FURTHER REVISION QUESTIONS ON STATISTICS (SAMPLE PROPORTION and CONFIDENCE INTERVALS)

Question 1

The company JCrew advertises that 95% of its online orders ship within two working days. A random sample of 200 of the 10,000 orders received over the past month is selected for an audit. The audit reveals that 180 of these orders shipped on tme.

a) What is the sample proportion of orders shipped on time?

$$\hat{p} = \frac{180}{200} = 0.9$$

b) Show that the sample data satisfies the conditions necessary for the sample proportion to follow an approximately normal distribution.

$$np = 200 \times 0.95 = 190 > 5$$

$$n(1-p) = 200 \times 0.05 = 10 > 5$$

$\therefore$ Conditions are satisfied.

c) What is the mean and standard deviation of the sampling proportion, $\hat{p}$?

$$E(\hat{p}) = p = 0.95$$

$$\sigma(\hat{p}) = \sqrt{\frac{0.95 \times 0.05}{200}} \simeq 0.01541$$

d) If JCrew really does ship 95% of its orders on time, what is the probability that the proportion in a random sample of 200 orders is as small or smaller than the proportion in the audit? (Use the normal approximation)

$$\Pr(\hat{p} \leq 0.9) \simeq 0.0006$$

$$\hat{p} \overset{d}{\simeq} N(\mu = 0.95, \sigma = 0.01541)$$

e) If instead this problem were treated as a Binomial distribution, how would this be set up? That is, what would we be trying to find the probability of?

$$X \overset{d}{=} Bi(n = 200, p = 0.95)$$

Let X = no. of orders shipped on time

$X = 0.90 \times 200 = 180$

Find: $\Pr(X \leq 200)$

So, if we were using the Binomial probability distribution instead of the Normal approximation to solve this problem, we would calculate: $\Pr(X \leq 200)$ where $X \overset{d}{=} Bi(n = 200, p = 0.95)$

Question 2

A Gallup poll found that for those Australians who have lost weight, 31% believed that the most effective strategy involved exercise. What is the probability that, for a random sample of 300 Australians, the sample proportion falls between 29% and 33%?

a) Use the Normal approximation to determine this probability.

$n = 300,\ p = 0.31 \qquad Pr(0.29 \leq \hat{p} \leq 0.33)$

$\hat{p} \overset{d}{\approx} N(\mu = 0.31, \sigma = 0.0267)$

$\sigma = \sqrt{\frac{0.31 \times 0.69}{300}}$

$Pr(0.29 \leq \hat{p} \leq 0.33)$

$\simeq 0.5461$

$\sigma = 0.02670206$

b) Use the Binomial distribution to determine this probability.

$0.29 \times 300 = 87 \ ; \ 0.33 \times 300 = 99$

X = no. of people who believe in exercise

$X \overset{d}{=} Bi(n = 300, p = 0.31)$

$Pr(87 \leq X \leq 99) \simeq 0.5828$

Question 3

Suppose that the true proportion of Melburnians who support an increase of $400 per week in the minimum wage is 65%. If we took a random sample of 805 Melburnians and calculated the sample proportion of those who support this increase, what would be the mean and the standard deviation of the sample proportion? Give your answers where appropriate to three decimal places.

$E(\hat{p}) = 0.65$

$\sigma(\hat{p}) = \sqrt{\frac{0.65 \times 0.35}{805}} \simeq 0.0192$

Question 4

A research company conducted a poll and asked a random sample of 805 Melburnians if they supported an increase of $400 to the minimum weekly wage. They found a sample proportion of 68%.

i. Determine a 95% confidence interval for the true proportion of Melburnians who support such a wage increase. $n = 805, \quad x = 0.68 \times 805 = 547.4 \simeq 547$

Confidence interval: (0.647, 0.712)

$CL = 0.95$

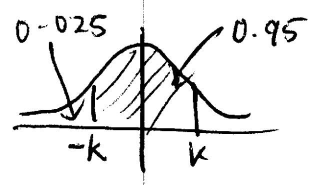
ii. Determine a 90% confidence interval for this true proportion.

(0.652,0.707)

Question 5

A survey estimated that 20% of all Americans aged 16 to 20 drove under the influence of drugs or alcohol. A similar survey is planned for New Zealand. They want a 95% confidence interval to have a margin of error of 0.04.

(a) Find the necessary sample size if they expect to find results similar to those in the United States.



$\hat{p} = 0.2$

$\therefore 0.04 = k\sqrt{\frac{0.2 \times 0.8}{n}}$

$-k = \text{invNorm}(0.025, 0, 1)$

$-k = -1.95996$

$\therefore 0.04 = 1.95996\sqrt{\frac{0.2 \times 0.8}{n}}$

Solving: $n = 384.146$ $\therefore n = 385$

(b) Suppose instead they used the conservative formula based on $\hat{p} = 0.5$. What is now the required sample size?

$0.04 = k \times \sqrt{\frac{0.5 \times 0.5}{n}}$ where $k = 1.95996$

Solving: $n = 600.2255$ $\therefore n \simeq 601$