

Question 3

Regular customers at a hairdressing salon can choose to have their hair cut by Shirley, Jen or Narj.

The salon has 600 regular customers who get their hair cut each month.

In June, 200 customers chose Shirley (S) to cut their hair, 200 chose Jen (J) to cut their hair and 200 chose Narj (N) to cut their hair.

The regular customers' choice of hairdresser is expected to change from month to month as shown in the transition matrix, T , below.

$$T = \begin{matrix} & \begin{matrix} \text{this month} \\ S & J & N \end{matrix} \\ \begin{matrix} S \\ J \\ N \end{matrix} & \begin{bmatrix} 0.75 & 0.10 & 0.10 \\ 0.10 & 0.75 & 0.15 \\ 0.15 & 0.15 & 0.75 \end{bmatrix} \end{matrix} \begin{matrix} S \\ J \\ N \end{matrix} \text{ next month}$$

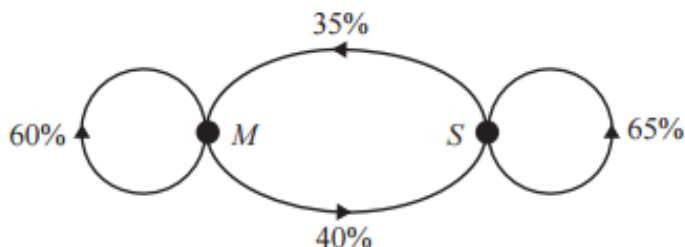
In the long term, the number of regular customers who are expected to choose Shirley is closest to

- A. 150
- B. 170
- C. 185
- D. 195
- E. 200

Question 4

Two hundred and fifty people buy bread each day from a corner store. They have a choice of two brands of bread: Megaslice (M) and Superloaf (S).

The customers' choice of brand changes daily according to the transition diagram below.



On a given day, 100 of these people bought Megaslice bread while the remaining 150 people bought Superloaf bread.

The number of people who are expected to buy each brand of bread the next day is found by evaluating the matrix product

A.

$$\begin{matrix} M & S \\ \begin{bmatrix} 0.60 & 0.40 \\ 0.35 & 0.65 \end{bmatrix} \end{matrix} \begin{bmatrix} 100 \\ 150 \end{bmatrix} \begin{matrix} M \\ S \end{matrix}$$

B.

$$\begin{matrix} M & S \\ \begin{bmatrix} 0.60 & 0.40 \\ 0.65 & 0.35 \end{bmatrix} \end{matrix} \begin{bmatrix} 100 \\ 150 \end{bmatrix} \begin{matrix} M \\ S \end{matrix}$$

C.

$$\begin{matrix} M & S \\ \begin{bmatrix} 0.60 & 0.35 \\ 0.40 & 0.65 \end{bmatrix} \end{matrix} \begin{bmatrix} 100 \\ 150 \end{bmatrix} \begin{matrix} M \\ S \end{matrix}$$

D.

$$\begin{matrix} M & S \\ \begin{bmatrix} 0.65 & 0.40 \\ 0.35 & 0.60 \end{bmatrix} \end{matrix} \begin{bmatrix} 100 \\ 150 \end{bmatrix} \begin{matrix} M \\ S \end{matrix}$$

E.

$$\begin{matrix} M & S \\ \begin{bmatrix} 0.60 & 0.65 \\ 0.40 & 0.35 \end{bmatrix} \end{matrix} \begin{bmatrix} 100 \\ 150 \end{bmatrix} \begin{matrix} M \\ S \end{matrix}$$

Question 9

A fast-food stand at the football sells pies (P) and chips (C).

Each week, 300 customers regularly buy either a pie or chips, but not both, from this stand.

For the first five weeks, the customers' choice of pie or chips is expected to change weekly according to transition matrix T_1 , where

$$T_1 = \begin{matrix} & \begin{matrix} \text{this week} \\ P & C \end{matrix} \\ \begin{matrix} \text{next week} \\ P \\ C \end{matrix} & \begin{bmatrix} 0.65 & 0.25 \\ 0.35 & 0.75 \end{bmatrix} \end{matrix}$$

After the first five weeks, due to expected cold weather, the customers' choice of pie or chips is change weekly according to the transition matrix T_2 , where

$$T_2 = \begin{matrix} & \begin{matrix} \text{this week} \\ P & C \end{matrix} \\ \begin{matrix} \text{next week} \\ P \\ C \end{matrix} & \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \end{matrix}$$

In week 1, 150 customers bought a pie and 150 customers bought chips.

Let S_1 be the state matrix for week 1.

The number of customers expected to buy a pie or chips in week 8 can be found by evaluating

- A. $T_2^7 S_1$
- B. $T_1^8 S_1$
- C. $T_2^3 (T_1^4 S_1)$
- D. $T_1^3 (T_2^4 S_1)$
- E. $T_1^3 (T_2^5 S_1)$

Question 3

A coffee shop sells three types of coffee, Brazilian (B), Italian (I) and Kenyan (K). The regular customers buy one cup of coffee each per day and choose the type of coffee they buy according to the following transition matrix, T .

$$T = \begin{matrix} & \begin{matrix} \text{choose today} \\ B & I & K \end{matrix} \\ \begin{matrix} \text{choose tomorrow} \\ B \\ I \\ K \end{matrix} & \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0 & 0.8 & 0.1 \\ 0.2 & 0.1 & 0.8 \end{bmatrix} \end{matrix}$$

On a particular day, 84 customers bought Brazilian coffee, 96 bought Italian coffee and 81 bought Kenyan coffee.

If these same customers continue to buy one cup of coffee each per day, the number of these customers who are expected to buy each of the three types of coffee in the long term is

- | | | | | | |
|-----------|----|-----------|-----|-----------|----|
| A. | | B. | | C. | |
| Brazilian | 85 | Brazilian | 87 | Brazilian | 88 |
| Italian | 85 | Italian | 58 | Italian | 86 |
| Kenyan | 91 | Kenyan | 116 | Kenyan | 87 |
| D. | | E. | | | |
| Brazilian | 89 | Brazilian | 116 | | |
| Italian | 89 | Italian | 89 | | |
| Kenyan | 83 | Kenyan | 58 | | |

Question 8

There are 30 children in a Year 6 class. Each week every child participates in one of three activities: cycling (C), orienteering (O) or swimming (S).

The activities that the children select each week change according to the transition matrix below.

$$T = \begin{array}{c} \text{this week} \\ C \quad O \quad S \\ \left[\begin{array}{ccc} 0.5 & 0.3 & 0.3 \\ 0.1 & 0.6 & 0.2 \\ 0.4 & 0.1 & 0.5 \end{array} \right] \begin{array}{l} C \\ O \\ S \end{array} \text{ next week} \end{array}$$

From the transition matrix it can be concluded that

- A. in the first week of the program, ten children do cycling, ten children do orienteering and ten children do swimming.
- B. at least 50% of the children do not change their activities from the first week to the second week.
- C. in the long term, all of the children will choose the same activity.
- D. orienteering is the most popular activity in the first week.
- E. 50% of the children will do swimming each week.

Question 7

A new colony of several hundred birds is established on a remote island. The birds can feed at two locations, A and B . The birds are expected to change feeding locations each day according to the transition matrix

$$T = \begin{array}{c} \text{this day} \\ A \quad B \\ \left[\begin{array}{cc} 0.4 & 0.3 \\ 0.6 & 0.7 \end{array} \right] \begin{array}{l} A \\ B \end{array} \text{ next day} \end{array}$$

In the beginning, approximately equal numbers of birds feed at each site each day.

Which of the following statements is **not** true?

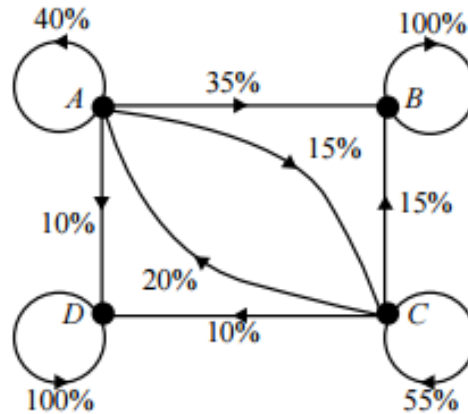
- A. 70% of the birds that feed at B on a given day will feed at B the next day.
- B. 60% of the birds that feed at A on a given day will feed at B the next day.
- C. In the long term, more birds will feed at B than at A .
- D. The number of birds that change feeding locations each day will decrease over time to zero.
- E. In the long term, some birds will always be found feeding at each location.

The following information relates to Questions 7, 8 and 9.

A large population of mutton birds migrates each year to a remote island to nest and breed. There are four nesting sites on the island, *A*, *B*, *C* and *D*.

Researchers suggest that the following transition matrix can be used to predict the number of mutton birds nesting at each of the four sites in subsequent years. An equivalent transition diagram is also given.

$$T = \begin{matrix} & \begin{matrix} \textit{this year} \\ A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0.4 & 0 & 0.2 & 0 \\ 0.35 & 1 & 0.15 & 0 \\ 0.15 & 0 & 0.55 & 0 \\ 0.1 & 0 & 0.1 & 1 \end{bmatrix} \end{matrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix} \textit{ next year}$$



Question 7

Two thousand eight hundred mutton birds nest at site *C* in 2008.

Of these 2800 mutton birds, the number that nest at site *A* in 2009 is predicted to be

- A. 560
- B. 980
- C. 1680
- D. 2800
- E. 3360

Question 8

This transition matrix predicts that, in the long term, the mutton birds will

- A. nest only at site *A*.
- B. nest only at site *B*.
- C. nest only at sites *A* and *C*.
- D. nest only at sites *B* and *D*.
- E. continue to nest at all four sites.

Question 9

Six thousand mutton birds nest at site *B* in 2008.

Assume that an equal number of mutton birds nested at each of the four sites in 2007. The same transition matrix applies.

The total number of mutton birds that nested on the island in 2007 was

- A. 6 000
- B. 8 000
- C. 12 000
- D. 16 000
- E. 24 000

Question 7

A transition matrix, T , and a state matrix, S_2 , are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \quad S_2 = \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$$

If $S_2 = TS_1$, the state matrix S_1 is

- A. $\begin{bmatrix} 200 \\ 250 \\ 150 \end{bmatrix}$ B. $\begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$ C. $\begin{bmatrix} 300 \\ 0 \\ 300 \end{bmatrix}$ D. $\begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$ E. undefined

Question 2

A new shopping centre called Shopper Heaven (S) is about to open. It will compete for customers with Easttown (E) and Noxland (N).

Market research suggests that each shopping centre will have a regular customer base but attract and lose customers on a weekly basis as follows.

- 80% of Shopper Heaven customers will return to Shopper Heaven next week
- 12% of Shopper Heaven customers will shop at Easttown next week
- 8% of Shopper Heaven customers will shop at Noxland next week
- 76% of Easttown customers will return to Easttown next week
- 9% of Easttown customers will shop at Shopper Heaven next week
- 15% of Easttown customers will shop at Noxland next week
- 85% of Noxland customers will return to Noxland next week
- 10% of Noxland customers will shop at Shopper Heaven next week
- 5% of Noxland customers will shop at Easttown next week

- a. Enter this information into transition matrix T as indicated below (express percentages as proportions, for example write 76% as 0.76).

$$T = \begin{array}{c} \begin{matrix} & \text{this week} & & & \\ & S & E & N & \\ \left[\begin{array}{ccc} & & \\ & & \\ & & \end{array} \right] & & & & \\ & S & E & N & \\ \end{matrix} \end{array} \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \begin{array}{l} \\ \\ \text{next week} \\ \\ \end{array}$$

2 marks

During the week that Shopper Heaven opened, it had 300 000 customers.

In the same week, Easttown had 120 000 customers and Noxland had 180 000 customers.

- b. Write this information in the form of a column matrix, K_0 , as indicated below.

$$K_0 = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{array}{l} S \\ E \\ N \end{array}$$

- c. Use T and K_0 to **write** and **evaluate** a matrix product that determines the number of customers expected at each of the shopping centres during the following week.

2 marks

- d. Show by calculating at least two appropriate state matrices that, in the long term, the number of customers expected at each centre each week is given by the matrix

$$K = \begin{bmatrix} 194983 \\ 150513 \\ 254504 \end{bmatrix}$$