ANSWERS

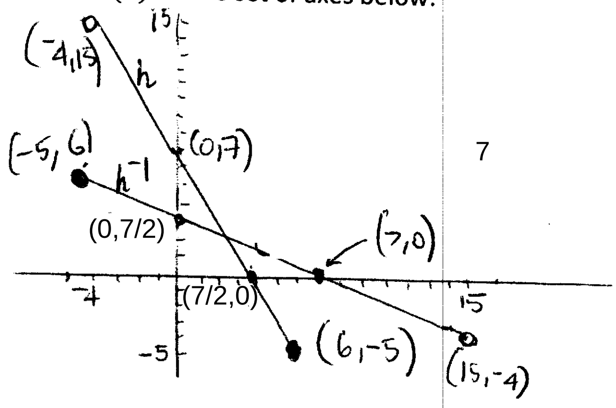
Exercise 5G Exam Style Questions

Question 1

Let $h: (-4,6] \to R, h(x) = -2x + 7$

a. Sketch the graph of $h(x)$ on the set of axes below:

$h(-4) = 15,\ h(6) = -5$



Make sure you include all intercepts for both functions

Make sure you label h and its inverse clearly

Make sure you include all endpoint coordinates

b. Complete the domain range table below:

Dom(h) $(-4,6]$	Ran (h) $[-5,15)$
Dom(h^{-1}) $[-5,15)$	Ran(h^{-1}) $(-4,6]$

c. Specify the inverse function h^{-1}.

$$y = -2x + 7$$
$$x = -2y + 7$$
$$2y = -x + 7$$
$$y = -\frac{x}{2} + \frac{7}{2}$$
$$h^{-1}: [-5, 15) \to R,\ h^{-1}(x) = -\frac{x}{2} + \frac{7}{2}$$

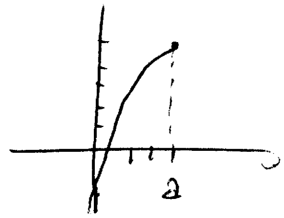
d. Find the co-ordinates of the point where h and h^{-1} intersect.

$$y = x$$
$$y = -2x + 7$$
$$\therefore\ x = -2x + 7$$
$$3x = 7$$
$$x = \frac{7}{3}$$

$\therefore$ Point of intersection: $\left(\frac{7}{3}, \frac{7}{3}\right)$

Question 2

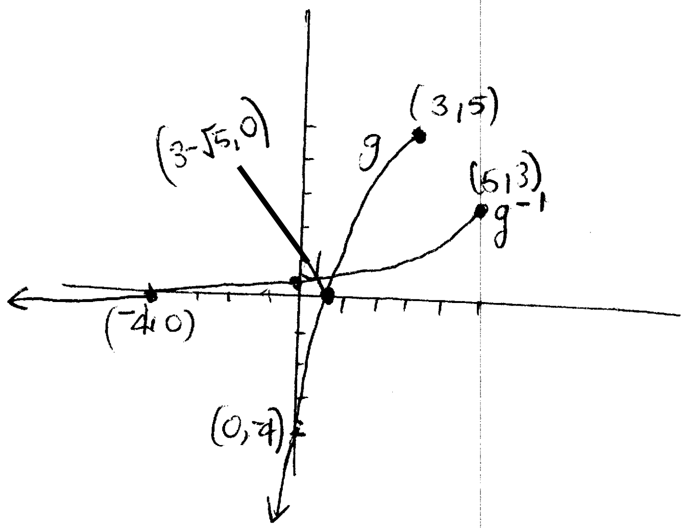
Let $u: (-\infty, a] \rightarrow R, u(x) = -(x-3)^2 + 5$



a. Find the largest value of *a* for which the function *u* has an inverse function.

$a = 3$

b. Sketch the graph of *u*(x) on the diagram below.



$-(x-3)^2+5=0$

$(x-3)^2=5$

$x-3=\pm\sqrt{5}$

$x=3\pm\sqrt{5}$

c. Fill out the domain range table below:

Dom(u)	Ran (u)
$(-\infty, 3]$	$(-\infty, 5]$
Dom(u^{-1})	**Ran(u^{-1})**
$(-\infty, 5]$	$(-\infty, 3]$

d. Sketch the graph of $u^{-1}(x)$ on the same set of axes.

$u(x)$

x-intercept: $(3-\sqrt{5}, 0)$

y-intercept: $(0, -4)$

$u^{-1}(x)$

x-intercept: $(-4, 0)$

y-intercept: $(0, 3-\sqrt{5})$

If you can't comfortably fit all intercept co-ordinates on graph, simply list them

$$y = x$$
$$y = -(x-3)^2 + 5$$

e. Find $\{x: u(x) = u^{-1}(x)\}$

$$x = -(x-3)^2 + 5$$
$$(x-3)^2 + x - 5 = 0$$
$$x^2 - 6x + x + 4 = 0$$
$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$
$$x = 4, 1$$

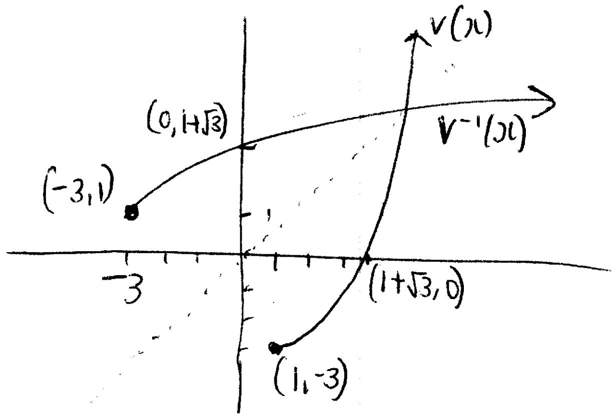
But from diagram we must reject $x = 4$
(Also, since $\mathrm{dom}(u) = (-\infty, 3]$, $x = 4$ lies outside its domain)

$\therefore$ x = 1

Question 3

Let $v: [-3, \infty) \to R, v(x) = \sqrt{x+3} + 1$

a. Sketch the graph of v.



b. Explain why v has an inverse function. It is a one-to-one function

c. Create a domain/range table, displaying the domains and ranges of both v and its inverse function.

$\mathrm{dom}(v)$ $[-3, \infty)$	$\mathrm{ran}(v)$ $[1, \infty)$
$\mathrm{dom}(v^{-1})$ $[1, \infty)$	$\mathrm{ran}(v^{-1})$ $[-3, \infty)$

d. Sketch the graph of v^{-1} on the same diagram above.

e. Specify v^{-1}.

$$y = \sqrt{x+3} + 1$$

$$\downarrow$$

$$x = \sqrt{y+3} + 1$$

$$x - 1 = \sqrt{y+3}$$

$$(x-1)^2 = y + 3$$

$$y = (x-1)^2 - 3$$

$$v^{-1}: [1, \infty) \to R, \quad v^{-1}(x) = (x-1)^2 - 3$$

f. Find the exact co-ordinates of the point where *v* and its inverse intersect.

$$y = x$$

$$y = (x-1)^2 - 3$$

$$(x-1)^2 - 3 = x$$

$$x^2 - 2x + 1 - 3 = x$$

$$x^2 - 3x - 2 = 0$$

Use Quadratic Formula

$$a = 1, \; b = -3, \; c = -2$$

$$x = \frac{--3 \pm \sqrt{(-3)^2 - 4 \times 1 \times -2}}{2}$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

From diagram, the value of x must be $\frac{3+\sqrt{17}}{2}$

since $\mathrm{dom}(v^{-1}) = [1, \infty)$

$\therefore$ Intersection point: $\left(\frac{3+\sqrt{17}}{2}, \frac{3+\sqrt{17}}{2}\right)$