

**Question 2** (18 marks)

Rebecca's Robotics manufactures three types of components for robots: sensors, motors and controllers. The manufacturing processes for each type of component are independent.

It is known that 8% of all of the sensors manufactured are defective.

- a. A random sample of five sensors is selected.

Find, correct to four decimal places, the probability that

- i. exactly two of these selected sensors are defective

2 marks

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- ii. exactly two of these selected sensors are defective, given that at most two sensors in the sample are defective.

2 marks

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- b. A random sample of 50 sensors is selected and it is found that the proportion of defective sensors in this sample is 0.08

Determine an approximate 90% confidence interval for the proportion of defective sensors, correct to four decimal places.

2 marks

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A hole is drilled into each motor. The depth of the hole is normally distributed with a mean of 20 mm and a standard deviation of 0.3 mm.

- c. What is the probability that, for a randomly selected motor, the depth of the hole is greater than 20.6 mm? Give your answer correct to four decimal places.

1 mark

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The depth of the hole drilled into a motor must be within 0.5 mm of the mean, otherwise the motor is defective.

- d. What is the probability that a motor is defective, correct to four decimal places? 2 marks

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- e. Rebecca delivers an order for five sensors and five motors.

What is the probability that the order contains exactly two defective components? Give your answer correct to three decimal places.

3 marks

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- f. A knob is attached to each controller. The height of a knob is normally distributed with a mean of 30 mm. If the knob on a controller has a height greater than 30.4 mm or less than 29.6 mm, then the controller is defective.

Rebecca wants to ensure that less than 2% of all controllers manufactured are defective.

What is the maximum standard deviation of the height of a knob, in millimetres, that can be attached to a controller so that less than 2% of controllers are defective? Give your answer correct to two decimal places.

2 marks

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The weight,  $w$ , in grams, of controllers is modelled by the following probability density function.

$$C(w) = \begin{cases} \frac{3}{640000}(330 - w)^2(w - 290) & 290 \leq w \leq 330 \\ 0 & \text{elsewhere} \end{cases}$$

- g.** Determine the mean weight, in grams, of the controllers. 2 marks

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- h.** Determine the probability that a randomly selected controller weighs less than the mean weight of the controllers. Give your answer correct to four decimal places. 2 marks

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