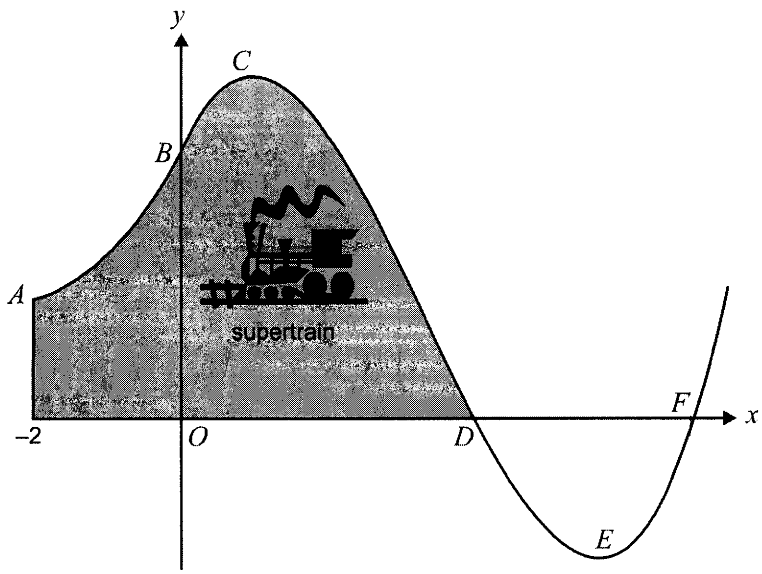
Question 4



A part of the track for Tim's model train follows the curve passing through A, B, C, D, E and F shown above. Tim has designed it by putting axes on the drawing as shown. The track is made up of two curves, one to the left of the y-axis and the other to the right.

B is the point $(0, 7)$.

The curve from B to F is part of the graph of $f(x) = px^3 + qx^2 + rx + s$ where p, q, r and s are constants and $f'(0) = 4.25$.

a. **i.** Show that $s = 7$.

$f(0) = 7 \quad \therefore 7 = p(0)^3 + q(0)^2 + r(0) + s$

$\therefore s = 7$

ii. Show that $r = 4.25$.

$f'(x) = 3px^2 + 2qx + r$

$f'(0) = 4.25$

$\therefore 4.25 = r$

1 + 1 = 2 marks

The furthest point reached by the track in the positive y direction occurs when $x = 1$. Assume $p > 0$.

b. **i.** Use this information to find q in terms of p.

$f'(1) = 0 \quad \therefore 3p + 2q + 4.25 = 0$

$\therefore 2q = -4.25 - 3p$

$q = -2.125 - \frac{3p}{2}$

ii. Find $f(1)$ in terms of p.

Use CAS to assist!

Defining $f(x) = px^3 + qx^2 + 4.25 + 7$

and $q = \frac{-4.25 - 3p}{2}$

we find: $f(1) = \frac{73}{8} - \frac{p}{2}$

iii. Find the value of a in terms of p for which $f'(a) = 0$ where $a > 1$.

Solving $f'(x) = 0$ gives

$x = \frac{17}{12p}$ or $x = 1$

Since $a > 1$, $a = \frac{17}{12p}$

iv. If $a = \frac{17}{3}$, show that $p = 0.25$ and $q = -2.5$.

$\frac{17}{3} = \frac{17}{12p}$

$\therefore 12p = 3$

$\therefore p = \frac{1}{4}$ and $q = \frac{-4.25 - 3 \times 0.25}{2}$

$= -2.5$

2 + 1 + 1 + 2 = 6 marks

For the following assume $f(x) = 0.25x^3 - 2.5x^2 + 4.25x + 7$.

c. Find the exact coordinates of D and F.

$f(x) = 0.25x^3 - 2.5x^2 + 4.25x + 7$

At D, F, $f(x) = 0$

$x = -1, 4, 7$

$\therefore D = (4, 0), \quad F = (7, 0)$

2 marks

d. Find the greatest distance that the track is from the x-axis, when it is below the x-axis, correct to two decimal places.

Let $f'(x) = 0$

$x = 1, \frac{17}{3}$

At $t, x = \frac{17}{3}$

$f\left(\frac{17}{3}\right) = \frac{-109}{27}$

1 mark

$\therefore$ Greatest distance below x-axis $= 3.70$ km