PRACTICE EXAM ON NEW SECTIONS OF STUDY DESIGN

Question 1

Lat Long

A flight from Rome (41.9°N, 12.5°E) to Dubai (25.0°N, 55.3°E) was scheduled to depart at 2:10 pm on Tuesday.

a. i. What is the time difference between Rome and Dubai? Give your answer to the nearest hour.

$55.3° - 12.5° = 42.8°$

$42.8 \div 15 = 2.85 \approx 3$ hours

1 mark

ii. What time is it in Dubai when it is 2: 10 pm on Tuesday in Rome?

Dubai is AHEAD of Rome, so it is later in Dubai

2:10 pm + 3 hours = 5:10 pm

1 mark

(b) The flight departed Rome on time and arrived in Dubai at 12:15 am on Wedenesday, Dubai time. How long was the flight?

1 mark

	Rome	Dubai
Departure time	2:10pm	5:10pm
Arrival time		12:15 AM

∴ Length of flight

= 12:15 AM Wednesday − 5:10pm Tuesday

= 7 hours 5 minutes

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Question 2

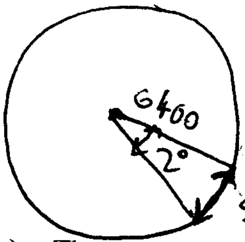
DO NOT WRITE IN THIS MARGIN

A light aircraft is scheduled to fly directly from Wagga Wagga to Sydney. Due to bad weather, the aircraft must fly another route.

(a) The first leg of the journey has the aircraft travelling **due north** from Wagga Wagga 35°S, 147°E to Point A 33°S, 147°E

Determine the distance from Wagga Wagga to Point A, to the nearest km.

(Take the radius of the earth to be 6,400 km)



$35^\circ - 33^\circ = 2^\circ$

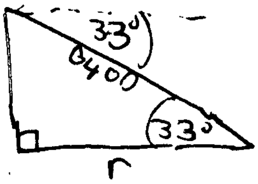
$S = \dfrac{\pi R \theta}{180}$

$\therefore S = \dfrac{\pi \times 6400 \times 2}{180}$

$= 223$ km

(b) The second leg of the journey has the aircraft travelling **due east** from Point A (33°S, 147°E) to Sydney (33°S, 152°E).

i. Calculate the radius of the circle of latitude where Sydney is located. Give your answer to the nearest km.



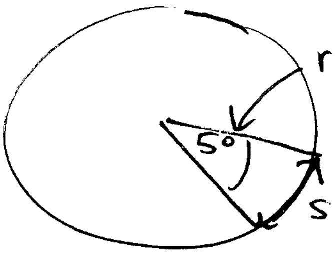
$r = 6400 \cos(33^\circ)$

$r \simeq 5367$ km

ii. Calculate the distance between Point A and Sydney flying **due east.** Give your answer to the nearest km.

Flies along circle of latitude

$152^\circ - 147^\circ = 5^\circ$



2 marks

$S = \dfrac{\pi r \theta}{180}$

$S = \dfrac{\pi \times 5367 \times 5}{180}$

$S \simeq 468$ km

Question 3

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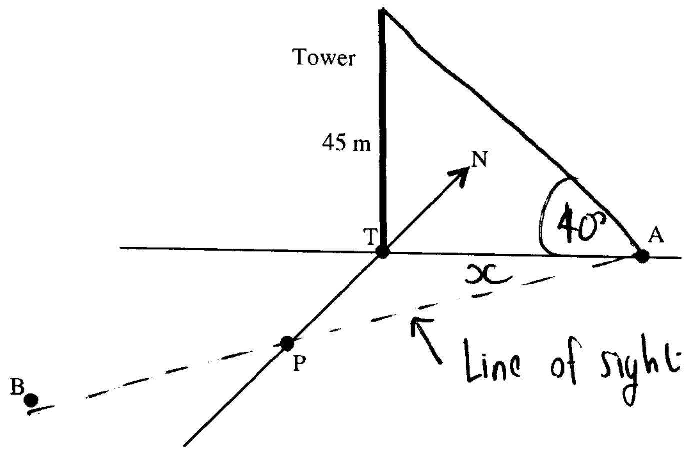


The diagram above shows a flat section of land with a 45 m high tower. Point A is directly east of the tower.

(a) The angle of elevation from Point A to the top of the tower is 40°.

Determine the horizontal distance from Point A to the base of the tower (T).

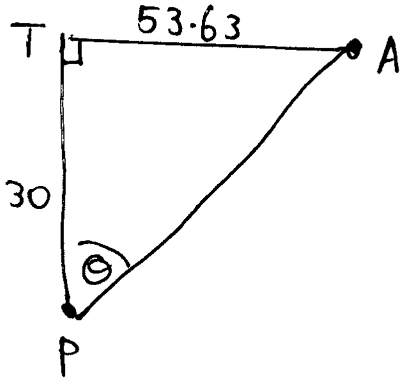
$$\tan(40°) = \frac{45}{x}$$

$$x = \frac{45}{\tan(40°)}$$

$$x \simeq 53.63 \text{ m}$$

(b) Point B is 30 m directly south of the tower. Point P is in a direct line of sight from Point A when viewed from point B. Determine the bearing of Point A **from** Point B.
Give your answer to the nearest degree.

Bearing from B to A is same as from P to A



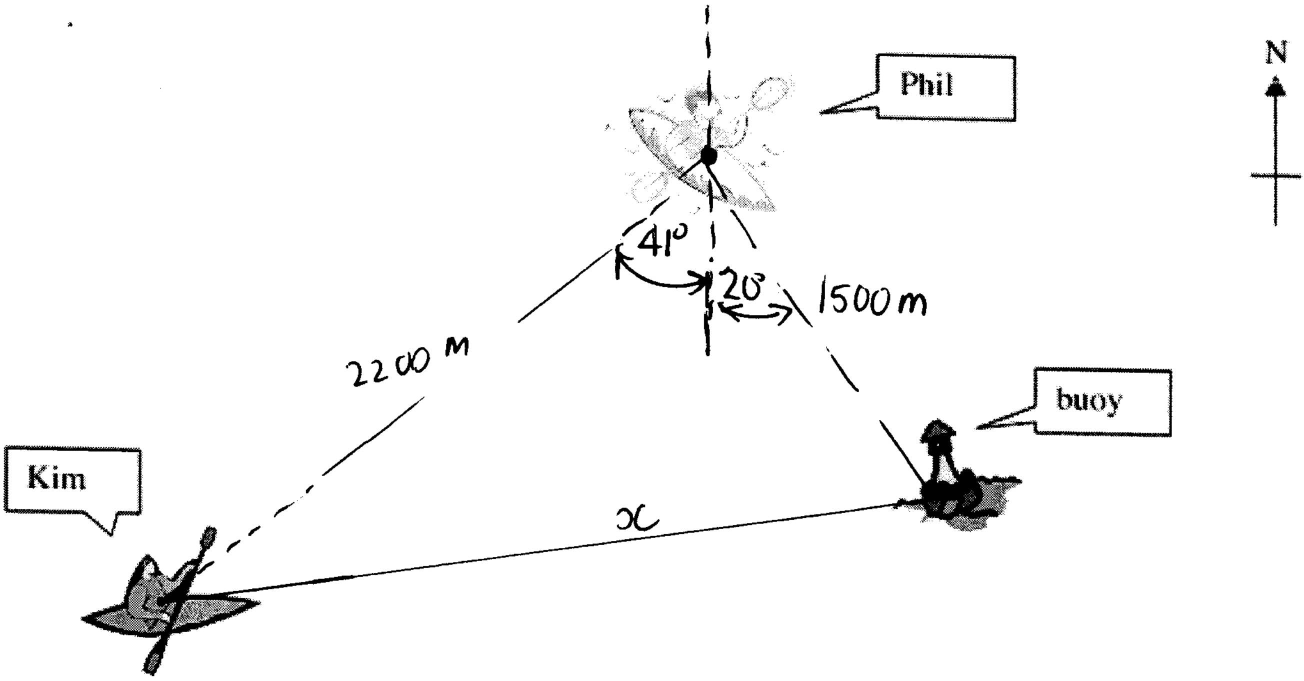
2 marks

$$\tan(\theta) = \frac{53.63}{30} \quad \therefore \theta = \tan^{-1}\left(\frac{53.63}{30}\right).$$

$$\simeq 60.8°$$

$\therefore$ Bearing = 061° T

Two canoeists at sea are aiming for the same point (a buoy). Phil is 1 500 m from the buoy at a bearing of 160°T Phil also sees Kim, another canoeist, 2 200 m away at a bearing of 221°T from him.



(a) Determine the distance Kim is from the buoy. Give your answer in metres correct to four significant figures.

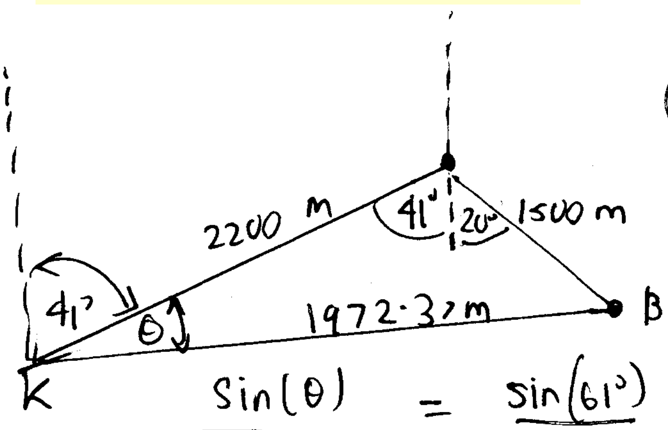
$$x^2 = 2200^2 + 1500^2 - 2 \times 2200 \times 1500 \cos(61°)$$

$$x = \sqrt{2200^2 + 1500^2 - 2 \times 2200 \times 1500 \cos(61°)}$$

$$= 1972.37 \approx 1972 \text{ m}$$

2 marks

(b) Determine the bearing that Kim will have to travel on to reach the buoy. Give your answer to the nearest degree.



$$\frac{\sin(\theta)}{1500} = \frac{\sin(61°)}{1972.37}$$

$$\sin(\theta) = \frac{1500 \sin(61°)}{1972.37}$$

$$\theta = \sin^{-1}\left(\frac{1500 \sin(61°)}{1972.37}\right)$$

$$\theta = 41.7° \approx 42°$$

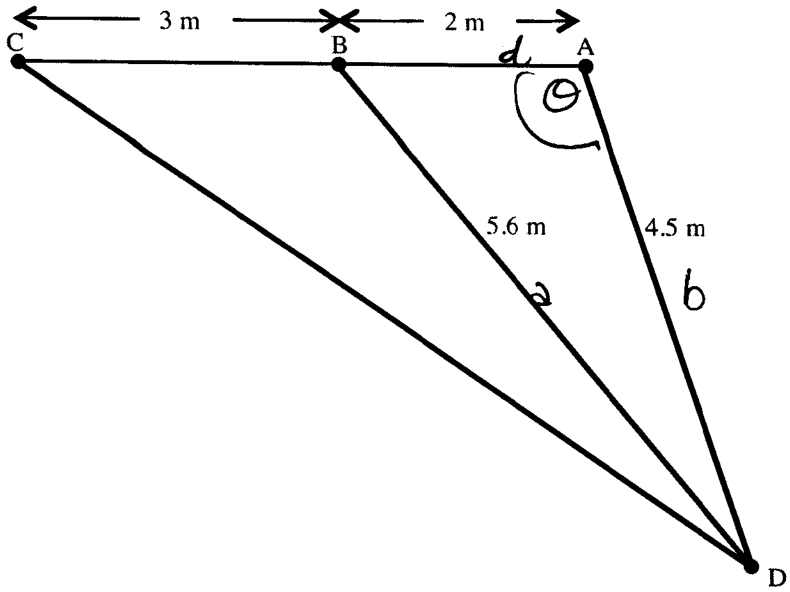
∴ Bearing of Buoy from K is $41° + 42° = 083°T$

2 marks

Question 4

For Marker Use Only

The diagram of a steel construction is shown below.



Determine the length of the beam CD.

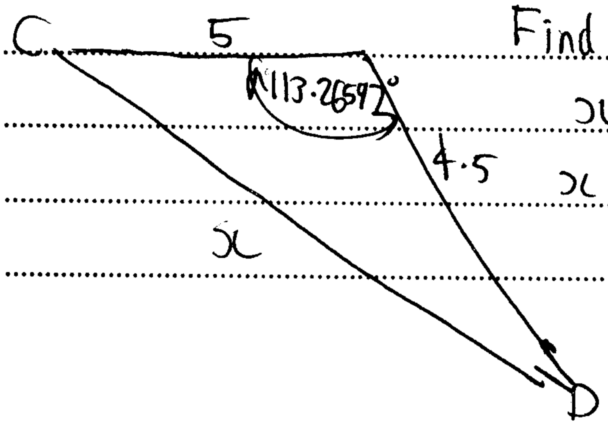
t C B A

Find θ

$$\cos(\theta) = \frac{2^2 + 4.5^2 - 5.6^2}{2 \times 2 \times 4.5}$$

$$\theta = \cos^{-1}\left(\frac{2^2 + 4.5^2 - 5.6^2}{2 \times 2 \times 4.5}\right)$$

$$\theta \simeq 113.26597^\circ$$



Find $\overline{CD}$:

$$x^2 = 5^2 + 4.5^2 - 2 \times 5 \times 4.5 \cos(113.26\ldots)$$

$$x = \sqrt{5^2 + 4.5^2 - 2 \times 5 \times 4.5 \cos(113.26597)}$$

$$x \approx 7.94 \text{ m}$$

(three significant figures)

Question 5

In order to buy a second-hand scooter, Kim obtained a personal loan of \$5000 with monthly repayments of \$440 to be paid at the end of each month. The table below shows the amount owing at the start of each month, the interest payable for that month, the repayment and the amount owing at the end of each month for the first six months.

Month	Amount owing at the start of the month (\$)	Interest (\$)	Repayment (\$)	Amount owing at the end of the month (\$)
1	5000	40	440	4600
2	4600	36.80	440	4196.80
3	4196.80	33.57	440	3790.37
4	3790.37	30.32	440	3380.70
5	3380.70	27.05	440	2967.74
6	2967.74	23.74	440	2551.48

(a) Calculate the annual interest rate. (2 marks)

Monthly rate $= \frac{40}{5000} \times 100\ \% = 0.8\%$

$\therefore$ Annual rate $= 0.8 \times 12 = 9.6\%$ per annum

(b) Write a recursive rule to determine the amount owing at the end of each month. (2 marks)

$R = 1 + \frac{r/n}{100}$

$= 1 + \frac{9.6/12}{100}$

$= 1.008$

$$\boxed{\begin{array}{l} V_{n+1} = 1.008 V_n - 440 \\ V_0 = 5000 \end{array}}$$

(c) In which month would Kim pay off the loan? (1 mark)

N = ?
I = 9.6
PMT = −440
FV = 0
PV = 5000
P/Y = 12
C/Y = 12

gives N = 11.96

$\therefore$ Pays it off in the 12 th month.

(d) How much is Kim's final repayment? Give your answer to the nearest cent. (2 marks)

N = 12
I = 9.6
PMT = −440
FV = ?
PV = 5000
P/Y = 12
C/Y = 12

gives FV = 16.93

Final payment
= \$440 − \$16.93
= \$423.07

Alternative method:
N = 11
I = 9.6
PMT = −440
FV = ?
PV = 5000
P/Y = C/Y = 12
gives FV = −419.71

$\therefore$ Final payment
= 419.71 × 1.008
= \$423.07

(e) How much did Kim actually pay for the scooter? (2 marks)

Total paid by Kim
= 11 × 440 + 423.07
= \$5263.07

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 6

same

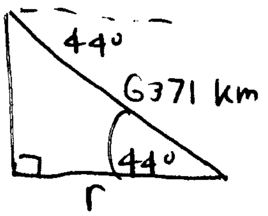
A ship leaves from the harbour of Christchurch, New Zealand (44°S 173°E) and sails **due east** to the entrance of the harbour of Castro, Chile (44°S 74°W).

DO NOT WRITE IN THE MARGIN

(a) Calculate the distance covered in this journey, to the nearest kilometre. Assume that the radius of the Earth is 6371 km. **Working must be shown.**

1

due east → along circle of latitude 44°S



$r = 6371\cos(44°)$

$r = 4582.91$ km

continued

3 marks

(b)

(i) Difference in longitude $= 173° - -74°$

$= 173° + 74°$

$= 257°$

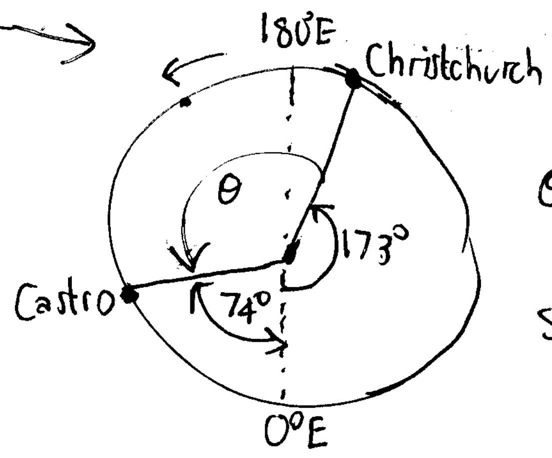
Time difference $= \frac{257}{15} \approx 17.13 \simeq 17$ hours

1 mark

(ii) Departed : 8:30 am Christchurch time (Monday)

8:30am − 17 hours = 3:30pm Sunday in Castro.

Q 6 (a) [Cont]



$\theta = 360° - 74° - 173°$

$= 113°$

$S = \frac{\pi r \theta}{180}$ $\theta = 113°$ $r = 4582.91$

$S = \frac{\pi \times 4582.91 \times 113}{180}$

$= 9039$ km

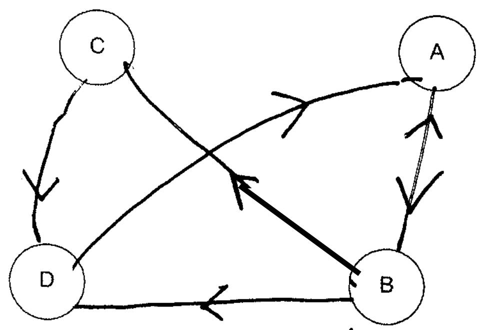
Question 7

Four rural towns (A, B, C, and D) each have their own water storage for use in emergency bushfire situations. Water can also be pumped between the towns if necessary, as shown in the connectivity matrix, M, below:

$$M = \begin{array}{cc} & \textit{pumped to} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ \textit{pumped from} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

NOTE: In this matrix, the element in row 2 column 1 is equal to 1. This means that water can be pumped **from B to A.**

(a) Using the nodes below, show these connections as a network diagram.



(2 marks)

(b) (i) (1) Find $P = M + M^2$.

$$\begin{array}{cc} & \textit{pumped to} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ \textit{pumped from} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

(2 marks)

(2) Why are the main diagonal elements ($P_{1,1}$, $P_{2,2}$, $P_{3,3}$, and $P_{4,4}$) not useful in this context?

They are redundant connections (end back at the original starting point)

(1 mark)

(ii) Using your result from part (b)(i)(1), or otherwise, explain which town or towns would be most at risk in a bushfire situation.

Towns C and B are most at risk because B has water pumped to it from only two other towns (A and D) while C has water pumped to it from A and B only.

Town A however has water pumped to it from all three other towns, including two connections from B (one direct and one a two-step connection).

Town D has water pumped to it from all three other towns as well, including two connections from B (one direct and one a two-step connection)

PLEASE TURN OVER

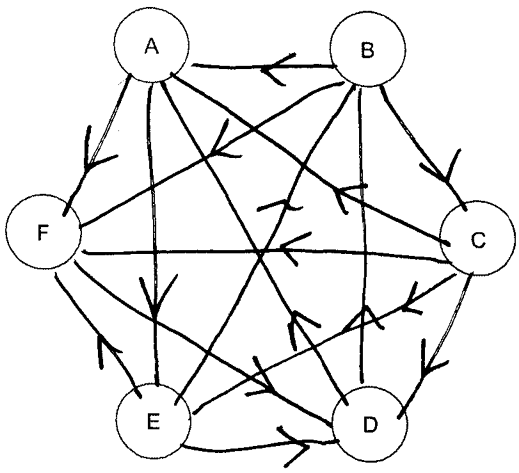
Question 8

Six friends — Anna (A), Betty (B), Chris (C), Dennis (D), Enzo (E), and Fred (F) — decided to spend the weekend playing a 'round robin' lawn bowls tournament in which each of the friends would play a match against each of the others.

By lunchtime on Sunday:

- Anna had defeated Enzo and Fred
- Betty had defeated Anna, Chris, and Fred
- Chris had won against Anna, Dennis, and Fred and Enzo
- Dennis had won against Betty and Anna
- Enzo had defeated Dennis, Fred, and Betty
- Fred had defeated Dennis.

(a) Using the nodes in the diagram below, draw a network diagram to show this information.



(2 marks)

(b) Complete the dominance matrix D below

$$D = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

(2 marks)

(c) (i) Calculate $T = D + D^2$ to rank the players from first to last on the matches played so far.

List the players from first to last.

losers

TOTAL 1 step and 2 step dominances

$$D+D^2 = \text{winner} \quad \begin{array}{c|cccccc|c} & A & B & C & D & E & F & \\ \hline A & 0 & 1 & 0 & 2 & 1 & 2 & 6 \\ B & 2 & 0 & 1 & 2 & 2 & 3 & 10 \\ C & 2 & 2 & 0 & 3 & 2 & 3 & 12 \\ D & 2 & 1 & 1 & 0 & 1 & 2 & 7 \\ E & 2 & 2 & 1 & 2 & 0 & 2 & 9 \\ F & 1 & 1 & 0 & 1 & 0 & 0 & 3 \end{array}$$

2 marks

1st : Chris

2nd : Betty

3rd : Enzo

4th: Dennis

5th : Anna

6th: Fred

Question 9

A car is initially purchased for \$24 000 and depreciates by \$17 00~~0~~ per year.

a. Write a recursive relation that gives the value of the car in dollars after $n + 1$ years, in terms of its value after n years. Write both parts of the rule, including for V_0, on the same line, separated by a comma.

$V_0 = 24000$
$V_{n+1} = V_n - 1700n$

b. Write down a rule that will give the value of the car after n years.

$V_n = 24000 - 1700n$

During which year

c. ~~After how many years~~ will the value of the car fall below \$10,100 ?

$10\,100 = 24000 - 1700n$

Solving: $n = 8.17$

$\therefore$ During 9th year.

(1 + 1 + 1 = 3 marks)

Question 10

Tim is starting up his own business. He has saved \$15 000 to buy equipment and he borrows another \$50 000 from the bank. He is charged interest at the rate of 4.5% per annum, compounding monthly, and makes regular monthly repayments of \$400.

a. Write down a calculation from which the amount that Tim owes at the end of the first month can be evaluated.

$50\,000 \times 1.00375 - 400$

$R = 1 + \frac{4.5/12}{100}$
$= 1.00375$

b. Write a recurrence relation that gives the balance B_{n+1} in terms of the balance in the preceding month B_n.

$\boxed{\begin{aligned} B_{n+1} &= 1.00375 B_n - 400 \\ B_0 &= 50\,000 \end{aligned}}$

c. To the nearest month, how many months does it take for him to pay off his loan?

$N = ?$
$I = 4.5$
$PV = 50000$
$PMT = -400$
$FV = 0$
$P/Y = C/Y = 12$

gives: $N = 168.9887$

$\therefore$ 169 months.

d. What is the value of Tim's final repayment? Give your answer to the nearest cent

N = 169 P/y = C/y = 12
I = 4.5
PV = 50000 gives FV = 4.49
PMT = −400 ∴ Final payment = 400 − 4.49
FV = ? = \$395.51

e. How much in total does Tim pay for his equipment?

Tim pays:
15000 + 168 × 400 + 395.51
= \$82595.51

(1 + 1 + 1 + 1 + 1 = 5 marks)

Question 11

A new car depreciates in value each year according to the recursion relation:

$$V_{n+1} = 0.89V_n, V_0 = 21\ 000$$

a. How much was the car purchased for?

\$21000

1 mark

b. As a percentage, what was the annual depreciation rate?

1 − 0.89 = 0.11 ∴ 11%

1 mark

c. Determine the value of the car after 9 years. Give your answer to the nearest dollar.

$$V_n = \left(1 - \frac{11}{100}\right)^n \times 21000$$

$$V_9 = 0.89^9 \times 21000 = \$7357$$

2 marks

d. When the value of the car reaches \$600, it is considered useful only for parts. At the end of which year will it be considered useful only for parts?

$$0.89^n \times 21000 = 600$$

Solving : n = 30.51

∴ At end of 31st year

2 marks

Question 12

After one year, the value of a company's machinery has depreciated by $16 020 from $89 000.

a. At what rate did the machinery depreciate? Give your answer as a percentage.

$$\frac{16020}{89000} \times \frac{100}{1} = 18\%$$

b. What will the machinery be worth at the end of the second year? Give your answer to the nearest cent. (Assume reducing balance depreciation)

$$V_n = 89000 \times (1 - 0.18)^n$$

$$\therefore V_2 = 89000 \times 0.82^2 = \$59843.60$$

c. Write a recursive relation that gives the value V_{n+1} of the machinery after $n+1$ years in terms of its value after n years, and its initial value V_0.

$$\boxed{\begin{aligned} V_{n+1} &= 0.82\,V_n \\ V_0 &= 89000 \end{aligned}}$$

d. The company bought this machinery at the end of 2011. When its value falls below $3000, the company will invest in new machinery. In which year will this occur?

$$3000 = 89000 \times 0.82^n$$

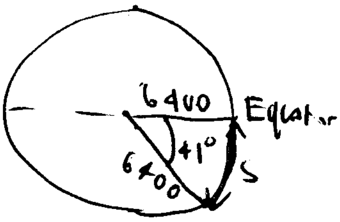
Solving: $n = 17.08$

$\therefore$ It will occur in 18th year.

(1 + 1 + 1 + 2 = 5 marks)

Question 13

A location has co-ordinates $(41°S, 85°W)$. Find its distance from the equator, assuming that the earth's radius is 6 400 km. Give your answer to the nearest kilometre.



$S = \dfrac{\pi R \theta}{180}$ $\quad R = 6400$

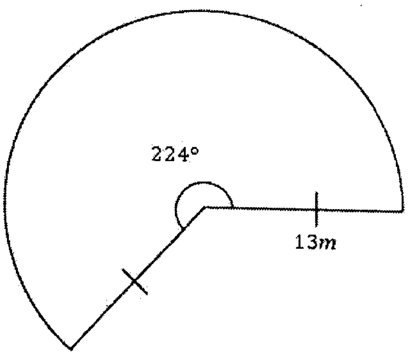
$\theta = 41°$

$\therefore S = \dfrac{\pi \times 6400 \times 41}{180} \approx 4580 \text{ km}$

2 marks

Question 14

Calculate the area of the sector shown in the diagram below, in square metres, correct to one decimal place.



$A = \dfrac{\pi r^2 \theta}{360}$ $\quad \theta = 224°$

$r = 13 \text{ m}$

$\therefore A = \dfrac{\pi \times 13^2 \times 224}{360}$

$= 330.356 \text{ m}^2$

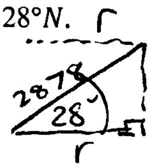
$\simeq 330.4 \text{ m}^2$

2 marks

Question 15

Mercury has a radius of 2 878 km.

i. Calculate the radius of the small circle on Mercury that lies at a latitude of 28°N. Give your answer correct to three significant figures.

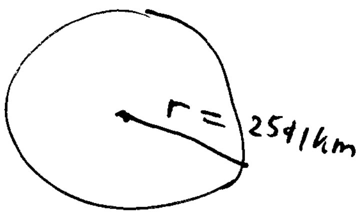


$r = 2878 \cos(28°)$

$r = 2541 \text{ km} \simeq 2540 \text{ km}$ (3 sig figures)

ii. Calculate the total distance around this small circle, correct to the nearest km.

(2 + 2 = 4 marks)



$C = 2\pi r$

$\therefore C = 2\pi \times 2541.12 = 15966 \text{ km}$

(if using $r = 2540$, then $C = 15959$ km would be your answer)

Core

Question 1

The stemplot in Figure 1 shows the distribution of the average age, in years, at which women first marry in 17 countries.

average age, in years, of women at first marriage

key: 27|3 represents 27.3 years

24						
25	0					
26	6					
27	1	1	3	4	7	
28	2	2	2	3	3	6
29	1	1				
30	1	4				
31						

Figure 1

a. For these countries, determine

i. the lowest average age of women at first marriage

25·0

ii. the median average age of women at first marriage.

28·2

1 + 1 = 2 marks

The stemplot in Figure 2 shows the distribution of the average age, in years, at which men first marry in 17 countries.

average age, in years, of men at first marriage

key: 32|5 represents 32.5 years

25							
26	0						
27		Q1					
28	9						
29	0	9	9				
30	0	0	3	5	6	7	9
31	0	0	2				
32	5	9					
33	Q3						

Median

Figure 2

b. For these countries, determine the interquartile range (IQR) for the average age of men at first marriage.

$Q_3 = 31\cdot0$, $Q_1 = 29\cdot9$

1 mark

$IQR = Q_3 - Q_1$

$= 31\cdot0 - 29\cdot9$

$= 1\cdot1$

Core – Question 1 – continued

c. If the data values displayed in Figure 2 were used to construct a boxplot with outliers, then the country for which the average age of men at first marriage is 26.0 years would be shown as an outlier.

Explain why this is so. Show an appropriate calculation to support your explanation.

Lower fence $= Q_1 - 1.5 \times IQR$

$= 29.9 - 1.5 \times 1.1$

$= 28.25$

Since $26.0 < 28.25$, 26.0 is an outlier

2 marks

Question 2

Table 1 shows information about a particular country. It shows the percentage of women, by age at first marriage, for the years 1986, 1996 and 2006.

Table 1

Age of women at first marriage	Year of marriage		
	1986	**1996**	**2006**
19 years and under	8.5%	3.7%	2.0%
20 to 24 years	42.1%	31.3%	21.5%
25 to 29 years	23.4%	31.7%	34.5%
30 years and over	26.0%	33.3%	42.0%

a. Of the women who first married in 1986, what percentage were aged 20 to 29 years inclusive?

42.1% + 23.4% = 65.5%

1 mark

b. Does the information in Table 1 support the opinion that, for the years 1986, 1996 and 2006, the age of women at first marriage was associated with year of marriage?
Justify your answer by quoting appropriate percentages. It is sufficient to consider one age group only when justifying your answer.

Yes, it does support the opinion that age of first marriage was associated with year of marriage

In 1986, 42.1% of women were aged 20-24 at the time of their first marriage; in 1996, only 31.3% were aged 20-24 and by 2006, only 21.5% were aged 20-24 years old.

Question 4

The average age of women at first marriage in years (*average age*) and average yearly income in dollars per person (*income*) were recorded for a group of 17 countries.

The results are displayed in Table 2. A scatterplot of the data is also shown.

Table 2

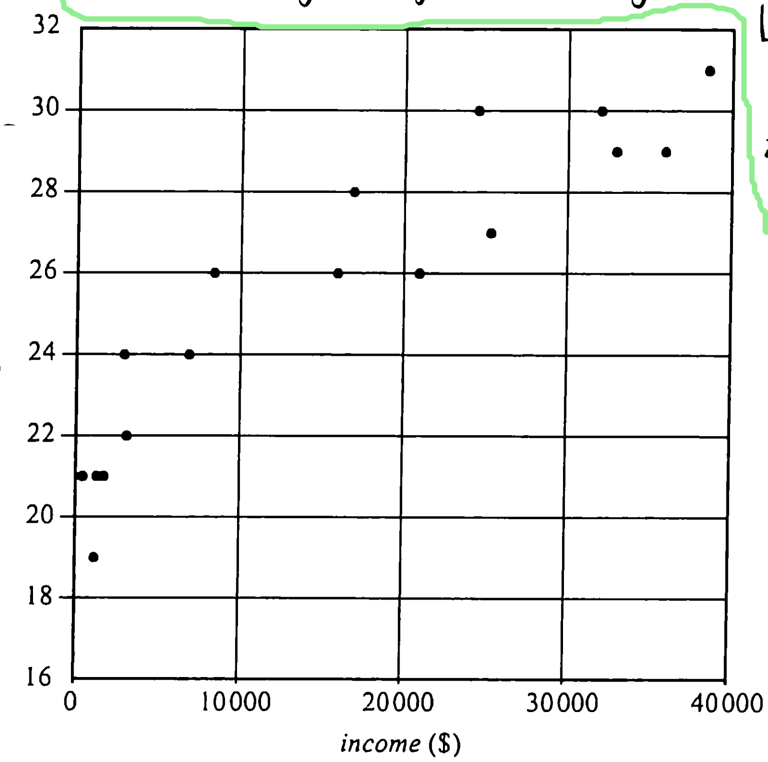
average age (years)	*income* ($)
21	1 750
22	3 200
26	8 600
26	16 000
28	17 000
26	21 000
30	24 500
30	32 000
31	38 500
29	33 000
27	25 500
29	36 000
19	1 300
21	600
24	3 050
24	6 900
21	1 400

logincome

3.423

3.505

CAS: Use logincome as the x variable and average age as the y-variable, linear regression a+bx



The relationship between *average age* and *income* is nonlinear.

A **log transformation** can be applied to the variable *income* and used to linearise the scatterplot.

a. Apply this log transformation to the data and determine the equation of the least squares regression line that allows *average age* to be predicted from log (*income*).

Write the coefficients for this equation, correct to two decimal places, in the spaces provided.

average age = $\boxed{2.39}$ + $\boxed{5.89}$ × log (*income*)

2 marks

b. Use this equation to predict the average age of women at first marriage in a country with an average yearly income of $20 000 per person.

Write your answer correct to one decimal place.

average age $= 2.39 + 5.89 \times \log(20000)$

$\simeq 27.7$

Predicted average age = 27.7 years

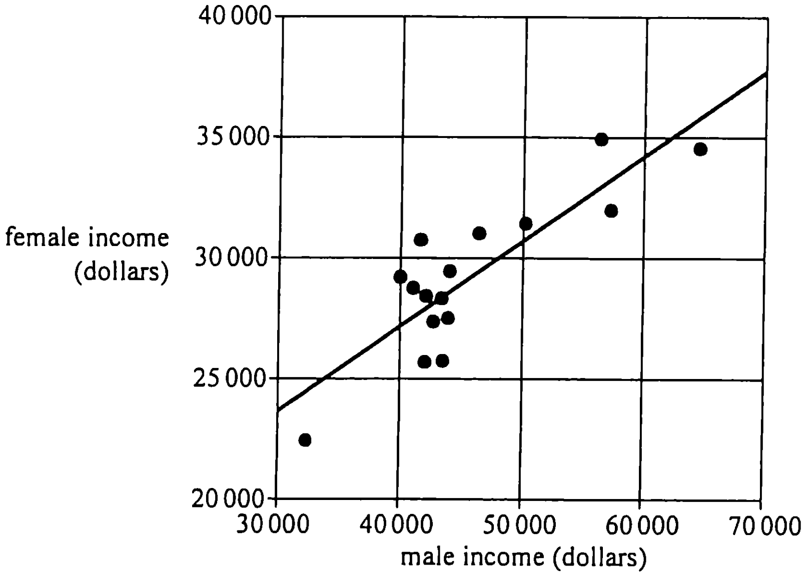
1 mark

Total 15 marks

END OF CORE

Question 2

In the scatterplot below, average annual *female income*, in dollars, is plotted against average annual *male income*, in dollars, for 16 countries. A least squares regression line is fitted to the data.



The equation of the least squares regression line for predicting female income from male income is

female income = 13 000 + 0.35 × *male income*

a. What is the explanatory variable?

male income

1 mark

b. Complete the following statement by filling in the missing information.

From the least squares regression line equation it can be concluded that, for these countries, on average, female income increases by \$ 350 for each \$1000 increase in male income.

1 mark

c. **i.** Use the least squares regression line equation to predict the average annual female income (in dollars) in a country where the average annual male income is \$15 000.

female income = 13000 + 0·35×15000 = \$18,250

ii. The prediction made in **part c. i.** is not likely to be reliable.

Explain why.

It is extrapolation (prediction outside of the data set).

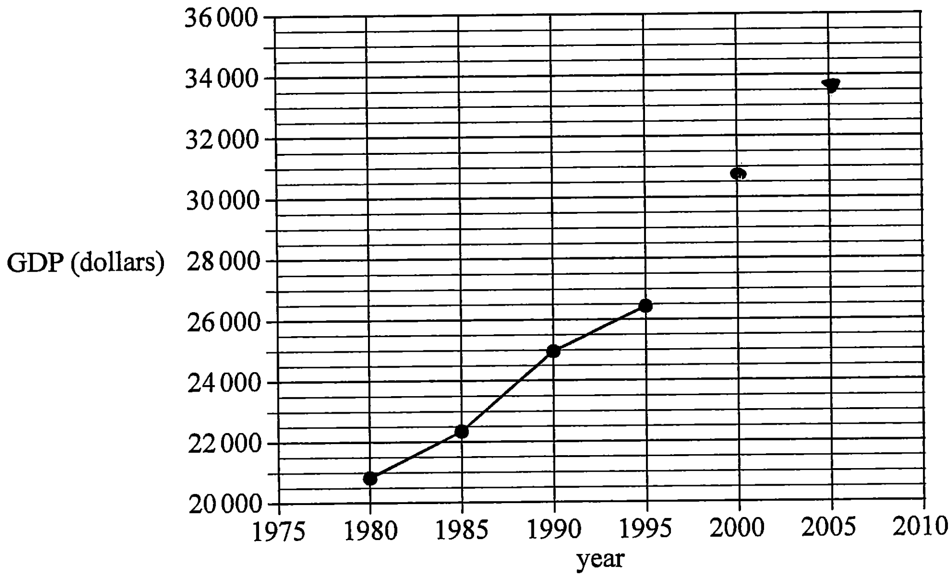
1 + 1 = 2 marks

Question 3

Table 2 shows the Australian gross domestic product (GDP) per person, in dollars, at five yearly intervals for the period 1980 to 2005.

Table 2

Year	1980	1985	1990	1995	2000	2005
GDP	20 900	22 300	25 000	26 400	30 900	33 800



a. Complete the time series plot above by plotting the GDP for the years 2000 and 2005.

1 mark

b. Briefly describe the general trend in the data.

An increasing trend

1 mark

In Table 3, the variable *year* has been rescaled using 1980 = 0, 1985 = 5 and so on. The new variable is *time*.

Table 3

Year	1980	1985	1990	1995	2000	2005
Time	0	5	10	15	20	25
GDP	20 900	22 300	25 000	26 400	30 900	33 800

c. Use the variables *time* and *GDP* to write down the equation of the least squares regression line that can be used to predict *GDP* from *time*. Take *time* as the independent variable.

GDP = 20000 + 524 × time

2 marks

d. In the year 2007, the *GDP* was $34 900. Find the error in the prediction if the least squares regression line calculated in **part c.** is used to predict *GDP* in 2007.

Predicted GDP:

GDP = 20000 + 524 × 27 = 34,148

Residual = Actual − Predicted

= 34900 − 34148 = $752

2 marks

Total 15 marks

END OF CORE

TURN OVER