SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (12 marks)

Consider the region A bounded by the curve $y = 4x - x^2$ and the x-axis for $x \in [0, 4]$.

a. Find the area of region A. 1 mark

b. Region A can be subdivided into two regions, B and C, by the line $y = x$. The points in B are above the line $y = x$ and the points in region C are below it.

Find the coordinates of the points of intersection of the line $y = x$ with the curve $y = 4x - x^2$. 1 mark

c. Find the area of region B. 2 marks

 TEVMMU34EX2_QA_2015.FM

d. Consider the equation $y = mx$, where m is a real constant such that $0 < m < 4$. This equation defines a whole family of lines. For each value of m, the line will intersect the curve $y = 4x - x^2$ at the origin and also at one other point, P_m. The line $y = mx$ will divide the region A into two regions, B_m and C_m (with points in region B_m above the line $y = mx$).

Find the coordinates of the point P_m as a function of m. 2 marks

e. Find the area of region B_m as a function of m. 2 marks

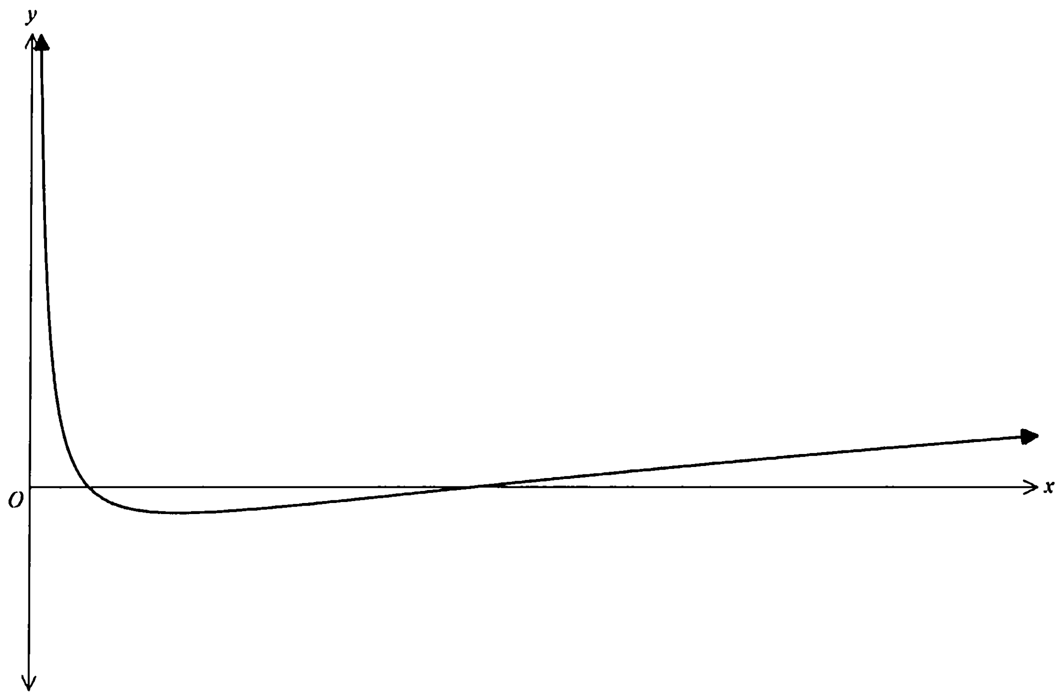
f. Hence, find an expression for the area of region C_m in terms of m. 2 marks

g. Find the particular value of m for which the two regions B_m and C_m are equal in area. 2 marks

Question 4 (13 marks)

Consider the family of functions $f_a : R^+ \to R$, which is defined by $f_a(x) = \frac{a}{x} + \sqrt{x} - 3$, where a is a real number and $a > 0$.

Part of the graph of f_a is shown below.



a. State the interval for which the graph of f_a is strictly increasing, in terms of a.

2 marks

b. Determine the absolute minimum value of f_a, in terms of a.

2 marks

c. Show that the equation of the tangent to the graph of f_a at the point when $x=4$ is $y=-\frac{(a-4)}{16}x+\frac{(a-4)}{2}$.

3 marks

The function $f_a(x)$ is transformed to form $g_a(x)$, where $g_a(x)$ is defined as $g_a(x) = f_a(x) + b$.

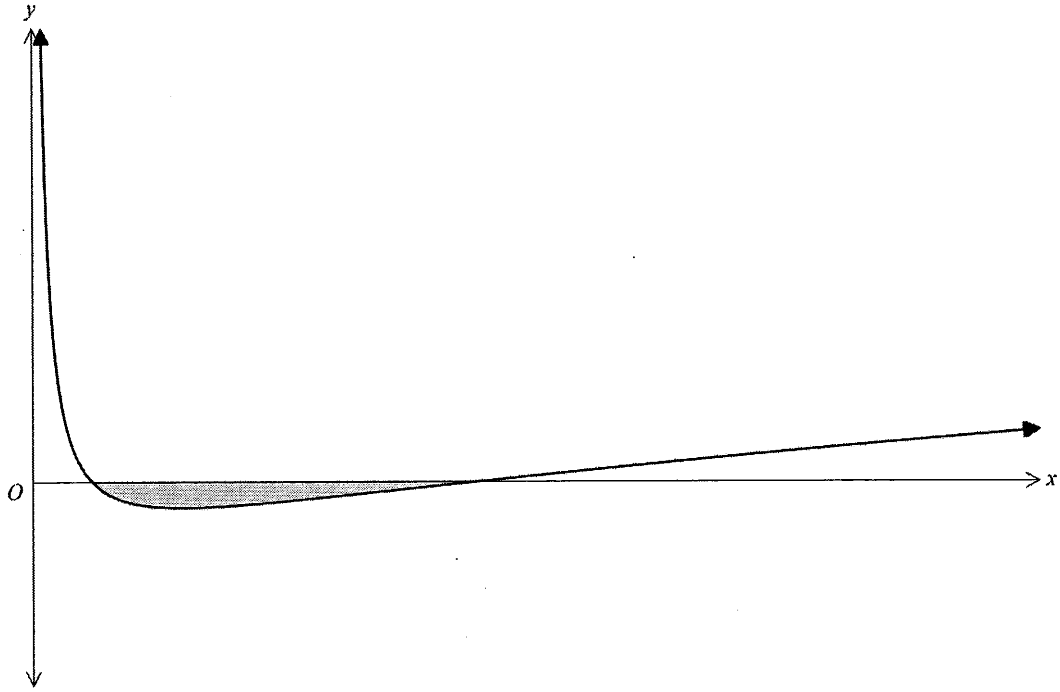
d. Find the value of b, in terms of a, such that the tangent drawn to the curve of $g_a(x)$ at $x = 4$ passes through the origin.

2 marks

Consider the graph of $f_a(x)$ for $a = 2$.
Let $h_a(x) = f_a(2x)$.

e. **i.** Find the value of the area that is bounded by the x-axis and the graph of $y = f_a(x)$ for $a = 2$, as shaded in the diagram below, correct to 3 decimal places.

2 marks



ii. Hence, find the area that is bounded by the x-axis and the graph of $y = h_a(x)$ for $a = 2$, correct to 3 decimal places.

2 marks

Question 3

a. The gradient of a curve at any point is given by $\frac{dh}{dt} = \sin\left(\frac{\pi t}{6}\right)$. The depth of water in a tidal pool can be modelled by h, where h metres is the depth of the water and t is the number of hours after 6 a.m. on a given day. Find an expression for h in terms of t, if there is no water in the pool at 6 a.m.

3 marks

b. It is only safe to swim in the tidal pool when the water level is at the most two metres. Between what hours, over a 24-hour period, is it safe to swim? Give the time to the nearest minute.

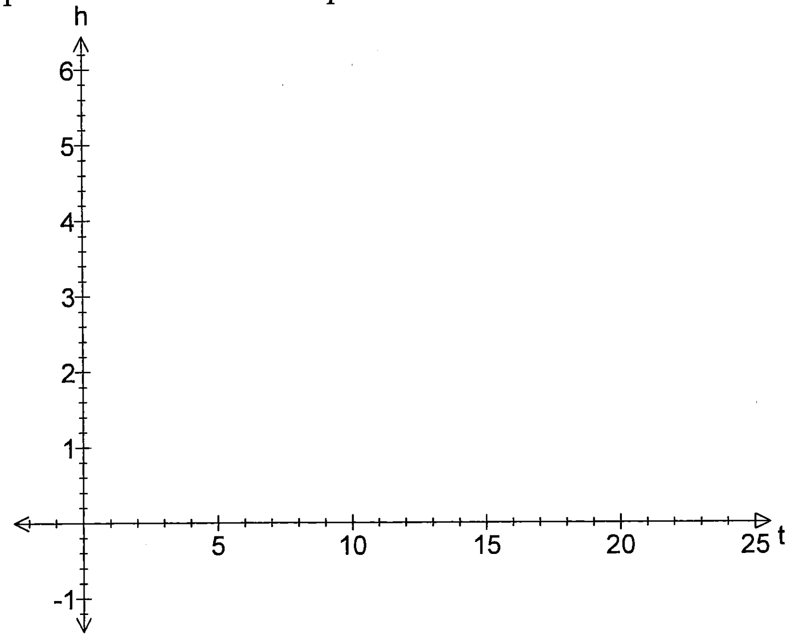
2 marks

c. What is the exact average depth of water in the tidal pool over the 24-hour period?

2 marks

TURN OVER

d. Sketch the graph of h and construct a rectangle whose area is the same as the area under the graph of h over the 24-hour period.



3 marks

e. Describe the transformations required to transform the graph of $y = \cos(t)$ into the graph of h.

____

____

____

____

2 marks

f. Find the general solution to the equation $0 = \frac{-6}{\pi}\cos\left(\frac{\pi t}{6}\right) + \frac{6}{\pi}$.

____

____

1 mark

g. Consider the functional equation $k(t+\pi) = k(t)$. Show that the function with rule $k(t) = \cos(2t)$ satisfies this functional equation. Give reasons for your answer.

2 marks

Total 15 marks

Question 4

The total surface area of a rectangular box with a square base and open at the top, is 675 cm^2.

a. Express h in term of x.

2 marks

b. **i.** Find the dimensions of the box so that its volume is a maximum, and hence state the volume of the box.

4 marks