MATRICES

A matrix (plural *matrices*) is a rectangular array of numbers (*elements*) enclosed by square brackets

The order of a matrix is : *number of rows* × *number of columns*. A matrix is usually denoted by a capital letter

Examples:

$A = \begin{bmatrix} 1 & -5 \\ 0 & 17 \end{bmatrix}$ Order: (A is a s**quare** matrix)

$B = \begin{bmatrix} 7 \\ 15 \\ -13 \end{bmatrix}$ Order: (B is called a **column matrix** or **column vector**)

$C = \begin{bmatrix} 4 & 0 \\ -6 & 10 \\ -13 & 8 \end{bmatrix}$ Order:

ADDITION and SUBTRACTION

You can only add/subtract matrices of the same order:

$A = \begin{bmatrix} 3 & 19 \\ -10 & -16 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 8 \\ -13 & 14 \end{bmatrix}$

Find:

i. $A + B$	ii. $A - B$	iii. $A + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

MULTIPLICATION BY A SCALAR

You simply multiply each element by the scalar!

Example: $X = \begin{bmatrix} 4 & 3 & 11 \\ 9 & 0 & 6 \\ -1 & 5 & -8 \end{bmatrix}$ Find $8X$

Example:

If $P = \begin{bmatrix} 3 & 5 \\ 8 & 7 \end{bmatrix}$ and $Q = \begin{bmatrix} 4 & 2 \\ 1 & 6 \end{bmatrix}$ find: $6P - 5Q$

MULTIPLICATION OF MATRICES by EACH OTHER

When multiplying two matrices together, you multiply the rows of the first by the columns of the second to generate the product matrix.

Example:

$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 9 & 11 \end{bmatrix}$ =

$\begin{bmatrix} 1 & 0 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 8 \end{bmatrix}$ =

$\begin{bmatrix} 2 \\ 8 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 6 & 5 \end{bmatrix}$

Multiplication of two matrices is only defined if the no. of columns in the first matrix = no. of rows in the second matrix.

In general, when multiplying matrices, $A.B \neq B.A$

Question: $L = [2 \ -1]$ and $X = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$.

a. Find LX

b. Find XL.

Question: $M = \begin{bmatrix} 4 & 5 \\ -1 & 11 \end{bmatrix}$ and let $N = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$.

a. Find MN

b. Find NM.

IDENTITY 2×2 MATRIX

The identity two by two matrix is: $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The identity matrix is special because, for it, it is true to say: $AI = IA = A$

Example: Calculate:

a. $\begin{bmatrix} 4 & 5 \\ -3 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ -3 & 7 \end{bmatrix}$

INVERSE MATRICES

Given a square two by two matrix A, if there exists another square matrix B such that $AB = BA = I$ then B is called the inverse of A and is given the symbol: A^{-1}

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then, if an inverse matrix exists, $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc$ is known as the **determinant** of A, called det(A)

A does not have an inverse if $ad - bc = 0$ (such matrices are known as **singular** matrices)

NOTE: Any square matrix A has an inverse ONLY if det (A) $\neq \mathbf{0}$

Example:

Let $A = \begin{bmatrix} 3 & -1 \\ 4 & 1 \end{bmatrix}$

a. Find A^{-1}

b. Verify that $A.A^{-1} = I$ and $A^{-1}.A = I$

Example: Find: $\begin{bmatrix} 11 & 9 \\ 5 & 2 \end{bmatrix}^{-1}$

Example:

Let A= $\begin{bmatrix} 3 & 2 \\ 1 & 6 \end{bmatrix}$, let $B = \begin{bmatrix} 4 & -1 \\ 2 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 4 \\ 2 & 6 \end{bmatrix}$

a. Find X such that: $AX + B = C$

b. Find Y such that: $YA + B = C$

SIMULTANEOUS EQUATIONS solved WITH MATRICES

Matrices can be used to solve simultaneous equations.

Consider the simultaneous equations:
$-x + 2y = -1$
$-x + 4y = 2$

Set these simultaneous equations up in matrix form and use matrix methods to solve.

$$\begin{bmatrix} & \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

This is of the form:

A is called the coefficient matrix

To solve the simultaneous equations, we can always use the algorithm:
$X = A^{-1}K$ (provided that A is **not singular**!!)

Example: Use matrix methods to solve the simultaneous equations:
$2x + 5y = -10$
$y = x + 4$

Questions

1. Two children spend their pocket money buying books and CDs. One child spends \$120 and buys four books and four CDs. The other child buys three CDs and five books and pays \$114. Set up simultaneous equations and use matrix methods to determine the price of one CD and of one book.

2. Find the 2×2 matrix such that: $A.\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 12 & 14 \end{bmatrix}$

3. Let $A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$
   a. Find A^{-1}.
   b. If $AX = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$, find X.
   c. If $YA = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$, find Y.
4. If $\begin{bmatrix} 1 & 2 \\ 4 & x \end{bmatrix}$ is a singular matrix, find the value of *x*.
5. Let $M = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$.
   a. Find the matrix M^2 $(M.M)$
   b. Find M^{-1}
   c. Find x and y, given that: $M\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.
6. Consider the simultaneous equations:
$$2x - 3y = 3$$
$$4x + y = 5$$
   a. Write these equations in the matrix form: *A.X = K*
   b. Find det(*A*) and A^{-1}.
   c. Solve the simultaneous equations.
7. Consider the simultaneous equations:
$$2x + y = 3$$
$$4x + 2y = 8$$
   a. Write these equations in the form: *A.X = K*
   b. Find det(A) and explain why A^{-1} cannot be found.
   c. Interpret this result geometrically.