

**Question 7**

The random variable  $X$  has this probability distribution.

$X$	0	1	2	3	4
$\Pr(X = x)$	0.1	0.2	0.4	0.2	0.1

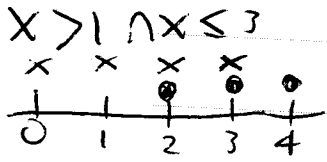
Find

a.  $\Pr(X > 1 | X \leq 3)$

$$\Pr(X > 1 | X \leq 3) = \frac{\Pr(X > 1 \cap X \leq 3)}{\Pr(X \leq 3)}$$

$$= \frac{\Pr(X=2) + \Pr(X=3)}{\Pr(X \leq 3)}$$

$$= \frac{0.6}{0.9} = \frac{2}{3}$$



b.  $\text{Var}(X)$ , the variance of  $X$ .

2 marks

$$\text{var}(X) = E(X^2) - \mu^2$$

$$\begin{aligned} E(X^2) &= 0^2 \times 0.1 + 1^2 \times 0.2 + 2^2 \times 0.4 + 3^2 \times 0.2 + 4^2 \times 0.1 \\ &= 0.2 + 4 \times 0.4 + 9 \times 0.2 + 16 \times 0.1 \\ &= 0.2 + 1.6 + 1.8 + 1.6 \\ &= 5.2 \end{aligned}$$

$$\begin{aligned} E(X) &= 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.2 + 4 \times 0.1 \\ &= 0.2 + 0.8 + 0.8 + 0.4 \\ &= 2 \end{aligned}$$

3 marks

$$\begin{aligned} \therefore \text{var}(X) &= 5.2 - 2^2 \\ &= 5.2 - 4 \\ &= 1.2 \end{aligned}$$

TURN OVER

**Question 8**

The discrete random variable  $X$  has the probability distribution

$x$	-1	0	1	2
$\Pr(X=x)$	$p^2$	$p^2$	$\frac{p}{4}$	$\frac{4p+1}{8}$

Find the value of  $p$ .

$$p^2 + p^2 + \frac{p}{4} + \frac{4p+1}{8} = 1$$

$$\therefore p^2 + p^2 + \frac{p}{4} + \frac{p}{2} + \frac{1}{8} = 1$$

$$\therefore 2p^2 + \frac{3p}{4} - \frac{7}{8} = 0$$

$$16p^2 + 6p - 7 = 0$$

$$(8p+7)(2p-1) = 0$$

$$p = -\frac{7}{8}, \frac{1}{2}$$

$$\therefore p = \frac{1}{2} \text{ (since } p > 0)$$

3 marks

**Question 7**

A biased coin is tossed three times. The probability of a head from a toss of this coin is  $p$ .

a. Find, in terms of  $p$ , the probability of obtaining

i. three heads from the three tosses

$$X = \text{no. of heads} \quad X \stackrel{d}{=} \text{Bi}(n=3, p)$$

$$\text{Pr}(X=3)$$

$$= \binom{3}{3} p^3 (1-p)^0 = p^3$$

ii. two heads and a tail from the three tosses.

$$\text{Pr}(X=2) = \binom{3}{2} p^2 (1-p)$$

$$= 3p^2 (1-p)$$

1 + 1 = 2 marks

b. If the probability of obtaining three heads equals the probability of obtaining two heads and a tail, find  $p$ .

$$p^3 = 3p^2 (1-p)$$

$$\therefore p = 3(1-p)$$

$$p = 3 - 3p$$

$$4p = 3$$

$$p = \frac{3}{4}$$

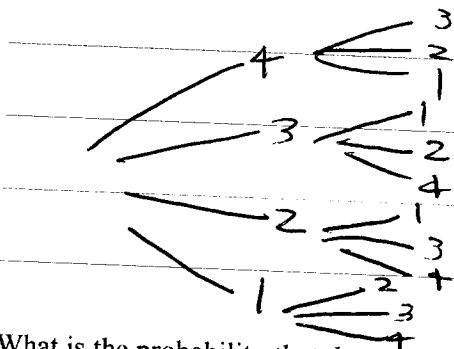
2 marks

**TURN OVER**

**Question 5**

Four identical balls are numbered 1, 2, 3 and 4 and put into a box. A ball is randomly drawn from the box, and not returned to the box. A second ball is then randomly drawn from the box.

- a. What is the probability that the first ball drawn is numbered 4 and the second ball drawn is numbered 1?



$$\Pr(\text{first 4 and second 1})$$

$$= \Pr(4, 1)$$

$$= \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

1 mark

- b. What is the probability that the sum of the numbers on the two balls is 5?

$$\Pr(4, 1) + \Pr(3, 2) + \Pr(2, 3) + \Pr(1, 4)$$

$$= \frac{4}{12}$$

$$= \frac{1}{3}$$

1 mark

- c. Given that the sum of the numbers on the two balls is 5, what is the probability that the second ball drawn is numbered 1?

$$A = \text{"sum of numbers is 5"}$$

$$B = \text{"second ball is numbered 1"}$$

$$\Pr(B|A) = \frac{\Pr(B \cap A)}{\Pr(A)}$$

$$= \frac{\Pr(4, 1)}{\frac{1}{3}} = \frac{\frac{1}{12}}{\frac{1}{3}} = \frac{1}{4}$$

2 marks

**TURN OVER**