SOLUTIONS

Question 1

An investment that earns simple interest can be modelled by the following recursion relation:

$$V_{n+1} = V_n + 2840, V_0 = 40\,000$$

Where V_n is the value of the investment after n years.

a. What was the initial amount invested?

$40,000

1 mark

b. What amount of interest was paid each year?

$2840

1 mark

c. Calculate the annual percentage interest rate that the investment was earning. Give your answer correct to ~~two~~ one decimal places

$\frac{2840}{40000} \times \frac{100}{1} = 7.1\%$ per annum

1 mark

Question 2

Sally has $4 000 to invest. She deposits this in an account that earns simple interest at the rate of 7% per annum. Her sister, Eva has $6000 to invest and she deposits in an account that earns 3% per annum simple interest.

a. Write a recursive relation that gives the value of Sally's investment after *n* years.

$V_{n+1} = V_n + D, V_0 = 4000$ $\quad D = \frac{4000 \times 7}{100} = 280$

$\therefore V_{n+1} = V_n + 280, V_0 = 4000$

1 mark

b. Write a recursive relation that gives the value of Eva's investment after n years.

$V_{n+1} = V_n + D, V_0 = 6000$

$\therefore V_{n+1} = V_n + 180, V_0 = 6000$ $\quad D = \frac{6000 \times 3}{100} = 180$

1 mark

c. Determine the number of years after which Eva's investment will be equal in value to Sally's investement

$4000 + 280n = 6000 + 180n$

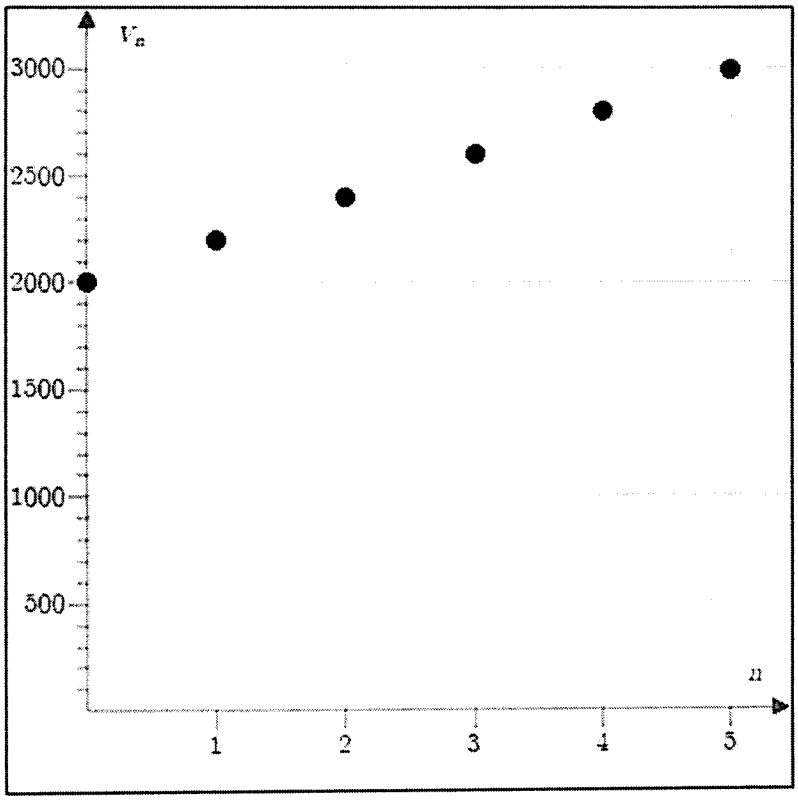
$\therefore 100n = 2000$

$n = 20$ $\therefore$ 20 years

2 marks

Question 3

The value, in dollars, at the end of each year of an investment that pays simple interest annually is graphed below.



$V_0 = 2000$

$V_5 = 3000$

$\therefore D = \frac{3000 - 2000}{5}$

$= 200$

a. Use the graph to determine a recursive relation that gives the value V_{n+1} of the investment after n+1 years in terms of V_n and V_0.

$V_{n+1} = V_n + 200, \quad V_0 = 2000$

1 mark

b. What is the annual interest rate?

$\frac{200}{2000} \times 100 = 10\%$ per annum

1 mark

Question 4

$12 000 is invested in a simple interest account earning interest at 3.6% per annum. Interest is paid at the end of each **month.** No withdrawals or deposits to the account are made.

a. How much interest is paid each month?

Monthly rate $= \frac{3.6}{12}\% = 0.3\%$

$\therefore$ Interest per month $= \frac{12000 \times 0.3}{100} = \$36$

1 mark

b. Write down a recurrence relation which gives the value V_{n+1} of the investment after n+1 months, in terms of V_n and V_0.

$V_{n+1} = V_n + 36, \quad V_0 = 12000$

1 mark

c. At the end of which month does the value of the investment first go past $20,000 ?

$12000 + 36n = 20000$

$\therefore 8000 = 36n$

$n = 222.22$

1 mark

∴ At end of 223rd month

Question 5

$3 000 is invested at the beginning of the year in an account that earns 18% per annum compound interest, compounding monthly.

a. Write down a recursive relation that gives the value V_{n+1} of the investment after n+1 months, in terms of V_n and V_0.

$R = 1 + \frac{r/n}{100}$

$= 1 + \frac{18/12}{100} = 1.015$

$V_{n+1} = 1.015 V_n, \quad V_0 = 3000$

1 mark

b. Write down a rule that gives the value V of the investment after n months.

$V = (1.015)^n \times 3000$

1 mark

c. Calculate the value of the investment after 2 years. Give your answer to the nearest cent.

After 2 years, $n = 24$

$V = 1.015^{24} \times 3000$

$= \$4288.51$

1 mark

Question 6

The following spreadsheet shows the balance of a savings account in 2015, where interest is compounded monthly.

	A	B	C	D
1	Month	Balance at beginning of month	Interest	Balance at end of month
2	July	6000	120	X
3	August	6120	122.40	6242.40
4	September	6242.40	Y	6367.25
5	October	Z	127.35	6494.60
6	November	6494.60	129.89	6624.49

a. Calculate the value of X.

$X = 6120$

1 mark

b. Determine the **annual** interest rate being paid.

$\frac{120}{6000} \times 100\% = 2\%$ per month

$= 24\%$ per annum

1 mark

c. Calculate the value of Y.

$Y = 6367.25 - 6242.40$

$= \$124.85$

1 mark

d. Write a recursive relation that will give the balance B_{n+1} after n+1 months in terms of B_n and B_0.

$B_{n+1} = 1.02 \times B_n, \quad B_0 = 6000$

1 mark

Question 7

$8 000 is invested in an investment account that pays 6% per annum compound interest, compounded quarterly.

a. The value V_n of the investment after n quarters can be expressed in the form: $V_n = R^n \times V_0$, where $V_0 = 8000$ is the initial amount invested.

State the value of R.

$R = 1 + \frac{6/4}{100} = 1.015$

1 mark

b. Determine the value of the investment at the end of 3 years. Give your answer to the nearest cent.

$V_n = 1.015^n \times 8000$

After 3 years, $n = 12$

$\therefore V_{12} = 1.015^{12} \times 8000 = \$9564.95$ 2 marks

c. Determine the amount of interest that was earned in the third year. Give your answer to the nearest cent.

After 2 years, $V_8 = 1.015^8 \times 8000 = \$9011.94$

$\therefore$ Interest earned $= 9564.95 - 9011.94 = \$553.01$ 2 marks

Question 8

James invests $2000 at 2.3% per annum, compounded daily.

a. Calculate the amount of interest that he will earn at the end of 1 year, correct to the nearest cent.

$R = 1 + \frac{\frac{2.3}{365}}{100} = 1.000063\ldots$

$\therefore V = 2000 \times (1.000063\ldots)^n$, n = no. of days

When $n = 365$

$V = 2000 \times (1.000063\ldots)^{365} = \$2046.53$ 1 mark

b. Hence state the effective annual interest rate, correct to two decimal places.

Interest earned in 1 year

$= 2046.53 - 2000$

$= \$46.53$

1 mark

$\therefore$ Effective interest rate $= \frac{46.53}{2000} \times \frac{100}{1} = 2.33\%$ per annum

Alternatively:

use CAS: $\text{eff}(2.3, 365) = 2.33\%$ per annum

Question 9

An investment earns 5.2% per annum compounding quarterly. What is the effective interest rate, correct to two decimal places?

eff (5.2, 4) ≈ 5.30% per annum

Note you must write 5.30% with the zero at the end to show that it is rounded to two decimal places.

1 mark

Question 10

Pauline invests $190000 at a rate of 12% per annum compounded quarterly. She makes regular withdrawals from the account at the end of each quarter. What is the biggest quarterly withdrawal that Pauline can make if she wants her annuity to last 30 years? Give your answer to the nearest cent.

N = 120 P/y = 4
I = 12 C/y = 4
PV = −190000 gives PMT = 5869.08
FV = 0
PMT = ?

∴ Biggest possible quarterly payment
= $5869.08.

2 marks

Question 11

Valentina invests $110000 in an annuity that earns 6% per annum, compounding monthly. At the end of each month, she withdraws $2550 from the account after interest is paid.

a. Let V_{n+1} be the value of the annuity at the end of n +1 months. Write a recursive relation which gives V_{n+1} in terms of V_n and V_0.

$R = 1 + \frac{r/n}{100}$
$= 1 + \frac{6/12}{100}$
$= 1.005$

$V_{n+1} = 1.005 V_n - 2550, \quad V_0 = 110{,}000$

1 mark

b. How long will Valentina's annuity last? Give your answer to the nearest month.

N = ?
I = 6
PV = −110000
PMT = 2550
FV = 0
P/y = 12
C/y = 12
gives: N = 48.7

→ Lasts about 49 months.

2 marks

Question 12

The table below shows the balance in an investment in which regular deposits are made at the end of each year.

Year	Beginning Balance	Interest	Deposit	End Balance
1	y	180	700	9880
2	9880	197.60	700	10777.60
3	10777.60	x	700	11693.15
3	11693.15	233.86	700	w

a. i. Determine the value of *w*

$w = 11693.15 + 233.86 + 700 = 12627.01$

ii. Determine the value of *x*.

$x + 700 = 11693.15 - 10777.60$

$x = 915.55 - 700 = 215.55$

iii. Determine the value of *y*.

$y = 9000$

3 marks

b. Calculate the annual interest rate.

$\frac{180}{9000} \times \frac{100}{1} = 2\%$ per annum

1 mark

c. Write a recursive relation which gives V_{n+1} in terms of V_n and V_0, where V_n is the value of the investment after n years.

$V_{n+1} = 1.02 \times V_n + 700, \; V_0 = 9000$

1 mark

d. The total amount of the investment is withdrawn after 25 years. Calculate the total amount of **interest** earned over these 25 years. Give your answer to the nearest cent.

N = 25
I = 2
PV = −9000
PMT = −700
FV = ?
P/Y = 1
C/Y = 1

gives FV = 37186.66

∴ $37186.66 will be withdrawn

2 marks

Question 13

To save up for a car, Gwen opens up a savings account that earns 5% per annum, compounding monthly. She initially deposits $1 400 in the account at the beginning of the first month, and then deposits $195 at the end of each month. Calculate the amount of money that Gwen will have in her savings account at the end of 3 years. Give your answer to the nearest cent.

$N = 36$ $P/y = 12$
$I = 5$ $C/y = 12$
$PV = -1400$ gives $FV = 9182.96$
$PMT = -195$ $\therefore$ She will have $9182.96.
$FV = ?$

2 marks

Question 14

Tara needs to save $71 000 to start a small business. A bank offers her a savings account for this purpose, offering 14 % per annum, compounded monthly. Tara will initially deposit $6000 and can afford to contribute $900 to the account at the end of every month. How long, to the nearest month, will it take for Tara to achieve her goal?

$N = ?$ $FV = 71000$ gives $N = 49.798$
$I = 14$ $P/y = 12$ $\therefore \approx 50$ months
$PV = -6000$ $C/y = 12$
$PMT = -900$

2 marks

Question 15

Ray is saving for a deposit to buy a house. He can initially afford to deposit $14 000 in a savings account thanks to a donation from his parents, and then will make regular deposits of $1 100 at the end of each month. Calculate annual interest rate he will require to achieve a target of $80 000 in savings at the end of 4 years, if interest is compounded monthly. Give your answer correct to two decimal places.

$N = 48$
$I = ?$
$PV = -14000$
$PMT = -1100$
$FV = 80000$
$P/y = 12$
$C/y = 12$
gives $I = 7.38$
$\therefore$ Requires 7.38% per annum.

2 marks

Question 16

Robert has just won a $300 000 lottery and decides to invest in a perpetuity that pays 6% per annum, compounding monthly.

a. What monthly payment will Robert receive?

$Q = \frac{Pr}{100}$ $P = 300000$ $r = \frac{6}{12} = 0.5$

$Q = \frac{300000 \times 0.5}{100}$

$= \$1500$

1 mark

b. What will be the balance in his perpetuity at the end of 3 years?

$300,000

(the balance of a perpetuity remains constant).

1 mark

Question 17

Eric wants to set up a scholarship where the top student each year receives a prize of $8 000. If the interest on the initial investment averages 2% per annum, compounding annually, how much should his initial investment be?

$Q = \frac{Pr}{100}$

$Q = 8000$

$P = ?$

$r = 2$

1 mark

$\therefore 8000 = \frac{P \times 2}{100}$

$P = 400,000$

$\therefore$ Initial investment $= \$400,000.$

Question 18

$20 000 is invested in a perpetuity at 9% per annum, compounding monthly. A constant amount is withdrawn at the end of each month. This perpetuity can be defined recursively as:

$A_{n+1} = a \times A_n - b, A_0 = c$ where A_{n+1} is the value of the perpetuity after n months.

a. What is the value of a ?

9% p.a $= \frac{9}{12} = 0.75\%$ per month.

$a = 1 + \frac{9/12}{100}$

$= 1.0075$

$a = 1.0075$

b. What is the value of b ?

b = interest generated in 1 month $= \frac{0.75}{100} \times 20000$

$= 150$

c. What is the value of c ?

$C = 20000.$

3 marks

Check: $20000 \times 1.0075 - 150$

$= 20,000.$

$\therefore$ Balance remains constant.

Question 19

Jack takes out a reducing balance loan of $100 000. The following table lists his first few repayments, and shows how much of each repayment was interest, and how much went to reducing the principal:

Starting balance (before any repayment): $100, 000

Period	Interest	Principal	Balance
1	$583.33	$191.97	$99,808.03
2	$582.21	$193.09	$99,614.95
3	$581.09	$194.21	$99,420.74
4	$579.95	$195.34	$99,225.39
5	$578.81	$196.48	$99,028.91
6	$577.67	$197.63	$98,831.28
7	$576.52	$198.78	$98,632.50
8	$575.36	$199.94	$98,432.55
9	$574.19	$201.11	$98,231.44
10	$573.02	$202.28	$98,029.16
11	$571.84	$203.46	$97,825.70
12	$570.65	$204.65	$97,621.05

a. How much does Jack pay each month as his regular repayment?

$583.33 + 191.97 = \$775.30$

b. What is the annual interest rate being charged?

In 1st month, interest charged was $583.33

$\frac{583.33}{100000} \times \frac{100}{1} = 0.5833\%$ per month $\simeq 0.5833 \times 12\%$

$= 7\%$ per annum

c. How much of Jack's 25th repayment will go towards reducing the principal?

N=24 FV=? gives FV = −95070.098

I=7 PMT=P/Y=12 now. find FV

PMT = −775.30 C/Y=12 when N = 25

PV = 100000

Principal has been reduced by:

95070.098 − 94849.374

= \$220.72

N=25

I=7

PMT=−775.30

PV=100000

FV=?

P/Y=C/Y=12

gives FV = −94849.374

d. At the end of which month will Jack make his final repayment?

N = ?
I = 7
PV = 100000
PMT = −775.30
P/y = 12
C/y = 12
FV = 0

gives N = 239.999
≈ 240

∴ At 240th month, Jack makes final repayment.

e. What is the value of his final repayment, correct to the nearest cent?

N = 240
I = 7
PV = 100000
PMT = −775.30
P/y = 12
C/y = 12
FV = ?

gives FV = 0.55446

∴ Final payment
= 775.30 − 0.55
= $774.75

f. Correct to the nearest dollar, how much interest does Jack pay on this loan?

Total paid = 239 × 775.30 + 774.75
= $186071.45

∴ Interest = 186071.45 − 100,000
≈ $86071