirley (S) to cut their hair, 200 chose Jen (J) to cut their hair and 200 chose Narj (N) to cut their hair.

The regular customers' choice of hairdresser is expected to change from month to month as shown in the transition matrix, T, below.

$$\begin{array}{c} \text{this month} \\ \begin{array}{cc} & \begin{array}{ccc} S & J & N \end{array} \\ T = & \begin{bmatrix} 0.75 & 0.10 & 0.10 \\ 0.10 & 0.75 & 0.15 \\ 0.15 & 0.15 & 0.75 \end{bmatrix} \begin{array}{l} S \\ J \\ N \end{array} \text{ next month} \end{array} \end{array}$$

In the long term, the number of regular customers who are expected to choose Shirley is closest to

A. 150
B. 170
C. 185
D. 195
E. 200

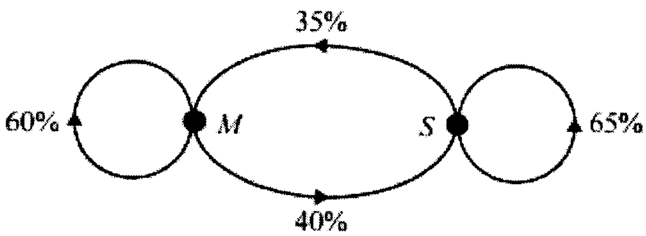
$$\begin{bmatrix} 0.75 & 0.10 & 0.10 \\ 0.10 & 0.75 & 0.15 \\ 0.15 & 0.15 & 0.75 \end{bmatrix}^{40} \begin{bmatrix} 200 \\ 200 \\ 200 \end{bmatrix} = \begin{bmatrix} 171.4 \\ 203.6 \\ 225 \end{bmatrix}$$

$$\begin{bmatrix} 0.75 & 0.10 & 0.10 \\ 0.10 & 0.75 & 0.15 \\ 0.15 & 0.15 & 0.75 \end{bmatrix}^{41} \begin{bmatrix} 200 \\ 200 \\ 200 \end{bmatrix} = \begin{bmatrix} 171.4 \\ 203.6 \\ 225 \end{bmatrix}$$

Question 4

Two hundred and fifty people buy bread each day from a corner store. They have a choice of two brands of bread: Megaslice (M) and Superloaf (S).

The customers' choice of brand changes daily according to the transition diagram below.



this day

$$\begin{array}{cc} & \begin{array}{cc} M & S \end{array} \\ \text{next day} \begin{array}{c} M \\ S \end{array} & \begin{bmatrix} 0.6 & 0.35 \\ 0.4 & 0.65 \end{bmatrix} \begin{bmatrix} 100 \\ 150 \end{bmatrix} \end{array}$$

On a given day, 100 of these people bought Megaslice bread while the remaining 150 people bought Superloaf bread.

The number of people who are expected to buy each brand of bread the next day is found by evaluating the matrix product

A.
$$\begin{array}{c} \begin{array}{cc} M & S \end{array} \\ \begin{bmatrix} 0.60 & 0.40 \\ 0.35 & 0.65 \end{bmatrix} \begin{bmatrix} 100 \\ 150 \end{bmatrix} \begin{array}{l} M \\ S \end{array} \end{array}$$

B.
$$\begin{array}{c} \begin{array}{cc} M & S \end{array} \\ \begin{bmatrix} 0.60 & 0.40 \\ 0.65 & 0.35 \end{bmatrix} \begin{bmatrix} 100 \\ 150 \end{bmatrix} \begin{array}{l} M \\ S \end{array} \end{array}$$

C.
$$\begin{array}{c} \begin{array}{cc} M & S \end{array} \\ \begin{bmatrix} 0.60 & 0.35 \\ 0.40 & 0.65 \end{bmatrix} \begin{bmatrix} 100 \\ 150 \end{bmatrix} \begin{array}{l} M \\ S \end{array} \end{array}$$

D.
$$\begin{array}{c} \begin{array}{cc} M & S \end{array} \\ \begin{bmatrix} 0.65 & 0.40 \\ 0.35 & 0.60 \end{bmatrix} \begin{bmatrix} 100 \\ 150 \end{bmatrix} \begin{array}{l} M \\ S \end{array} \end{array}$$

E.
$$\begin{array}{c} \begin{array}{cc} M & S \end{array} \\ \begin{bmatrix} 0.60 & 0.65 \\ 0.40 & 0.35 \end{bmatrix} \begin{bmatrix} 100 \\ 150 \end{bmatrix} \begin{array}{l} M \\ S \end{array} \end{array}$$

Question 9

A fast-food stand at the football sells pies (P) and chips (C).

Each week, 300 customers regularly buy either a pie or chips, but not both, from this stand.

For the first five weeks, the customers' choice of pie or chips is expected to change weekly acco transition matrix T_1, where

$$\begin{array}{cc} & \text{this week} \\ & \begin{array}{cc} P & C \end{array} \\ T_1 = & \begin{bmatrix} 0.65 & 0.25 \\ 0.35 & 0.75 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ P \\ C \end{array} \text{ next week}$$

After the first five weeks, due to expected cold weather, the customers' choice of pie or chips is change weekly according to the transition matrix T_2, where

$$\begin{array}{cc} & \text{this week} \\ & \begin{array}{cc} P & C \end{array} \\ T_2 = & \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ P \\ C \end{array} \text{ next week}$$

In week 1, 150 customers bought a pie and 150 customers bought chips.

Let S_1 be the state matrix for week 1.

The number of customers expected to buy a pie or chips in week 8 can be found by evaluating

A. $T_2^7 S_1$

B. $T_1^8 S_1$

C. $T_2^3 (T_1^4 S_1)$

D. $T_1^3 (T_2^4 S_1)$

E. $T_1^3 (T_2^5 S_1)$

$T_1 T_1 T_1 (T_1 S_1) = T_1^4 S_1$ is the state at end of week 5.
Then $T_2^3 \times T_1^4 S_1$ gives final resulting state.

Question 3

A coffee shop sells three types of coffee, Brazilian (B), Italian (I) and Kenyan (K). The regular customers buy one cup of coffee each per day and choose the type of coffee they buy according to the following transition matrix, T.

$$\begin{array}{cc} & \text{choose today} \\ & \begin{array}{ccc} B & I & K \end{array} \\ T = & \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0 & 0.8 & 0.1 \\ 0.2 & 0.1 & 0.8 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ B \\ I \\ K \end{array} \text{ choose tomorrow}$$

$T^{40} \begin{bmatrix} 84 \\ 96 \\ 81 \end{bmatrix} = \begin{bmatrix} 87 \\ 58 \\ 116 \end{bmatrix}$

$T^{41} \begin{bmatrix} 84 \\ 96 \\ 81 \end{bmatrix} = \begin{bmatrix} 87 \\ 58 \\ 116 \end{bmatrix}$

On a particular day, 84 customers bought Brazilian coffee, 96 bought Italian coffee and 81 bought Kenyan coffee.

If these same customers continue to buy one cup of coffee each per day, the number of these customers who are expected to buy each of the three types of coffee in the long term is

A.

Brazilian	85
Italian	85
Kenyan	91

B.

Brazilian	87
Italian	58
Kenyan	116

C.

Brazilian	88
Italian	86
Kenyan	87

D.

Brazilian	89
Italian	89
Kenyan	83

E.

Brazilian	116
Italian	89
Kenyan	58

Question 8

There are 30 children in a Year 6 class. Each week every child participates in one of three activities: cycling (C), orienteering (O) or swimming (S).

The activities that the children select each week change according to the transition matrix below.

$$\begin{array}{cccc} & & \textit{this week} & \\ & C & O & S \\ T = & \left[\begin{array}{ccc} 0.5 & 0.3 & 0.3 \\ 0.1 & 0.6 & 0.2 \\ 0.4 & 0.1 & 0.5 \end{array}\right] & \begin{array}{c} C \\ O \\ S \end{array} & \textit{next week} \end{array}$$

From the transition matrix it can be concluded that

A. in the first week of the program, ten children do cycling, ten children do orienteering and ten children do swimming.

B. at least 50% of the children do not change their activities from the first week to the second week.

C. in the long term, all of the children will choose the same activity.

D. orienteering is the most popular activity in the first week.

E. 50% of the children will do swimming each week.

Each percentage along leading diagonal is 50% or 60%.

Question 7

A new colony of several hundred birds is established on a remote island. The birds can feed at two locations, A and B. The birds are expected to change feeding locations each day according to the transition matrix

$$\begin{array}{ccc} & \textit{this day} & \\ & A \quad B & \\ T = & \left[\begin{array}{cc} 0.4 & 0.3 \\ 0.6 & 0.7 \end{array}\right] & \begin{array}{c} A \\ B \end{array} \ \textit{next day} \end{array}$$

In the beginning, approximately equal numbers of birds feed at each site each day.

Which of the following statements is **not** true?

A. 70% of the birds that feed at B on a given day will feed at B the next day.

B. 60% of the birds that feed at A on a given day will feed at B the next day.

C. In the long term, more birds will feed at B than at A.

D. The number of birds that change feeding locations each day will decrease over time to zero.

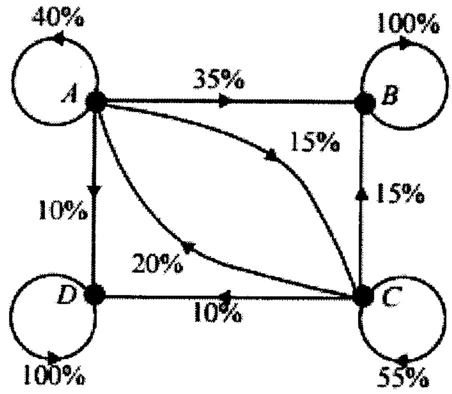
E. In the long term, some birds will always be found feeding at each location.

The following information relates to Questions 7, 8 and 9.

A large population of mutton birds migrates each year to a remote island to nest and breed. There are four nesting sites on the island, A, B, C and D.

Researchers suggest that the following transition matrix can be used to predict the number of mutton birds nesting at each of the four sites in subsequent years. An equivalent transition diagram is also given.

$$T = \begin{array}{c} \textit{this year} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0.4 & 0 & 0.2 & 0 \\ 0.35 & 1 & 0.15 & 0 \\ 0.15 & 0 & 0.55 & 0 \\ 0.1 & 0 & 0.1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ A \\ B \\ C \\ D \end{array} \quad \textit{next year}$$



Question 7

Two thousand eight hundred mutton birds nest at site C in 2008.

Of these 2800 mutton birds, the number that nest at site A in 2009 is predicted to be

A. 560
B. 980
C. 1680
D. 2800
E. 3360

$0.2 \times 2800 = 560$

Question 8

This transition matrix predicts that, in the long term, the mutton birds will

A. nest only at site A.
B. nest only at site B.
C. nest only at sites A and C.
D. nest only at sites B and D.
E. continue to nest at all four sites.

1's at position B to B and D to D mean that once at B, or D, birds will remain there.

Question 9

Six thousand mutton birds nest at site B in 2008.

Assume that an equal number of mutton birds nested at each of the four sites in 2007. The same transition matrix applies.

The total number of mutton birds that nested on the island in 2007 was

A. 6000
B. 8000
C. 12000
D. 16000
E. 24000

$$\begin{bmatrix} 0.4 & 0 & 0.2 & 0 \\ 0.35 & 1 & 0.15 & 0 \\ 0.15 & 0 & 0.55 & 0 \\ 0.1 & 0 & 0.1 & 1 \end{bmatrix} \begin{bmatrix} x \\ x \\ x \\ x \end{bmatrix} = \begin{bmatrix} - \\ 6000 \\ - \\ - \end{bmatrix}$$

$\therefore 0.35x + x + 0.15x + 0x = 6000$

$1.5x = 6000$

$x = \dfrac{6000}{1.5}$

$x = 4000$

$\therefore$ Total no of birds $= 4 \times 4000$
$= 16000$

Question 7

A transition matrix, T, and a state matrix, S_2, are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$$

$TS_1 = S_2$

$\therefore T^{-1}.T.S_1 = T^{-1}.S_2$

$\therefore I.S_1 = T^{-1}.S_2$

$\therefore S_1 = T^{-1}.S_2$

If $S_2 = TS_1$, the state matrix S_1 is

A. $\begin{bmatrix} 200 \\ 250 \\ 150 \end{bmatrix}$ **B.** $\begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$ **C.** $\begin{bmatrix} 300 \\ 0 \\ 300 \end{bmatrix}$ **D.** $\begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$ **E.** undefined

$S_1 = \begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$

Question 2

A new shopping centre called Shopper Heaven (S) is about to open. It will compete for customers with Eastown (E) and Noxland (N).

Market research suggests that each shopping centre will have a regular customer base but attract and lose customers on a weekly basis as follows.

80% of Shopper Heaven customers will return to Shopper Heaven next week
12% of Shopper Heaven customers will shop at Eastown next week
8% of Shopper Heaven customers will shop at Noxland next week

76% of Eastown customers will return to Eastown next week
9% of Eastown customers will shop at Shopper Heaven next week
15% of Eastown customers will shop at Noxland next week

85% of Noxland customers will return to Noxland next week
10% of Noxland customers will shop at Shopper Heaven next week
5% of Noxland customers will shop at Eastown next week

a. Enter this information into transition matrix T as indicated below (express percentages as proportions, for example write 76% as 0.76).

this week

$$T = \begin{array}{c} \begin{array}{ccc} S & E & N \end{array} \\ \begin{bmatrix} 0.8 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix} \end{array} \begin{array}{c} S \\ E \\ N \end{array} \quad \textbf{next week}$$

2 marks

During the week that Shopper Heaven opened, it had 300 000 customers.

In the same week, Eastown had 120 000 customers and Noxland had 180 000 customers.

b. Write this information in the form of a column matrix, K_0, as indicated below.

$$K_0 = \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} \begin{array}{c} S \\ E \\ N \end{array}$$

c. Use T and K_0 to **write** and **evaluate** a matrix product that determines the number of customers expected at each of the shopping centres during the following week.

$$K_1 = \begin{bmatrix} 0.8 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix} \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} = \begin{bmatrix} 268\,800 \\ 136\,200 \\ 195000 \end{bmatrix}$$

Shopper Heaven: 268,800
East Own : 136,200
Noxland : 195,000

2 marks

d. Show by calculating at least two appropriate state matrices that, in the long term, the number of customers expected at each centre each week is given by the matrix

$$K = \begin{bmatrix} 194983 \\ 150513 \\ 254504 \end{bmatrix}$$

$$K_{40} = T^{40} K_0 = \begin{bmatrix} 194982.9 \\ 150513.35 \\ 254503.75 \end{bmatrix}$$

NOTE That two consecutive calculations must be done to show a steady state has been reached.

$$T^{41} K_0 = \begin{bmatrix} 194982.9 \\ 150513.3 \\ 254503.8 \end{bmatrix}$$

∴ In the long term,

194983 at Shopper Heaven
150 513 at Eastown
254504 at Noxland

Note: Clear Statement of final answer

Question 2

The following transition matrix, T, is used to help predict class attendance of History students at the university on a lecture-by-lecture basis.

$$\begin{array}{cc} & \text{this lecture} \\ & \begin{array}{cc} \textit{attend} & \textit{not attend} \end{array} \\ T = & \begin{bmatrix} 0.90 & 0.20 \\ 0.10 & 0.80 \end{bmatrix} \begin{array}{l} \textit{attend} \\ \textit{not attend} \end{array} \text{ next lecture} \end{array}$$

S_1 is the attendance matrix for the first History lecture.

$$S_1 = \begin{bmatrix} 540 \\ 36 \end{bmatrix} \begin{array}{l} \textit{attend} \\ \textit{not attend} \end{array}$$

S_1 indicates that 540 History students attended the first lecture and 36 History students did not attend the first lecture.

a. Use T and S_1 to

i. determine S_2 the attendance matrix for the second lecture

$$S_2 = TS_1 = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}\begin{bmatrix} 540 \\ 36 \end{bmatrix} = \begin{bmatrix} 493.2 \\ 82.8 \end{bmatrix} \simeq \begin{bmatrix} 493 \\ 83 \end{bmatrix}$$

ii. predict the number of History students attending the fifth lecture.

NOTE: Clear statement of how many will attend

$$S_5 = T^4 S_1 = \begin{bmatrix} 421.456 \\ 154.544 \end{bmatrix}$$

$\therefore$ 421 predicted to attend.

1 + 1 = 2 marks

b. Write down a matrix equation for S_n in terms of T, n and S_1.

$$S_n = T^{n-1} S_1$$

1 mark

The History lecture can be transferred to a smaller lecture theatre when the number of students predicted to attend falls below 400.

c. For which lecture can this first be done?

$$S_8 = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}^7 \begin{bmatrix} 540 \\ 36 \end{bmatrix} = \begin{bmatrix} 396.85 \\ 179.15 \end{bmatrix}$$

8th lecture

1 mark

d. In the long term, how many History students are predicted to attend lectures?

$$S_{41} = T^{40} S_1 = \begin{bmatrix} 384 \\ 192 \end{bmatrix}$$

$$S_{42} = T^{41} S_1 = \begin{bmatrix} 384 \\ 192 \end{bmatrix}$$

$\therefore$ 384 predicted to attend

1 mark

Module 6: Matrices – continued

TURN OVER