

Module 2: Geometry and trigonometry

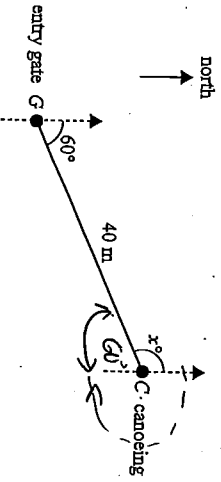
Question 1

In the plan below, the entry gate of an adventure park is located at point G.

A canoeing activity is located at point C.

The straight path GC is 40 metres long.

The bearing of C from G is 060° .



1. Write down the size of the angle that is marked x° in the plan above.

$$x = 120$$

1 mark

2. What is the bearing of the entry gate from the canoeing activity?

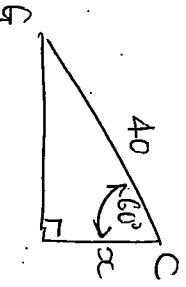
$$180^\circ + 60^\circ = 240^\circ$$

1 mark

3. How many metres north of the entry gate is the canoeing activity?

$$20 \text{ m}$$

1 mark



$$\cos(60^\circ) = \frac{x}{40}$$

$$x = 40 \cos(60^\circ)$$

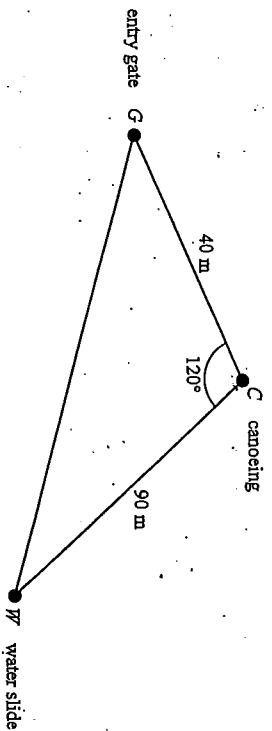
$$x = 20$$

Module 2: Geometry and trigonometry

GW is a 90 metre straight path between the canoeing activity and a water slide located at point W.

GW is a straight path between the entry gate and the water slide.

The angle GCV is 120° .



- d. i. Find the area that is enclosed by the three paths, GC, CV and GW.

Write your answer in square metres, correct to one decimal place.

$$A = 0.5 \times 40 \times 90 \sin(120^\circ)$$

$$= 1558.8 \text{ m}^2$$

$$\approx 1558.8 \text{ m}^2$$

- ii. Show that the length of path GW is 115.3 metres, correct to one decimal place.

$$C^2 = a^2 + b^2 - 2ab \cos(C)$$

$$\therefore C^2 = 40^2 + 90^2 - 2 \times 40 \times 90 \cos(120^\circ)$$

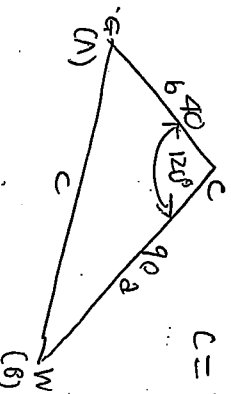
$$C^2 = 13300$$

$$C = \sqrt{13300}$$

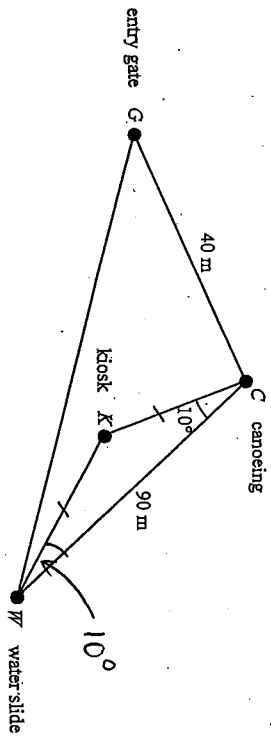
$$C = 115.326$$

$$\approx 115.3 \text{ m}$$

1 + 1 = 2 marks



Straight paths CK and WK lead to the kiosk located at point K.
These two paths are of equal length.
The angle KCV is 10° .

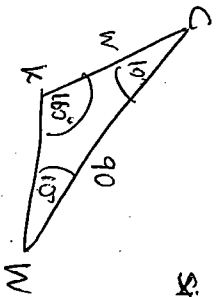


i. Find the size of the angle CKV.
 $180^\circ - 10^\circ - 10^\circ = 160^\circ$

ii. Find the length of path CK, in metres, correct to one decimal place.

$8 \quad 45.7 \text{ m}$

1 + 1 = 2 marks



$$\sin \frac{90}{\sin(160^\circ)} = \frac{W}{\sin(10^\circ)}$$

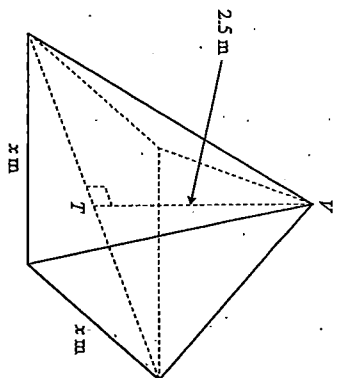
$$W = \frac{90 \sin(10^\circ)}{\sin(160^\circ)}$$

$$W \approx 45.69$$

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Question 3

A concrete square pyramid with volume 1.8 m^3 sits on the flat top of the hill.
The length of the square base of the pyramid is x metres. The height of the pyramid, YT , is 2.5 metres.



Find the value of x , correct to two decimal places.

$$V = \frac{Ah}{3}$$

$$\therefore 1.8 = \frac{x^2 \times 2.5}{3}$$

2 marks

Use solve on CAS.

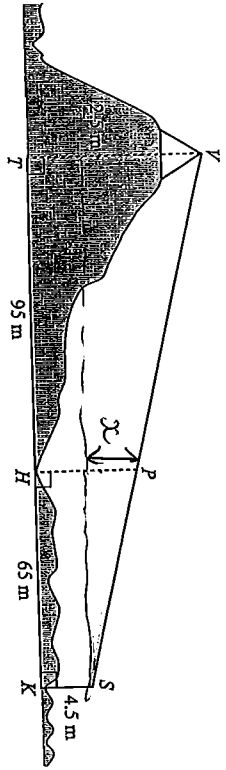
$$\text{OR! } x^2 = \frac{3 \times 1.8}{2.5}$$

$$x^2 = \frac{5.4}{2.5}$$

$$\therefore x = \sqrt{\frac{5.4}{2.5}}$$

$$\therefore x = 1.47$$

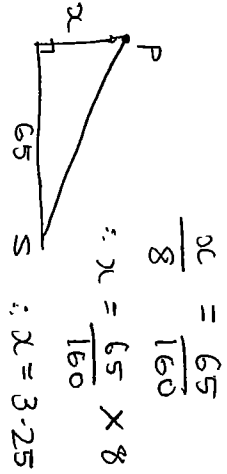
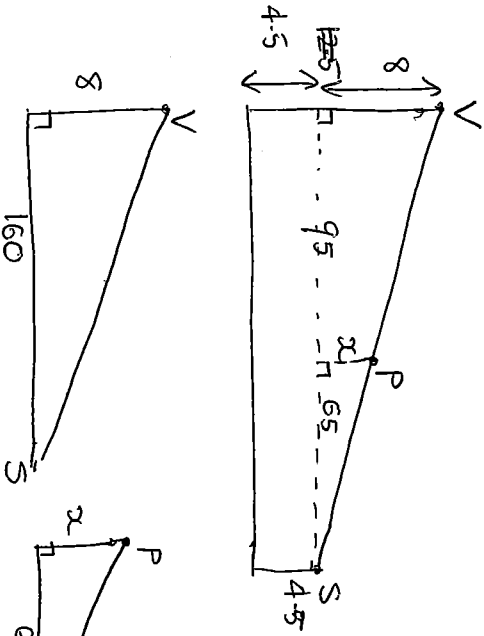
Question 4
A flying fox suspension wire begins at Y , 12.5 metres above T as shown in the diagram below. It ends at S , 4.5 metres above K .



At P , the flying fox wire passes over H .
The horizontal distances TH and HK are 95 metres and 65 metres respectively.
Calculate the vertical distance, PH , in metres.

$$\begin{aligned} PH &= 4.5 + 3.25 \\ &= 7.75 \text{ m} \end{aligned}$$

2 marks
Total 15 marks

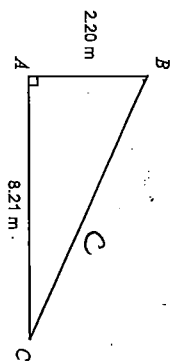


$$\begin{aligned} \frac{OC}{8} &= \frac{65}{160} \\ \therefore x &= \frac{65}{160} \times 8 \\ \therefore x &= 3.25 \end{aligned}$$

END OF MODULE 2

Module 2: Geometry and trigonometry

Question 1
Jane is landscaping her garden. A piece of shade cloth ABC has the dimensions as shown below.



2. Determine the length BC in metres. Write your answer correct to two decimal places.

$$\begin{aligned} C^2 &= 2.20^2 + 8.21^2 \\ C &= \sqrt{2.20^2 + 8.21^2} = 8.50 \end{aligned}$$

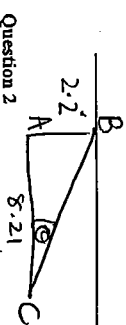
1 mark

b. Determine the angle ACB . Write your answer correct to the nearest degree.

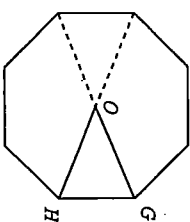
$$\tan(\theta) = \frac{2.2}{8.21}$$

$$\theta = \tan^{-1}\left(\frac{2.2}{8.21}\right) = 15^\circ$$

1 mark



Question 2
A paved area is constructed in the shape of a regular octagon as shown below.



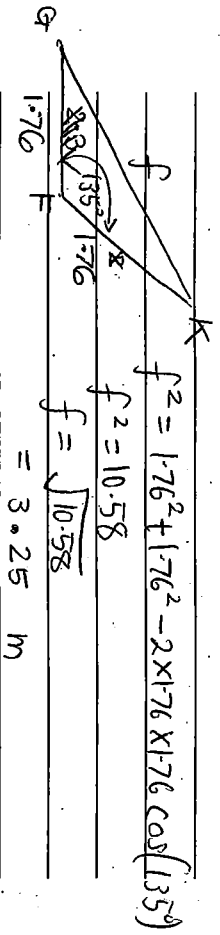
2. By calculation, show that the size of the angle GOH is 45° , where point O is the centre of the octagon.

$$\angle GOH = \frac{360}{8} = 45^\circ$$

1 mark

d. A straight wooden frame is to be built between points O and K for hanging baskets.

i. Calculate the length GK . Write your answer in metres, correct to two decimal places.

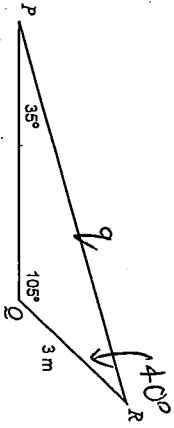


ii. Hence calculate the length OK . Write your answer in metres, correct to two decimal places.

$OK = 2.30 + 3.25 = 5.55 \text{ m}$

2 + 1 = 3 marks

A second piece of shade cloth PQR is also triangular and has dimensions as shown in the diagram below.



e. Calculate the length PR . Write your answer in metres, correct to two decimal places.

$q = \frac{3}{\sin(105^\circ)} \sin(35^\circ)$
 $q = \frac{3 \sin(35^\circ)}{\sin(105^\circ)}$
 $q = 5.05 \text{ m}$

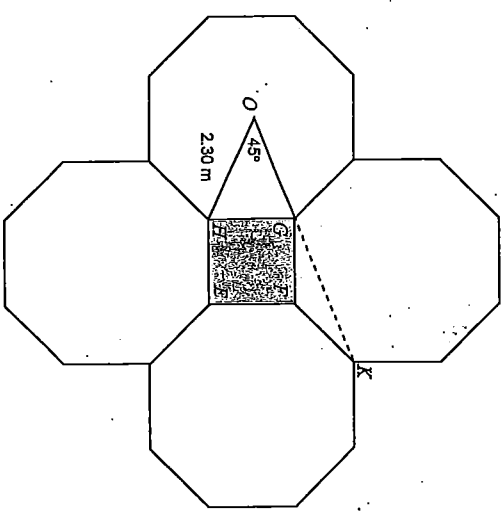
1 mark

b. The length $OG = OH = 2.30$ metres. Calculate the area of the octagonal paved area. Write your answer correct to the nearest square metre.

Area of triangle = $\frac{1}{2} \times 2.3 \times 2.3 \times \sin(45^\circ)$
 $= 1.8703$
 Total area of octagon = $1.8703 \times 8 \approx 15 \text{ m}^2$

2 marks

A square herb garden $ERGH$ is surrounded by four regular octagonal paved areas as shown in the diagram below.

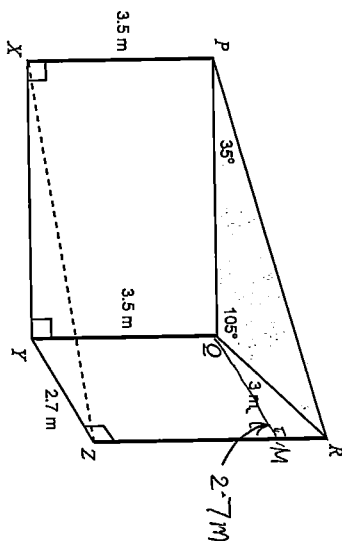


c. Calculate the side length GH of the square herb garden. Write your answer in metres, correct to two decimal places.

$\frac{x}{\sin(45^\circ)} = \frac{2.3}{\sin(67.5^\circ)}$
 $x = \frac{2.3 \sin(45^\circ)}{\sin(67.5^\circ)}$
 $x = 1.76$
 Length of square = 1.76 m

2 marks

The second piece of shade cloth PQR is attached to three vertical poles located at X , Y and Z as shown in the diagram below. Poles PX and QY are each 3.5 metres long. The horizontal distance YZ is 2.7 metres.



f. Calculate the length of the vertical pole RZ . Write your answer correct to the nearest centimetre.

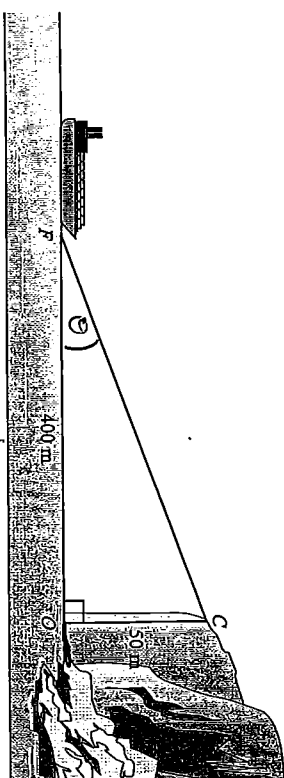
$a^2 = 3^2 - 2.7^2$
 $a = \sqrt{3^2 - 2.7^2}$
 $a = 1.30767$
 $\therefore RZ = 3.5 + 1.30767$
 $= 4.808 \text{ m}$
 $\approx 481 \text{ cm}$

2 marks

Module 2: Geometry and trigonometry

Question 1

A ferry, F , is 400 metres from point O at the base of a 50 metre high cliff, OC .



a. Show that the gradient of the line FC in the diagram is 0.125.

$m = \frac{\text{Rise}}{\text{Run}} = \frac{50}{400} = \frac{1}{8} = 0.125$

1 mark

b. Calculate the angle of elevation of point C from F .

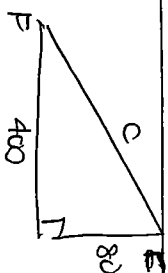
Write your answer in degrees, correct to one decimal place.
 $\tan(\theta) = \frac{50}{400}$
 $\theta = \tan^{-1}(0.125) \approx 7.1^\circ$

1 mark

c. Calculate the distance FC , in metres, correct to one decimal place.

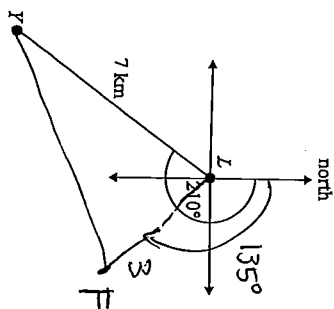
$c^2 = 80^2 + 400^2$
 $c = \sqrt{80^2 + 400^2}$
 $c = 407.922$
 $\approx 408 \text{ m}$

1 mark



Question 2

A yacht, Y, is 7 km from a lighthouse, L, on a bearing of 210° as shown in the diagram below.



a. A ferry can also be seen from the lighthouse. The ferry is 3 km from L on a bearing of 135° . On the diagram above, label the position of the ferry, F, and show an angle to indicate its bearing.

b. Determine the angle between LY and LF.

$$210^\circ - 135^\circ = 75^\circ$$

1 mark

c. Calculate the distance, in km, between the ferry and the yacht correct to two decimal places.

$$l^2 = 7^2 + 3^2 - 2 \times 7 \times 3 \cos(75^\circ)$$

$$l^2 = 47.1296$$

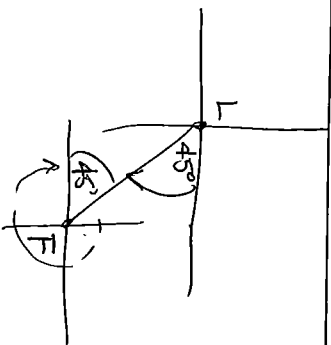
$$l = \sqrt{47.1296} \approx 6.87 \text{ km}$$

1 mark

d. Determine the bearing of the lighthouse from the ferry.

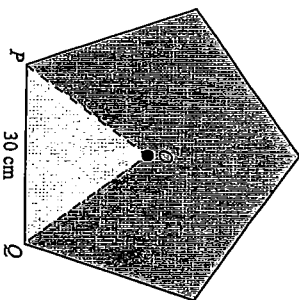
$$270^\circ + 45^\circ = 315^\circ \text{ T}$$

1 mark



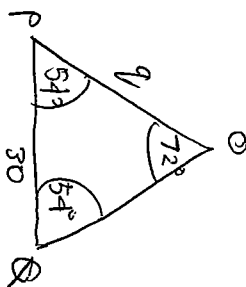
Question 3

The ferry has a logo painted on its side. The logo is a regular pentagon with centre O and side length 30 cm. It is shown in the diagram below.



a. Show that angle POQ is equal to 72° .

$$\angle POQ = \frac{360^\circ}{5} = 72^\circ$$



1 mark

b. Show that, correct to two decimal places, the length OP is 25.52 cm.

$$q = 30 \sin(54^\circ)$$

$$q \approx 25.52 \text{ cm}$$

1 mark

c. Find the area of the pentagon. Write your answer correct to the nearest cm^2 .

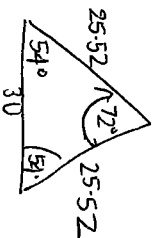
$$A = 0.5 \times 25.52 \times 30 \times \sin(72^\circ)$$

$$A \approx 309.697 \text{ cm}^2$$

$$\therefore \text{Area of pentagon} = 5 \times 309.697$$

$$\approx 1548 \text{ cm}^2$$

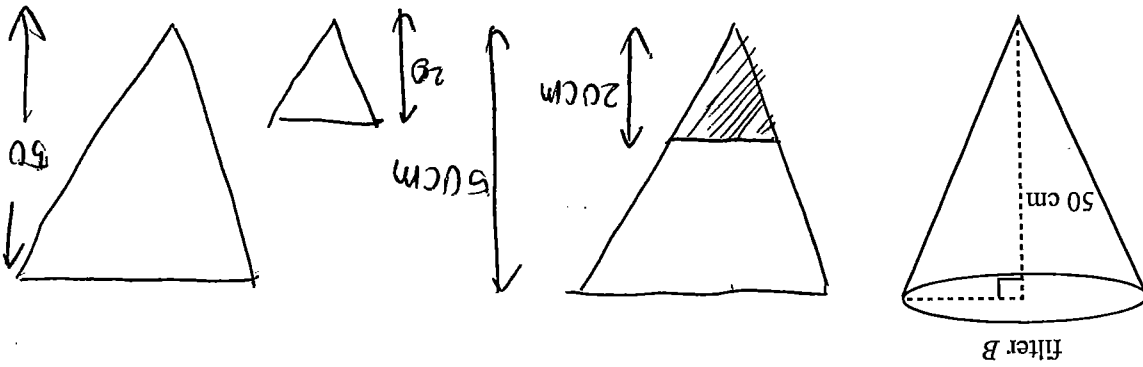
2 marks



2 marks
Total 15 marks

Similar cones: Linear scale factor: $k = \frac{50}{20} = 2.5$
 Volume scale factor = $(2.5)^3 = 15.625$
 The oil remaining, as a percentage of the volume of the entire filter = $0.064 \times 100 = 6.4\%$
 $\therefore 100\% - 6.4\% = 93.6\%$ was removed.

b. Originally filter B was full of oil, but some was removed. If the height of the oil in the cone is now 20 cm, what percentage of the original volume of oil was removed?



Filter B is a right cone with height 50 cm.

2 marks

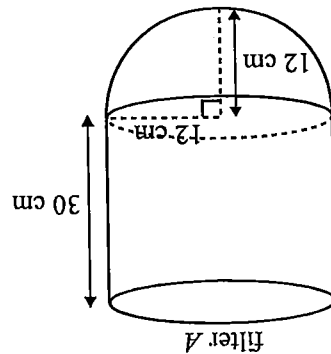
$$V = V_{\text{cylinder}} + V_{\text{hemisphere}}$$

$$= \pi r^2 h + \frac{2}{3} \pi r^3$$

$$= \pi \times 12^2 \times 30 + \frac{2}{3} \times \pi \times 12^3$$

$$\approx 17191 \text{ cm}^3$$

a. Calculate the volume of filter A. Write your answer correct to the nearest cm^3 .



The ferry has two fuel filters, A and B. Filter A has a hemispherical base with radius 12 cm. A cylinder of height 30 cm sits on top of this base.

Question 4