Question 18 (2008)

Let $f: \left[0, \frac{\pi}{2}\right] \to R$, $f(x) = \sin(4x) + 1$. The graph of f is transformed by a reflection in the x-axis followed by a dilation of factor 4 from the y-axis.
The resulting graph is defined by

A. $g: \left[0, \frac{\pi}{2}\right] \to R$ $\quad g(x) = -1 - 4\sin(4x)$

B. $g: [0, 2\pi] \to R$ $\quad g(x) = -1 - \sin(16x)$

C. $g: \left[0, \frac{\pi}{2}\right] \to R$ $\quad g(x) = 1 - \sin(x)$

D. $g: [0, 2\pi] \to R$ $\quad g(x) = 1 - \sin(4x)$

E. $g: [0, 2\pi] \to R$ $\quad g(x) = -1 - \sin(x)$

$f_1(x) = \sin(4x) + 1$

$f_2(x) = -f_1(x) = -(\sin(4x) + 1)$

$= -\sin(4x) - 1$

$f_3(x) = f_2\left(\frac{x}{4}\right) = -\sin(x) - 1$

Domain $\left[0, \frac{\pi}{2}\right]$ becomes

$\left[0 \times 4, \frac{1}{2} \times 4\right] = [0, 2\pi]$

Question 1 (2010)

The function with rule $f(x) = 4\tan\left(\frac{x}{3}\right)$ has period

A. $\frac{\pi}{3}$

B. 6π

C. 3

D. 3π

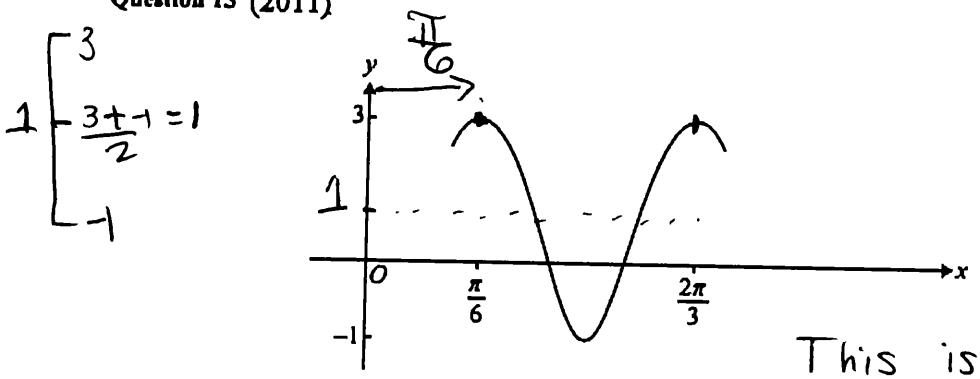
E. $\frac{2\pi}{3}$

Period $= \frac{\pi}{h}$

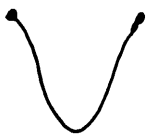
$= \frac{\pi}{1/3}$

$= 3\pi$

Question 15 (2011)



Basic cosine shape



The graph shown could have equation

A. $y = 2\cos\left(x + \frac{\pi}{6}\right) + 1$

B. $y = 2\cos 4\left(x - \frac{\pi}{6}\right) + 1$

C. $y = 4\sin 2\left(x - \frac{\pi}{12}\right) - 1$

D. $y = 3\cos\left(2x + \frac{\pi}{6}\right) - 1$

E. $y = 2\sin\left(4x + \frac{2\pi}{3}\right) - 1$

This is a cosine function which has been translated horizontally and vertically

$a = 3 - 1 = 2$

Period $= \frac{2\pi}{3} - \frac{\pi}{6} = \frac{\pi}{2}$

$\therefore \frac{2\pi}{n} = \frac{\pi}{2} \quad \therefore n = 4$

$\therefore y = 2\cos\left(4\left(x - \frac{\pi}{6}\right)\right) + 1$

Question 1 (2012)

The function with rule $f(x) = -3\sin\left(\frac{\pi x}{5}\right)$ has period

A. 3

B. 5

C. 10

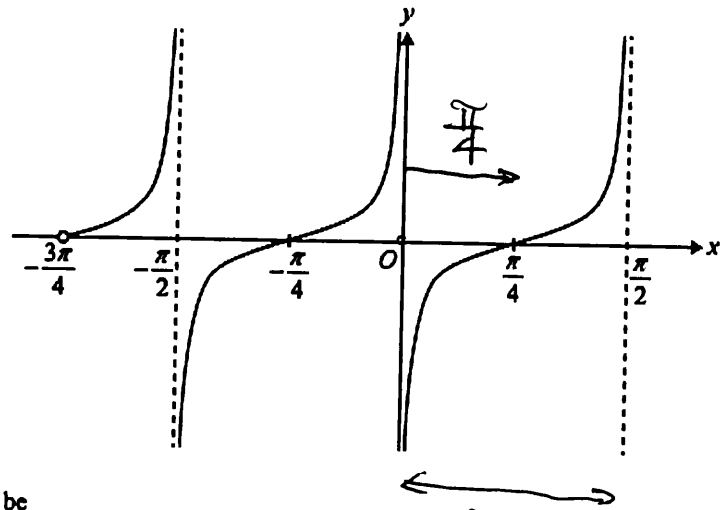
D. $\frac{\pi}{5}$

E. $\frac{\pi}{10}$

$\frac{2\pi}{\frac{\pi}{5}} = 2\pi \times \frac{5}{\pi} = 10$

Question 6 (2012)

A section of the graph of f is shown below.



The rule of f could be

A. $f(x) = \tan(x)$

B. $f(x) = \tan\left(x - \frac{\pi}{4}\right)$

C. $f(x) = \tan\left(2\left(x - \frac{\pi}{4}\right)\right)$

D. $f(x) = \tan\left(2\left(x - \frac{\pi}{2}\right)\right)$

E. $f(x) = \tan\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$

Period $= \frac{\pi}{2}$

$\therefore \frac{\pi}{n} = \frac{\pi}{2} \quad \therefore n = 2$

Translated $\frac{\pi}{4}$ to right

$\therefore y = \tan\left(2\left(x - \frac{\pi}{4}\right)\right)$

Question 7

The temperature, T °C, inside a building t hours after midnight is given by the function

$$f:[0, 24] \to R,\ T(t) = 22 - 10\cos\left(\frac{\pi}{12}(t-2)\right)$$

The average temperature inside the building between 2 am and 2 pm is

A. 10 °C

B. 12 °C

C. 20 °C

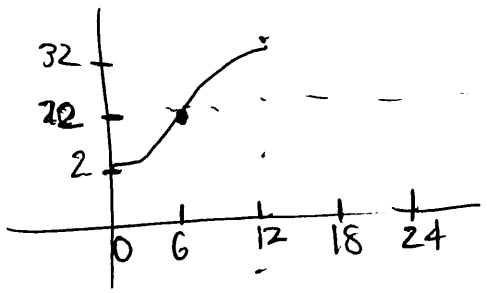
D. 22 °C

E. 32 °C

Graph on CAS

$t = 0$ 2 AM

$t = 12$ 2 PM



Average temperature is at $T = 22$ since exactly as much of graph above as below $T = 22$.

Question 1 (2013)

The function with rule $f(x) = -3\tan(2\pi x)$ has period

A. $\frac{2}{\pi}$

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

E. 2π

$\frac{\pi}{2\pi} = \frac{1}{2}$

Question 1 (12 marks) (2013)

Trigg the gardener is working in a temperature-controlled greenhouse. During a particular 24-hour time interval, the temperature (T °C) is given by $T(t) = 25 + 2\cos\left(\frac{\pi t}{8}\right)$, $0 \le t \le 24$, where t is the time in hours from the beginning of the 24-hour time interval.

a. State the maximum temperature in the greenhouse and the values of t when this occurs. **2 marks**

Max temp = 27°

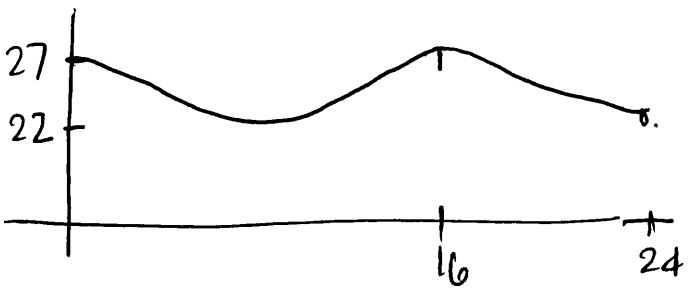
$t = 0, 16$

+2 → 27, 25

−2 → 23

b. State the period of the function T. **1 mark**

16 hours



Period $= \frac{2\pi}{\frac{\pi}{8}} = 16$ hours

Question 1 (7 marks)

The population of wombats in a particular location varies according to the rule $n(t) = 1200 + 400\cos\left(\frac{\pi t}{3}\right)$, where n is the number of wombats and t is the number of months after 1 March 2013.

a. Find the period and amplitude of the function n. **2 marks**

Period = $\frac{2\pi}{\pi/3} = \frac{2\pi}{1} \times \frac{3}{\pi} = 6$ months

Amp = 400

b. Find the maximum and minimum populations of wombats in this location. **2 marks**

Max = 1600

Min = 800

1600 ↑ +400
1200
800 ↓ -400

c. Find $n(10)$. **1 mark**

$n(10) = 1200 + 400\cos\left(\frac{10\pi}{3}\right)$

$= 1200 + 400 \times \frac{-1}{2} = 1000$

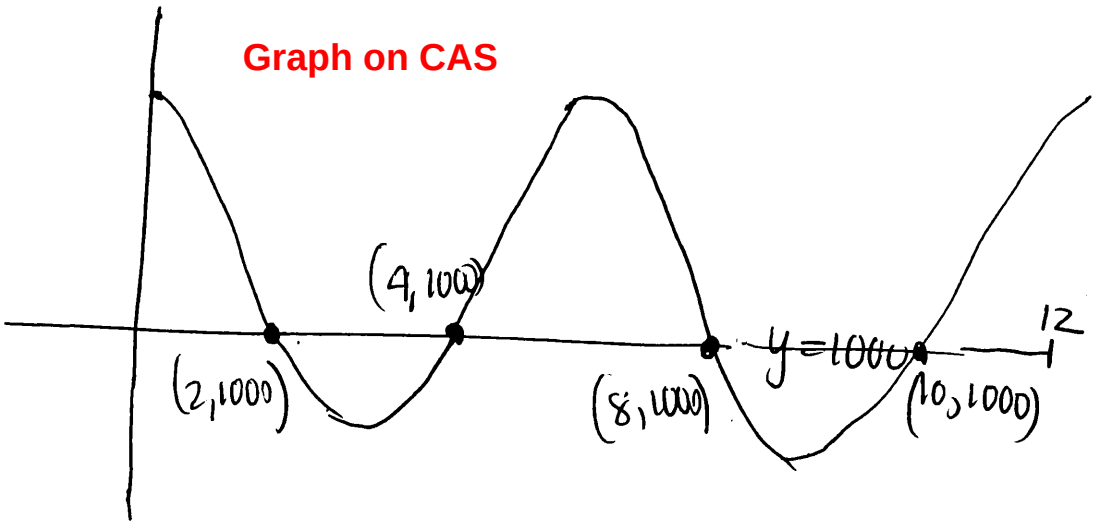
d. Over the 12 months from 1 March 2013, find the fraction of time when the population of wombats in this location was less than $n(10)$. **2 marks**

$n(t) < 1000$ for $t \in (2,4) \cup (8,10)$

$\therefore n(t) < 1000$ for a total of 4 months

$4/12 = \frac{1}{3}$ $\therefore$ Fraction required $= \frac{1}{3}$

Graph on CAS



Question 1 (2015)

Let $f: R \to R, f(x) = 2\sin(3x) - 3$.

The period and range of this function are respectively

(6)

A. period = $\frac{2\pi}{3}$ and range = $[-5, -1]$

B. period = $\frac{2\pi}{3}$ and range = $[-2, 2]$

C. period = $\frac{\pi}{3}$ and range = $[-1, 5]$

D. period = 3π and range = $[-1, 5]$

E. period = 3π and range = $[-2, 2]$

+2 ↑ −1
−3
−2 ↓ −5

Range: $[-5, -1]$

Period: $\frac{2\pi}{3}$

Question 2 (2016)

Let $f: R \to R, \ f(x) = 1 - 2\cos\left(\frac{\pi x}{2}\right)$.

The period and range of this function are respectively

A. 4 and $[-2, 2]$

B. 4 and $[-1, 3]$

C. 1 and $[-1, 3]$

D. 4π and $[-1, 3]$

E. 4π and $[-2, 2]$

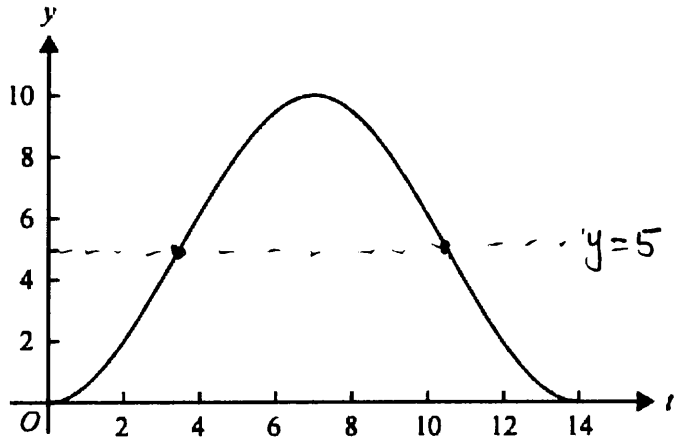
3 ↑ +2
1
−1 ↓ −2

Range: $[-1, 3]$

Period: $\frac{2\pi}{\frac{\pi}{2}} = 4$

Question 8

The UV index, y, for a summer day in Melbourne is illustrated in the graph below, where t is the number of hours after 6 am.



$\frac{10+0}{2} = 5$

$\frac{2\pi}{n} = 14$

$\therefore n = \frac{\pi}{7}$

Period = 14

Amplitude = 5

Reflected Cosine graph

Vertical translation of +5

$\therefore y = -5\cos\left(\frac{\pi x}{7}\right) + 5$

$y = 5 - 5\cos\left(\frac{\pi x}{7}\right)$

The graph is most likely to be the graph of

A. $y = 5 + 5\cos\left(\frac{\pi t}{7}\right)$

B. $y = 5 - 5\cos\left(\frac{\pi t}{7}\right)$

C. $y = 5 + 5\cos\left(\frac{\pi t}{14}\right)$

D. $y = 5 - 5\cos\left(\frac{\pi t}{14}\right)$

E. $y = 5 + 5\sin\left(\frac{\pi t}{14}\right)$

Question

Consider the function: $g(x) = 5\cos\left(2\left(x-\frac{\pi}{6}\right)\right)-1$

Period $= \frac{2\pi}{2} = \pi$

Translation: $\frac{\pi}{6}$ to right

Scale along x-axis: $\frac{\pi}{12}$ per unit

Quarter period $= \frac{\pi}{4}$

Write down the range of this function.

$+5 \uparrow \quad 4$

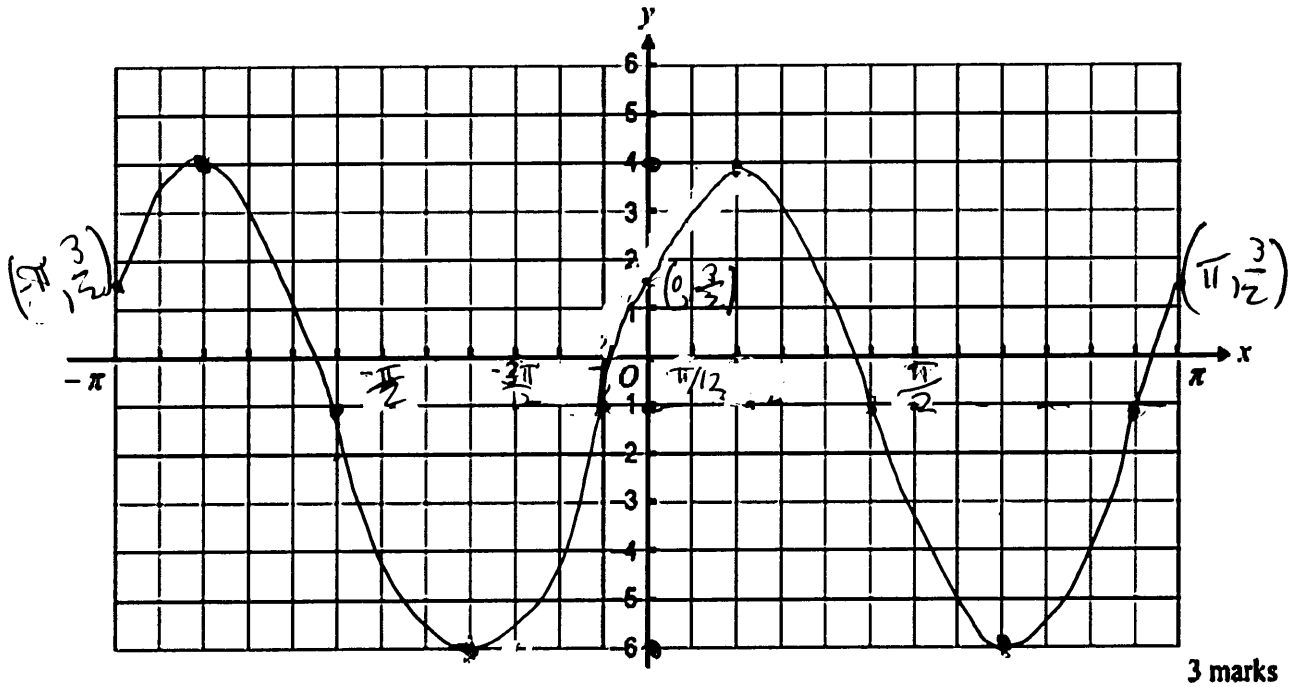
-1

$-5 \downarrow \quad -6$

$[-6, 4]$

2 marks

b. sketch the graph of the function f on the set of axes below. Label axes intercepts with their coordinates. Label endpoints of the graph with their coordinates.



3 marks

$5\cos\left(2\left(\pi-\frac{\pi}{6}\right)\right)-1$

$= 5\cos\left(2\times\frac{5\pi}{6}\right)-1$

$= 5\cos\left(\frac{5\pi}{3}\right)-1$

$= 5\times\frac{1}{2}-1$

$= \frac{3}{2}$

$g(0) = 5\cos\left(-\frac{\pi}{3}\right)-1$

$= 5\times\frac{1}{2}-1$

$= \frac{3}{2}$

$\therefore$ y-int: $\left(0, \frac{3}{2}\right)$

Question

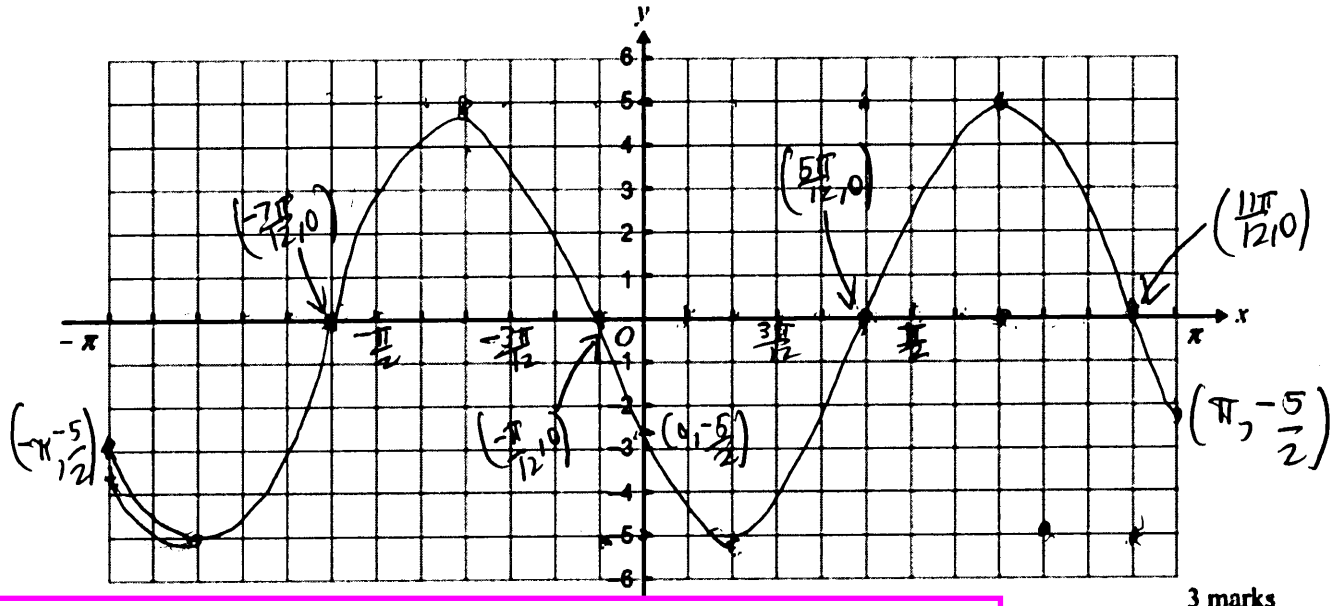
For the function $f:[-\pi, \pi] \to R$, $f(x) = 5\cos\left(2\left(x+\frac{\pi}{3}\right)\right)$

a. write down the amplitude and period of the function

$$\text{amp} = 5$$
$$\text{Period} = \frac{2\pi}{2} = \pi$$

2 marks

b. sketch the graph of the function f on the set of axes below. Label axes intercepts with their coordinates. Label endpoints of the graph with their coordinates.



3 marks

Horizontal shift: $\frac{\pi}{3}$ to right

Quarter period = $\frac{\pi}{4}$

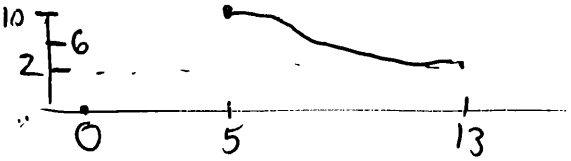
∴ Suitable scale along x-axis: $\frac{\pi}{12}$ per unit

y-int: $f(0) = 5\cos\left(\frac{2\pi}{3}\right) = -\frac{5}{2}$

Question The height of the water in a bay varies sinusoidally over time. On a certain day off the coast of Maine, a high tide of 10 feet occurred at 5:00 A.M. and a low tide of 2 feet occurred at 1:00 P.M. Write a model for the height h (in feet) of the water as a function of time t (in hours since midnight).

$\frac{2\pi}{n} = 16$

$n = \frac{\pi}{8}$



Half period = 8 hours

∴ Period = 16 hours

Cosine graph shifted 5 to right

Amp = 4 and 6 up

$h(t) = 6 + 4\cos\left(\frac{\pi}{8}(t-5)\right)$