DISCRETE RANDOM VARIABLES
CAS FREE

ANSWERS

Question 7

The random variable X has this probability distribution.

X	0	1	2	3	4
$\Pr(X = x)$	0.1	0.2	0.4	0.2	0.1

Find

a. $\Pr(X > 1 \mid X \leq 3)$

$$\Pr(x>1 \mid x \leq 3) = \frac{\Pr(x>1 \cap x \leq 3)}{\Pr(x \leq 3)} = \frac{\Pr(1 < x \leq 3)}{\Pr(x \leq 3)}$$

$$= \frac{0.4 + 0.2}{1 - 0.1}$$

$$= \frac{6}{9} = \frac{2}{3}$$

2 marks

b. $\mathrm{Var}(X)$, the variance of X.

$$\mathrm{Var}(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = 0^2 \times 0.1 + 1^2 \times 0.2 + 2^2 \times 0.4 + 3^2 \times 0.2 + 4^2 \times 0.1$$

$$= 0 + 1 \times 0.2 + 4 \times 0.4 + 9 \times 0.2 + 16 \times 0.1$$

$$= 0.2 + 1.6 + 1.8 + 1.6$$

$$= 5.2$$

3 marks

$$E(x) = 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.2 + 4 \times 0.1$$

$$= 0.2 + 0.8 + 0.6 + 0.4$$

$$= 2$$

$$\therefore \mathrm{Var}(x) = 5.2 - (2)^2$$

$$= 1.2$$

Question 8

The discrete random variable X has the probability distribution

x	-1	0	1	2
$\Pr(X = x)$	p^2	p^2	$\frac{p}{4}$	$\frac{4p+1}{8}$

Find the value of p.

$$p^2 + p^2 + \frac{p}{4} + \frac{4p+1}{8} = 1$$

$$2p^2 + \frac{2p}{8} + \frac{4p+1}{8} = 1$$

$$2p^2 + \frac{6p+1}{8} - 1 = 0$$

$$\therefore 16p^2 + 6p + 1 - 8 = 0$$

$$16p^2 + 6p - 7 = 0$$

$$(8p + 7)(2p - 1) = 0$$

$$\therefore p = \frac{1}{2} \quad (\text{since } p > 0)$$

3 marks

Question 4

On any given day, the number X of telephone calls that Daniel receives is a random variable with probability distribution given by

x	0	1	2	3
$\Pr(X = x)$	0.2	0.2	0.5	0.1

a. Find the mean of X.

$$E(x) = 0\times0.2 + 1\times0.2 + 2\times0.5 + 3\times0.2$$
$$= 0.2 + 1 + 0.3$$
$$= 1.5$$

2 marks

b. What is the probability that Daniel receives only one telephone call on each of three consecutive days?

$\Pr(x = 1) = 0.2$

Pr (only one phone call on 3 consecutive days)

$$= (0.2)^3$$
$$= 0.008$$

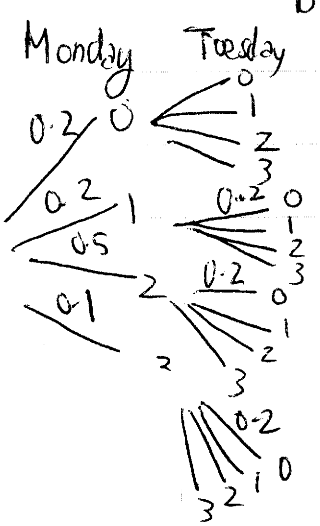
1 mark

c. Daniel receives telephone calls on both Monday and Tuesday.

What is the probability that Daniel receives a total of four calls over these two days?

Let A = Daniel receives phone calls on both M & T

B = Daniel receives a total of four calls



$$\Pr(B|A) = \frac{\Pr(B\cap A)}{\Pr(A)}$$

3 marks

$$\Pr(B\cap A) = \Pr(1,3) + \Pr(2,2) + \Pr(3,1)$$
$$= 0.2\times0.1 + 0.5\times0.5 + 0.1\times0.2$$
$$= 0.29$$

$$\Pr(A) = 1 - 0.2 - 0.2\times0.2 - 0.5\times0.2 - 0.1\times0.2$$
$$= 1 - 0.2 - 0.04 - 0.1 - 0.02 = 0.64$$

$$\therefore \Pr(B|A) = \frac{0.29}{0.64} = \boxed{\frac{29}{64}}$$

Question 7 (6 marks)

The probability distribution of a discrete random variable, X, is given by the table below.

x	0	1	2	3	4
$\Pr(X = x)$	0.2	$0.6p^2$	0.1	$1-p$	0.1

a. Show that $p = \frac{2}{3}$ or $p = 1$. 3 marks

$0.2 + 0.6p^2 + 0.1 + 1 - p + 0.1 = 1$

$0.6p^2 - p + 1.4 = 1$

$0.6p^2 - p + 0.4 = 0$

$6p^2 - 10p + 4 = 0$

$3p^2 - 5p + 2 = 0$

$(p-1)(3p-2) = 0$

$\therefore p = 1$ or $p = \frac{2}{3}$

$\frac{3}{5} \times \frac{4}{9} = \frac{4}{15}$

b. Let $p = \frac{2}{3}$.

i. Calculate $E(X)$. 2 marks

x	0	1	2	3	4
$\Pr(X=x)$	0.2	$0.6 \times \frac{4}{9} = \frac{4}{15}$	0.1	$\frac{1}{3}$	0.1

$$E(X) = 0 \times 0.2 + 1 \times \frac{4}{15} + 2 \times 0.1 + 3 \times \frac{1}{3} + 4 \times 0.1$$
$$= \frac{4}{15} + 0.2 + 1 + 0.4$$
$$= \frac{8}{30} + \frac{6}{30} + 1 + \frac{12}{30} = \frac{56}{30} = \frac{28}{15}$$

ii. Find $\Pr(X \geq E(X))$. 1 mark

$$\Pr\left(X \geq \frac{28}{15}\right) = \Pr(X > 1)$$
$$= \Pr(X=2) + \Pr(X=3) + \Pr(X=4)$$
$$= 0.1 + 0.1 + \frac{1}{3}$$
$$= \frac{1}{5} + \frac{1}{3}$$
$$= \frac{3}{15} + \frac{5}{15} = \frac{8}{15}$$

Question 9 (5 marks)

The number of customers, X, waiting to be served in a bakery at 9:00 am has the probability distribution given in the table below.

X	0	1	2	3	4
$p(x)$	$\frac{3k^2-1}{7}$	$\frac{3k}{7}$	$\frac{4k}{7}$	$\frac{2k}{7}$	$\frac{k}{7}$

a. Find the value of k. 3 marks

$$\frac{3k^2-1}{7} + \frac{3k}{7} + \frac{4k}{7} + \frac{2k}{7} + \frac{k}{7} = 1$$

$$\frac{3k^2-1}{7} + \frac{10k}{7} = 1$$

$$3k^2 - 1 + 10k = 7$$

$$3k^2 + 10k - 8 = 0$$

$$(3k-2)(k+4) = 0 \quad \therefore k = \frac{2}{3} \quad (k>0)$$

b. Calculate the probability that there is at least one customer in the shop at 9:00 am. 2 marks

$$\Pr(X \geq 1) = 1 - \Pr(X=0)$$

$$= 1 - \left(\frac{3k^2-1}{7}\right)$$

$$= 1 - \left(\frac{3\times\left(\frac{2}{3}\right)^2 - 1}{7}\right)$$

$$= 1 - \left(\frac{3\times\frac{4}{9} - 1}{7}\right)$$

$$= 1 - \frac{\frac{1}{3}}{7}$$

$$= 1 - \frac{1}{21} = \frac{20}{21}$$

Question 5 (4 marks)

The number of deliveries made to a business on any given work day can be represented by the random variable X.
The probability distribution of X is shown in the table below.

x	0	1	2	3
$\Pr(X=x)$	0.3	0.4	0.2	0.1

The mean number of deliveries is 1.1.

a. Find the variance of X.

2 marks

$$\begin{array}{r} 2.90 \\ -\ 1.21 \\ \hline 1.69 \end{array}$$

$\text{var}(X) = E(X^2) - 1.1^2$

$E(X^2) = 0^2 \times 0.3 + 1^2 \times 0.4 + 2^2 \times 0.4 + 3^2 \times 0.1$

$= 0 + 0.4 + 1.6 + 0.9 = 2.9$

$\therefore \text{var}(X) = 2.9 - (1.1)^2$

$= 2.9 - 1.21$

$= 1.69$

b. Find the probability that on a work day when there is a delivery, the number of deliveries is two or more.

2 marks

A = "there is a delivery"

B = "there are two or more deliveries"

$\Pr(B|A) = \dfrac{\Pr(B \cap A)}{\Pr(A)} \quad \dfrac{0.2 + 0.1}{1 - 0.3}$

$= \dfrac{3}{7}$

Question 7 (3 marks)

A discrete random variable X has a probability distribution given by

X	1	2
$\Pr(X = x)$	$\dfrac{1}{2e^k}$	$\dfrac{e^k}{4}$

Find the value(s) of k.

$$\frac{1}{2e^k} + \frac{e^k}{4} = 1$$

$$\frac{1}{2}e^{-k} + \frac{e^k}{4} = 1$$

$$\therefore 2e^{-k} + e^k = 4$$

Let $e^k = p$

$$\frac{2}{p} + p = 4$$

$$2 + p^2 = 4p$$

$$p^2 - 4p + 2 = 0$$

$$p = \frac{4 \pm \sqrt{8}}{2}$$

$$p = \frac{4 \pm 2\sqrt{2}}{2}$$

$$\therefore p = 2 \pm \sqrt{2}$$

$$\therefore e^k = 2 + \sqrt{2} \quad \text{or} \quad e^k = 2 - \sqrt{2}$$

$$k = \log_e(2 + \sqrt{2}) \quad \text{or} \quad k = \log_e(2 - \sqrt{2})$$

CAS ALLOWED

A parking building which is open for 7 hours a day has the following fee policy: 18 dollars per hour for the first 3 hours of parking, and 6 dollars for each additional hour. Many years of data shows that the number of hours of parking for a car, denoted X, is a discrete random variable with probability function

$$P(X=k)=\begin{cases}\frac{8-k}{28} & (k=1,2,\cdots,7)\\ 0 \text{ otherwise.}\end{cases}$$

What is the expected parking charge for a car in dollars under this policy?

X	1	2	3	4	5	6	7
$Pr(X=x)$	$\frac{7}{28}$	$\frac{6}{28}$	$\frac{5}{28}$	$\frac{4}{28}$	$\frac{3}{28}$	$\frac{2}{28}$	$\frac{1}{28}$

X = no. of hours.

Charges:

X	Charge C
1	18
2	36
3	54
4	60
5	66
6	72
7	78

C	18	36	54	60	66	72	78
$Pr(C=c)$	$\frac{7}{28}$	$\frac{6}{28}$	$\frac{5}{28}$	$\frac{4}{28}$	$\frac{3}{28}$	$\frac{2}{28}$	$\frac{1}{28}$

$$E(C) = 18\times\frac{7}{28} + 36\times\frac{6}{28} + 54\times\frac{5}{28} + 60\times\frac{4}{28} + 66\times\frac{3}{28} + \frac{72\times2}{28} + \frac{78\times1}{28}$$

$$= \frac{126}{28} + \frac{216}{28} + \frac{270}{28} + \frac{240}{28} + \frac{198}{28} + \frac{144}{28} + \frac{78}{28}$$

$$= \frac{1272}{28} \simeq \$45.43$$