

Question 8

(10 marks)

In order to buy a second-hand scooter, Kim obtained a personal loan of \$5000 with monthly repayments of \$440 to be paid at the end of each month. The table below shows the amount owing at the start of each month, the interest payable for that month, the repayment and the amount owing at the end of each month for the first six months.

Month	Amount owing at the start of the month (\$)	Interest (\$)	Repayment (\$)	Amount owing at the end of the month (\$)
1	5000	40	440	4600
2	4600	36.80	440	4196.80
3	4196.80	33.57	440	3790.37
4	3790.37	30.32	440	3380.70
5	3380.70	27.05	440	2967.74
6	2967.74	23.74	440	2551.48

- (a) Calculate the annual interest rate. (2 marks)

$$\text{Monthly rate} = \frac{40}{5000} \times 100 = 0.8\%$$

$$\therefore \text{Annual rate} = 12 \times 0.8 = 9.6\%$$

- (b) Write a recursive rule to determine the amount owing at the end of each month. (3 marks)

$$V_{n+1} = R V_n - D \qquad R = 1 + \frac{9.6/12}{100}$$

$$\therefore V_{n+1} = 1.008 V_n - 440 \qquad = 1.008$$

- (c) In which month would Kim pay off the loan? (1 mark)

$$N = ? \qquad FV = 0 \qquad \text{gives } N = 11.96$$

$$I = 9.6 \qquad \text{ply} = 12 \qquad \therefore \text{Pays it off with 12th payment.}$$

$$PMT = -440 \qquad \text{cly} = 12 \qquad \therefore \text{In 12th month}$$

$$PV = 5000$$

- (d) How much is Kim's final repayment? (2 marks)

$$N = 11 \qquad FV = ? \qquad \text{gives } FV = -419.70765$$

$$I = 9.6 \qquad \text{ply} = 12 \qquad \therefore \text{Interest charged on last month} = \frac{0.8}{100} \times 419.708 \approx 3.357$$

$$PMT = -440 \qquad \text{cly} = 12$$

$$PV = 5000 \qquad \therefore \text{Final payment}$$

- (e) How much did Kim actually pay for the scooter? (2 marks)

$$\text{Total paid} = 11 \times 440 + 423.07$$

$$= \$5263.07$$

$$= 419.708 + 3.357 = \$423.07$$

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## Question 2

A car is initially purchased for \$24 000 and depreciates by \$1 700 per year.

- a. Write a recursive relation that gives the value of the car in dollars after  $n + 1$  years, in terms of its value after  $n$  years. Write both parts of the rule, including for  $V_0$ , on the same line, separated by a comma.

$$V_{n+1} = V_n - 1700, V_0 = 24,000$$

- b. Write down a rule that will give the value of the car after  $n$  years.

$$V_n = 24000 - 1700n$$

- c. In which year will the value of the car fall below \$10,100?

$$10,100 = 24000 - 1700n$$

$$\text{Solving for } n: n \approx 8.17$$

$$\therefore \text{In 9th year.}$$

(1 + 1 + 1 = 3 marks)

## Question 3

Tim is starting up his own business. He has saved \$15 000 to buy equipment and he borrows another \$50 000 from the bank. He is charged interest at the rate of 4.5% per annum, compounding monthly, and makes regular monthly repayments of \$400.

- a. Write down a calculation from which the amount that Tim owes at the end of the first month can be evaluated.

$$50000 \times 1.00375 - 400$$
$$= \$49787.50$$

$$R = 1 + \frac{r/n}{100}$$
$$= 1 + \frac{4.5/12}{100} = 1.00375$$

- b. Write a recurrence relation that gives the balance  $B_{n+1}$  in terms of the balance in the preceding month  $B_n$ .

$$B_{n+1} = 1.00375 B_n - 400, B_0 = 50000$$

- c. To the nearest month, how many months does it take for him to pay off his loan?

$$N = ?$$
$$I = 4.5$$
$$PV = 50000$$
$$FV = 0$$
$$PMT = -400$$
$$PY = 12$$
$$CY = 12$$
$$\text{gives } N = 168.99$$
$$\approx 169 \text{ months}$$

d. What is the value of Tim's final repayment?

$N=168$     $ply=12$   
 $I=4.5$     $cy=12$   
 $PV=50000$   
 $FV=?$   
 $PMT=-400$

Interest for last month  
 $= 394.0292 \times 0.00375$   
 $= 1.4776$   
 $\therefore$  Total of last repayment  
 $= \$394.0292 + 1.4776$   
 $= \underline{\$395.51}$

e. How much in total does Tim pay for his equipment?

Total paid

$$= 15000 + 168 \times 400 + 395.51$$

$$= \underline{\$82,595.51}$$

(1+1+1+1+1 = 5 marks)

#### Question 4

A new car depreciates in value each year according to the recursion relation:

$$V_{n+1} = 0.89V_n, V_0 = 21\,000$$

a. How much was the car purchased for?

\$21,000

1 mark

b. As a percentage, what was the annual depreciation rate?

$$1 - 0.89 = 0.11 \quad \underline{11\% \text{ per annum}}$$

1 mark

c. Determine the value of the car after 9 years. Give your answer to the nearest dollar.

$$V = 21000 \times (0.89)^n$$

When  $n=9$ ,  $V = 21000 \times 0.89^9 = \underline{\$7357}$    2 marks

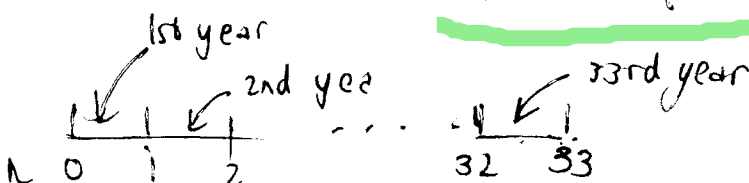
d. When the value of the car reaches \$500, it is considered useful only for parts. At the end of which year will it be considered useful only for parts?

$$21000 \times 0.89^n = 500$$

Solving:  $n = 32.07$

2 marks

$\therefore$  At end of 33rd year



### Question 5

After one year, the value of a company's machinery has depreciated by \$16 020 from \$89 000.

- a. At what rate did the machinery depreciate? Give your answer as a percentage.

$$r = \frac{16020}{89000} \times 100 = 18\%$$

- b. What will the machinery be worth at the end of the second year? Give your answer to the nearest cent.

$$V_0 = 89000$$

$$V_1 = 89000 \times 0.82 = \$72980$$

$$V_2 = 72980 \times 0.82 = \$59843.60$$

- c. Write a recursive relation that gives the value  $V_{n+1}$  of the machinery after  $n+1$  years in terms of its value after  $n$  years, and its initial value  $V_0$ .

$$V_{n+1} = \left(1 - \frac{18}{100}\right) \times V_n, V_0 = 89000$$

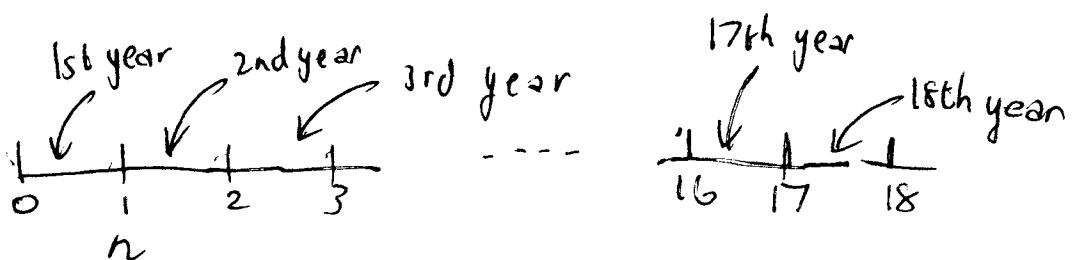
$$\therefore V_{n+1} = 0.82 \times V_n, V_0 = 89000$$

- d. The company bought this machinery at the end of 2011. When its value falls below \$3000, the company will invest in new machinery. In which year will this occur?

$$V = 0.82^n \times 89000$$

Solving:  $3000 = 89000 \times 0.82^n$  (1 + 1 + 2 = 5 marks)  
gives:  $n = 17.08$ .

$\therefore$  In 18th years, value of machinery falls below \$3,000.



### Question 6

Michael buys his first house for \$540,000. He pays a 5% deposit and receives a first home owners grant of \$15,000 from the government. He borrows the remainder from a bank. The interest on the loan is 4.8% per annum on the reducing monthly balance and Michael makes monthly repayments of \$3800.

a. How much is owed at the end of the first month? Give your answer correct to two decimal places.

$$540000 \times 0.05 = \$27000$$

$$R = 1 + \frac{4.8}{\frac{12}{100}} = 1.004$$

Amount borrowed

$$= 540000 - 27000 - 15000$$

$$= \$498,000$$

$\therefore$  Amount owed

$$= 498000 \times 1.004 - 3800$$

$$= \$496,192$$

b. Write a recursive relation, that gives the balance  $B_{n+1}$  after  $n+1$  months in terms of the balance  $B_n$  of the loan at the end of  $n$  months and the initial balance  $B_0$ .

$$B_{n+1} = 1.004 B_n - 3800,$$

$$B_0 = 498,000$$

c. How many repayments does it take for Michael to pay off the loan?

$$N = ? \quad \text{ply} = 12$$

$$I = 4.8 \quad \text{cly} = 12$$

$$PV = 498000 \quad \text{gives } N = 1.86 \cdot 07$$

$$PMT = -3800$$

$$FV = 0$$

$\therefore$  Takes 187 repayments

d. What is the value of Michael's final repayment?

$$N = 187$$

$$I = 4.8$$

$$PV = 498000$$

$$PMT = -3800$$

$$FV = ?$$

$$\text{ply} = 12$$

$$\text{cly} = 12$$

gives  $FV = 3548.37$

Final repayment

$$= 3800 - 3548.37$$

$$= \$251.63$$

e. How much did Michael actually pay for this house? Give your answer to the nearest dollar.

He pays Deposit + All Repayments

$$\begin{aligned} &= 27000 + 186 \times 3800 + 251.63 \\ &= \$734,051.63 \approx \underline{\underline{\$734,052}} \end{aligned}$$

f. If Michael had doubled his monthly repayments and everything else remained the same, how much less would Michael have paid for the house?

$$N = ?$$

$$I = 4.8$$

$$PV = 498,000$$

$$PMT = -7600$$

$$FV = 0$$

$$Ply = 12$$

To calculate final repayment:  
Cly = 12 gives  $N = 76.14$

$$N = 77$$

$$I = 4.8$$

$$PV = 498,000$$

$$PMT = -7600$$

$$FV = ?$$

$$Ply = 12$$

$$Cly = 12$$

$$\text{gives } FV = \$6531.13$$

$$\begin{aligned} \therefore \text{Final payment} &= \$7600 - \$6531.13 \\ &= \$1068.87 \end{aligned}$$

$\therefore$  Total paid

$$= 27000 + 76 \times 7600 + 1068.87$$

$$\begin{aligned} &= \$605,668.87 \therefore \text{Amount less} = 734,052 - 605,669 \\ &= \underline{\underline{\$128,383}} \end{aligned}$$