Question 1

(VCAA Q13, 2017)

Let $h:(-1, 1) \rightarrow R$, $h(x) = \dfrac{1}{x-1}$.

Which one of the following statements about h is **not** true?

A. $h(x)h(-x) = -h(x^2)$

B. $h(x) + h(-x) = 2h(x^2)$

C. $h(x) - h(0) = xh(x)$

D. $h(x) - h(-x) = 2xh(x^2)$

E. $(h(x))^2 = h(x^2)$

Question 2

(VCAA Q11, 2016)

The function f has the property $f(x) - f(y) = (y - x)f(xy)$ for all non-zero real numbers x and y.

Which one of the following is a possible rule for the function?

A. $f(x) = x^2$

B. $f(x) = x^2 + x^4$

C. $f(x) = x\log_e(x)$

D. $f(x) = \dfrac{1}{x}$

E. $f(x) = \dfrac{1}{x^2}$

Question 3

(VCAA 2014, Q10)

Which one of the following functions satisfies the functional equation $f(f(x)) = x$ for every real number x?

A. $f(x) = 2x$

B. $f(x) = x^2$

C. $f(x) = 2\sqrt{x}$

D. $f(x) = x - 2$

E. $f(x) = 2 - x$

Question 4

(VCAA 2013, Q13)

If the equation $f(2x) - 2f(x) = 0$ is true for all real values of x, then the rule for f could be

A. $\frac{x^2}{2}$

B. $\sqrt{2x}$

C. $2x$

D. $\log_e\left(\frac{x}{2}\right)$ (where $x > 0$)

E. $x - 2$

Question 5

(Q19, VCAA 2012)

A function f has the following two properties for all real values of θ.

$$f(\pi - \theta) = -f(\theta) \text{ and } f(\pi - \theta) = -f(-\theta)$$

A possible rule for f is

A. $f(x) = \sin(x)$

B. $f(x) = \cos(x)$

C. $f(x) = \tan(x)$

D. $f(x) = \sin\left(\frac{x}{2}\right)$

E. $f(x) = \tan(2x)$

Question 6

(Q12, VCAA 2008)

Let $f: R \to R$, $f(x) = e^x + e^{-x}$.

For all $u \in R$, $f(2u)$ is equal to

A. $f(u) + f(-u)$

B. $2f(u)$

C. $(f(u))^2 - 2$

D. $(f(u))^2$

E. $(f(u))^2 + 2$

Question 7

(Q17, VCAA 2007)

The function f satisfies the functional equation $f(f(x)) = x$ for the maximal domain of f.

The rule for the function is

A. $f(x) = x + 1$

B. $f(x) = x - 1$

C. $f(x) = \dfrac{x-1}{x+1}$

D. $f(x) = \log_e(x)$

E. $f(x) = \dfrac{x+1}{x-1}$

Question 8

(Q17, VCAA 2006)

The function f satisfies the functional equation $f\left(\dfrac{x+y}{2}\right) = \dfrac{f(x)+f(y)}{2}$ where x and y are any non-zero real numbers.

A possible rule for the function is

A. $f(x) = \log_e|x|$

B. $f(x) = \dfrac{1}{x}$

C. $f(x) = 2^x$

D. $f(x) = 2x$

E. $f(x) = \sin(2x)$