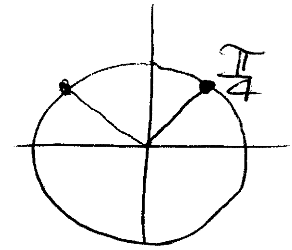
CAS FREE PRACTICE TEST

Question 1

Solve the equation:

$2\sin(x) - \sqrt{2} = 0 \; where \; -2\pi \le x \le \pi$

$\sin(x) = \frac{\sqrt{2}}{2}, \quad -2\pi \le x \le \pi$



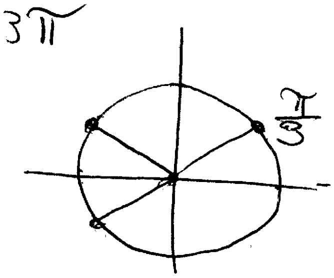
$x = -\frac{5\pi}{4}, -\frac{7\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

Question 2

Solve the equation: $2\cos(3x) + 1 = 0$, where $-\frac{\pi}{3} \le x \le \pi$

$\cos(3x) = \frac{-1}{2}, \quad \frac{-\pi}{3} \le x \le \pi$

$\cos(3x) = -\frac{1}{2}, \quad -\pi \le 3x \le 3\pi$



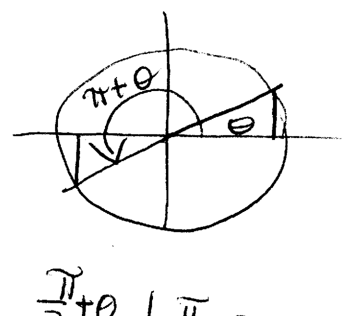
$3x = -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$

$\therefore x = -\frac{2\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$

Question 3

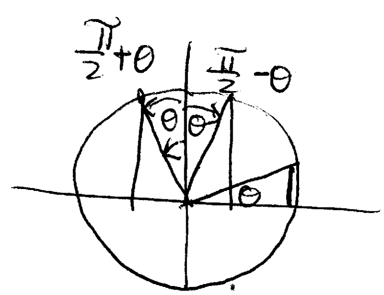
Simplify:

a) $\sin(\pi + \theta)$



$\sin(\pi + \theta) = -\sin(\theta)$

b) $\sin(\frac{\pi}{2} + \theta)$



$\sin\left(\frac{\pi}{2} + \theta\right) = \cos(\theta)$

Question 4

Consider the function: $y = -5\sin\left(\frac{\pi t}{8}\right) + 10$.

a. State the amplitude.

amplitude $= 5$

Note: the amplitude is POSITIVE !!

b. State the period.

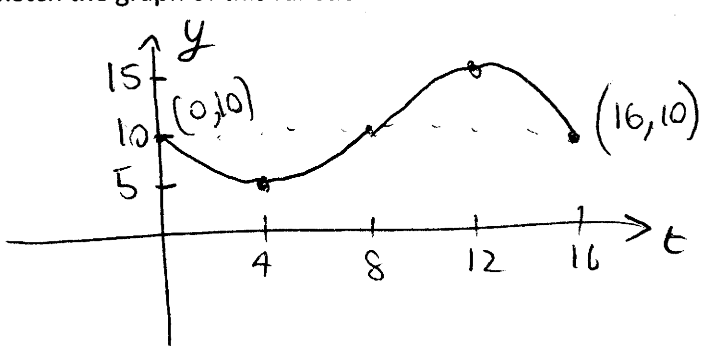
$$\frac{2\pi}{\frac{\pi}{8}} = \frac{2\pi}{1} \times \frac{8}{\pi} = 16$$

c. State the range.

15, 10, 5

$[5, 15]$

d. Sketch the graph of this function.



Give co-ordinates of endpoints

Question 5

If $\cos(\emptyset) = -\frac{3}{5}$ and $\frac{\pi}{2} < \emptyset < \pi$, find the value of $\sin(\emptyset)$.

$$\cos^2\phi + \sin^2\phi = 1$$

$$\therefore \left(\frac{-3}{5}\right)^2 + \sin^2\phi = 1$$

$$\sin^2\phi = 1 - \frac{9}{25}$$

$$\sin^2\phi = \frac{16}{25}$$

$$\therefore \sin\phi = \pm\sqrt{\frac{16}{25}}$$

Since $\frac{\pi}{2} < \phi < \pi$, $\sin\phi > 0$ $\therefore \sin\phi = \frac{4}{5}$.

CAS PRACTICE TEST

Question 1

The depth of the water in metres at a certain pier is given by the equation:

$d(t) = 2 + \frac{1}{2}\sin(\frac{\pi t}{6})$ where t is the number of hours after 10:00 am.

↑ 2.5
2
↓ 1.5

a. How deep will the water be at high tide?

2.5 m

b. Midtide occurs when the tide is exactly half way between high tide and low tide. From 10:00 am, what are the first two times when it is midtide?

$t = 6, 12$ $\therefore$ 4:00pm and 10:00pm

c. How many hours are there between two successive high tides?

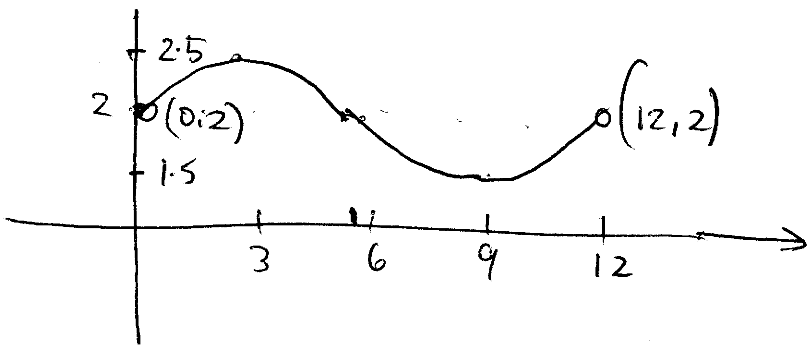
12 hours

Period $= \frac{2\pi}{\frac{\pi}{6}} = 12$

d. How many of these tide cycles are there in a 24 hour day?

$24 \div 12 = 2$ cycles

e. Sketch the graph of the depth of the water depth as a function of t for $0 < t < 12$.



Note: for this domain, endpoints must be shown open.

f. There is a rockshelf near to the pier that is only safe to visit when the depth of the water is 1.75m or less. If Dakota wants to visit the rockshelf in the evening, between what two times should she go?

$1.75 = 2 + \frac{1}{2}\sin\left(\frac{\pi t}{6}\right)$

$\therefore -\frac{1}{2} = \sin\left(\frac{\pi t}{6}\right)$

$\frac{\pi t}{6} = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\therefore t = 7, 11$

She should go between: 5:00pm and 9:00pm

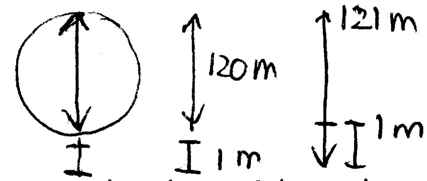
g. For what percentage of the time is the depth of the tide greater than 2.2 m? Give your answer correct to one decimal place.

$2.2 = 2 + \frac{1}{2}\sin\left(\frac{\pi t}{6}\right), 0 \le t \le 12$

Solving gives: $t = 0.7859, 5.2141$

Time interval $= 5.2141 - 0.7859 \simeq 4.4281$ hours

Required percentage $= \frac{4.4281}{12} \times 100\%$ $= \boxed{36.9\%}$



Question 2

A circular Ferris wheel is 120m in diameter and contains several carriages. Asher and Jack enter a carriage when it is at the lowest point and get off 24 minutes later, having gone around exactly 8 times. When a carriage is at the lowest point, it is 1 m off the ground.

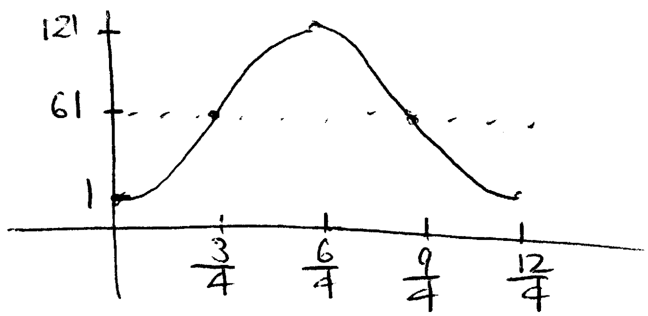
a. What is the maximum and minimum height of Asher and Jack's carriage?

Max = 121 m Min = 1 m

b. What is the period of the function $h(t)$, the function which gives the height of the carriage t minutes after it begins to move?

Period = $24 \div 8 = 3$ minutes

c. Sketch the graph of the function $h(t)$.



d. Find the equation of $h(t)$.

$$h(t) = -60\cos\left(\frac{2\pi t}{3}\right) + 61$$

Shape: reflected cosine

$a = 60$

$\frac{2\pi}{n} = 3$ $\therefore n = \frac{2\pi}{3}$

e. The ride suddenly stops 5 minutes and 30 ~~minutes~~ seconds after the ride starts. What is the height of Asher and Jack's carriage at this time?

When $t = 5.5$, $h(5.5) = -60\cos\left(\frac{11\pi}{3}\right) + 61$

$= -60\cos\left(4\pi - \frac{\pi}{3}\right) + 61$

$= -60 \times \frac{1}{2} + 61$ = 31 m

f. How long does it take for the carriage to first reach a height of 91 m?

$91 = -60\cos\left(\frac{2\pi t}{3}\right) + 61, \; 0 < t < 3$

Solving: $t = 1, 2$

$\therefore$ First reaches height of 91 m after 1 minute.