Question 1

Past records show that the times, in seconds, taken to run 100 m by children at a school can be modelled by a normal distribution with a mean of 16.12 and a standard deviation of 1.60

A child from the school is selected at random.

(a) Find the probability that this child runs 100 m in less than 15 s.

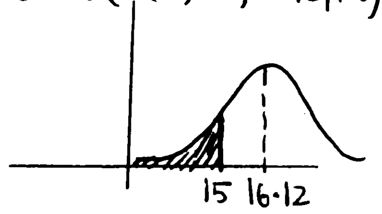
On sports day the school awards certificates to the fastest 30% of the children in the 100 m race.

(b) Calculate, to 2 decimal places, the slowest time taken to run 100 m for which a child will be awarded a certificate.

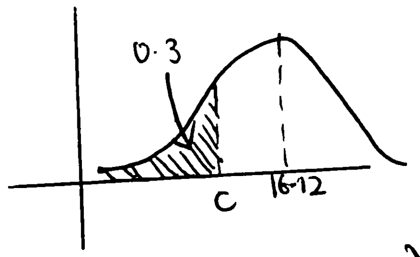
$T \stackrel{d}{=} N(\mu = 16.12, \sigma = 1.60)$

normalcdf$(-\infty, 15, 16.12, 1.6)$

(a) $\Pr(T < 15) = 0.2420$



(b) $\Pr(T < c) = 0.3$



invNorm$(0.3, 16.12, 1.60) = 15.28$

Never write CAS syntax on your solution. I have just included it to clarify what you type in CAS

$\therefore c \simeq 15.28$

Slowest time $= 15.28$ sec

Question 2

The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

(a) Find $P(L > 127)$.

(b) Find the value of d such that $P(L < d) = 0.10$

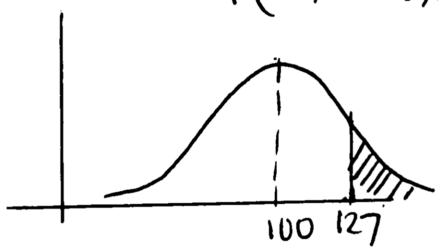
Alice is about to go on a 6 hour journey.
Given that it is 127 hours since Alice last charged her phone,

(c) find the probability that her phone will not need charging before her journey is completed.

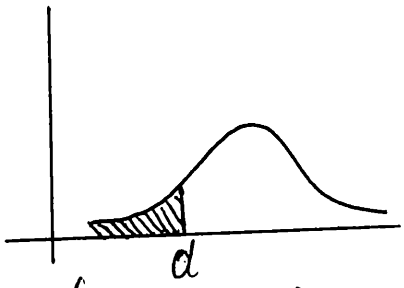
(a) $L \overset{d}{=} N(\mu = 100, \sigma = 15)$

$\Pr(L > 127) \simeq 0.0359$

normcdf$(127, \infty, 100, 15)$



(b)



$d = \text{invNorm}(0.1, 100, 15)$

$d \simeq 80.777$

$\therefore d = 80.777$ hours

(c) $\Pr(L > 133 \mid L > 127) = \dfrac{\Pr(L > 133 \cap L > 127)}{\Pr(L > 127)}$

$= \dfrac{\Pr(L > 133)}{\Pr(L > 127)} = \dfrac{0.013903}{0.035930}$

$\simeq 0.3870$

Question 3

A packing plant fills bags with cement. The weight X kg of a bag of cement can be modelled by a normal distribution with mean 50 kg and standard deviation 2 kg.

(a) Find P(X>53).

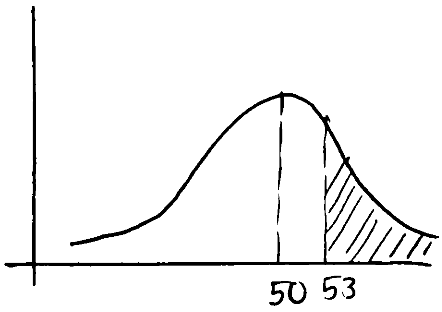
(b) Find the weight that is exceeded by 99% of the bags.

Three bags are selected at random.

(c) Find the probability that two weigh more than 53 kg and one weighs less than 53 kg.

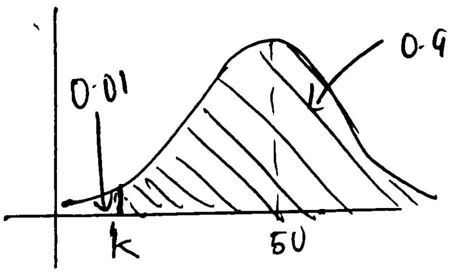
$X \stackrel{d}{=} N(\mu = 50, \sigma = 2)$

(a)



$Pr(X > 53) = 0.0668$

(b)



$Pr(X > k) = 0.99$

$k = \text{invNorm}(0.01, 50, 2)$

$k = 45.35$

$\therefore$ Required weight = 45.35 kg

(c) Let Y = no. of bags that weigh more than 53 kg $\quad Y \stackrel{d}{=} Bi(n = 3, p = 0.06681)$

$Pr(Y = 2) = 0.0125$

Question 4

The weights of bags of popcorn are normally distributed with mean of 200 g and 60% of all bags weighing between 190 g and 210 g.

(a) Find the standard deviation of the weights of the bags of popcorn.

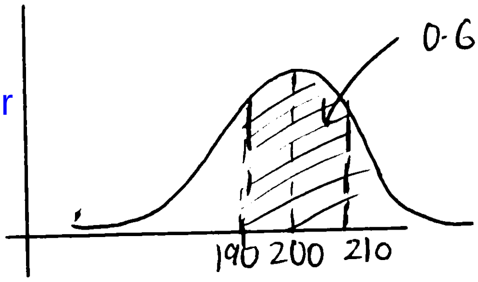
A shopkeeper finds that customers will complain if their bag of popcorn weighs less than 180 g.

(b) Find the probability that a customer will complain.

(a)

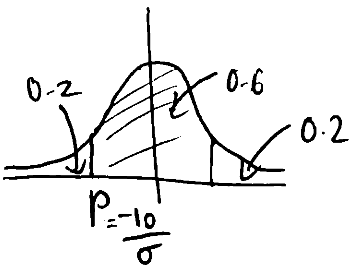
$W \stackrel{d}{=} N(\mu = 200, \sigma = ?)$

When either the standard deviation and/or the mean is unknown, we need to standardize



We require σ, so standardize: $Z = \dfrac{W - 200}{\sigma}$

$\Pr(190 < W < 210) = \Pr\left(-\dfrac{10}{\sigma} < Z < \dfrac{10}{\sigma}\right)$



$p = \text{invNorm}(0.2, 0, 1)$

$\therefore p = -0.8416$

$\therefore \dfrac{-10}{\sigma} = -0.8416$

$\therefore \sigma = \dfrac{10}{0.8416} \approx 11.88\text{ g}$

(b) $\Pr(\text{customer complains}) = \Pr(W < 180)$

≈ 0.0462

Question 5

The time, in minutes, taken to fly from London to Malaga has a normal distribution with mean 150 minutes and standard deviation 10 minutes.

(a) Find the probability that the next flight from London to Malaga takes less than 145 minutes.

The time taken to fly from London to Berlin has a normal distribution with mean 100 minutes and standard deviation **d** minutes.

Given that 15% of the flights from London to Berlin take longer than 115 minutes,

(b) find the value of the standard deviation **d**.

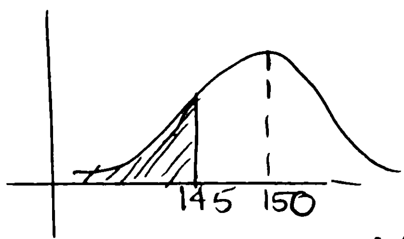
The time, **X** minutes, taken to fly from London to another city has a normal distribution with mean μ minutes.

Given that $P(X < \mu - 15) = 0.35$

(c) find $P(X > \mu + 15 \mid X > \mu - 15)$.

(a)

$\Pr(T<145) = 0.3085$

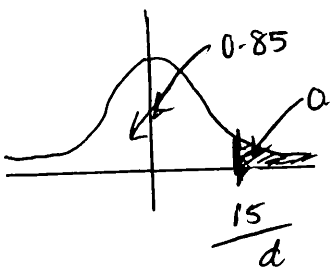


normalcdf$(-\infty, 145, 150, 10)$

(b) Need to find σ, so standardize.

$T \stackrel{d}{=} N(\mu = 100, \sigma = d)$

$\Pr(T > 115) = 0.15$ $\quad \therefore \Pr\left(Z > \frac{115-100}{d}\right) = 0.15$

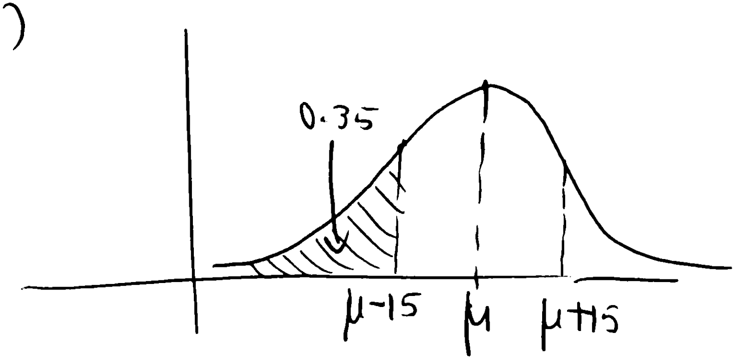


$\therefore \frac{15}{d} = \text{invNorm}(0.85, 0, 1)$

$\frac{15}{d} = 1.03643$

$\therefore d \simeq 14.47$ min

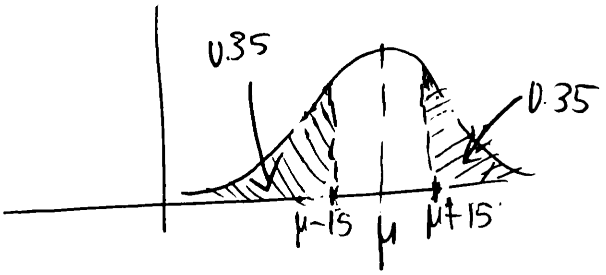
(c)



$$Pr(X < \mu - 15) = 0.35$$

$$Pr(X < \mu+15 \mid X > \mu - 15) = \frac{Pr(X<\mu+15 \cap X>\mu-15)}{Pr(X>\mu-15)}$$

$$= \frac{Pr(\mu-15 < X < \mu+15)}{Pr(X>\mu-15)}$$



$$= \frac{0.3}{0.65}$$

$$= \frac{30}{65} = \frac{6}{13}$$

Question 6

From experience a high-jumper knows that he can clear a height of at least 1.78 m once in 5 attempts. He also knows that he can clear a height of at least 1.65 m on 7 out of 10 attempts.

Assuming that the heights the high-jumper can reach follow a Normal distribution,

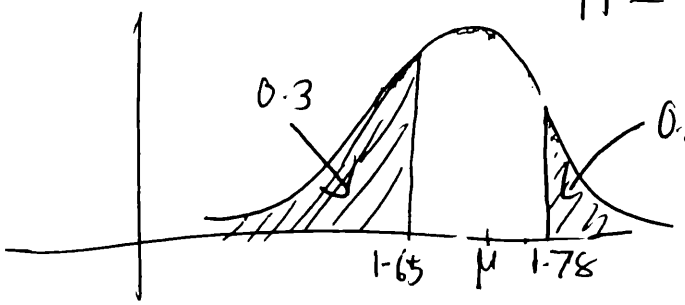
(a) draw a sketch to illustrate the above information,

(b) find, to 3 decimal places, the mean and the standard deviation of the heights the high-jumper can reach,

(c) calculate the probability that he can jump at least 1.74 m.

$H \overset{d}{=} N(\mu = ?, \sigma = ?)$

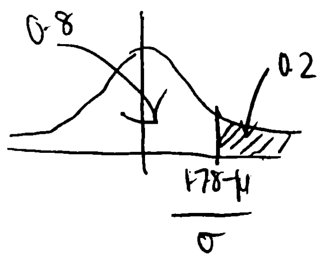
(a)



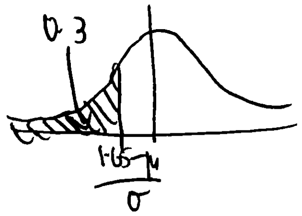
(b) $z = \dfrac{H - \mu}{\sigma}$

$\Pr(H > 1.78) = \Pr\left(z > \dfrac{1.78 - \mu}{\sigma}\right) = 0.2$

$\Pr(H < 1.65) = \Pr\left(z < \dfrac{1.65 - \mu}{\sigma}\right) = 0.3$



$\dfrac{1.78 - \mu}{\sigma} = \text{invNorm}(0.8, 0, 1) = 0.841621$



$\dfrac{1.65 - \mu}{\sigma} = \text{invNorm}(0.3, 0, 1) = -0.5244005$

Solving $\mu \simeq 1.6999055, \ \sigma = 0.0952$

$\therefore \mu = 1.700, \ \sigma = 0.095$

(c) $\Pr(H > 1.74) \simeq 0.337$ $\quad H \overset{d}{=} N(\mu = 1.700, \sigma = 0.0952)$