SOLUTIONS

Year 10
Department of Mathematics 2014

Name: _____

Parade College

Tutor Group: _____

TEST: Ch 9 Functions

Section	No. of questions	Marks	Your mark
A: Vocabulary knowledge	5	5	
B: Multiple Choice	10	10	
C: Short Answer	5	25	
D: Analysis Problem	2	10	
Total		50	

Your mark is

%

Instructions

- Read questions carefully
- CAS calculators may be used.
- No sharing of equipment, including calculators.
- Use a pencil when completing questions.

Section A: Vocabulary Knowledge (5 x 1 = 5 marks)

Complete the sentence by choosing the appropriate word from the word list below.

1	The axis of symmetry is the line that divides the parabola exactly in half.	axis of symmetry
2	The turning point on a parabola can be either a minimum or a maximum.	turning point
3	The asymptote is a line on the graph which the graph approaches but never cuts or touches.	asymptote
4	For the equation of a parabola, $y = a(x - h)^2 + k$ If the magnitude of a is greater than 1, the graph is narrower than the graph of $y = x^2$.	narrower
5	If the graph of $y = x^2$ is translated h units horizontally, the equation becomes $y = (x - h)^2$.	horizontally

WORD LIST

Turning Point	Asymptote	Maximum
Wider	Vertically	Axis of Symmetry
Narrower	Horizontally	Reflects

***Section B Multiple Choice* (10 marks)**

1- The graph of $y = x^2 + 6x + 2$ is equivalent to:

A $y = (x+6)^2 - 34$

B $y = (x-6)^2 - 34$

C $y = (x+3)^2 - 7$

D $y = (x-3)^2 - 7$

E $y = (x+6)^2 + 2$

$y = x^2 + 6x + 3^2 - 3^2 + 2$

$y = (x+3)^2 - 7$

2- The translation to turn $y = x^2$ into $y = (x-1)^2 - 7$ is:

A right 1, up 7

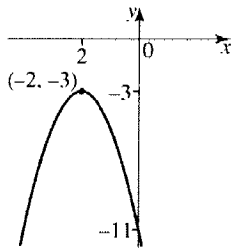
B left 1, up 7

C right 1, down 7

D left 1, down 7

E right 7, left 1

3- The equation of the graph below is:



A $y = -(x-2)^2 + 3$

B $y = -(x-2)^2 - 3$

C $y = -2(x+2)^2 - 3$

D $y = -2(x-2)^2 + 3$

E $y = -3(x-2)^2 + 3$

$y = a(x+2)^2 - 3$

When $x = 0$, $y = -11$

$-11 = a \times (0+2)^2 - 3$

$-8 = 4a$

$a = -2$ $\quad \therefore y = -2(x+2)^2 - 3$

4- Which graph has the x-intercepts closest together?

A $y = x^2 - 3x + 2$

B $y = x^2 - x - 2$

C $y = x^2 + x - 2$

D $y = 2x^2 + x - 2$

E $y = 3x^2 - 4x + 1$

A: $y = (x-2)(x-1)$

x-ints: $(1,0)$, $(2,0)$ Difference: $2-1 = 1$

B: $y = (x-2)(x+1)$

x-ints: $(2,0)$, $(-1,0)$ Difference $2 - -1 = 3$

C: $y = (x+2)(x-1)$

x-ints: $(-2,0)$, $(1,0)$ Difference: $1 - -2 = 3$

D: ~~$y = (2x-1)(x+1)$~~

Solve $2x^2 + x - 2 = 0$

$x = -1.28,\ 0.78$ Difference: 2.06

E:

Solve $(3x^2 - 4x + 1 = 0, x)$

$x = x = 1,\ 0.333\ldots$

Difference: $1 - 0.3\dot{3}$

$= 0.6\dot{6}$

5- For the graph $y=-2\left(x+\frac{1}{2}\right)^2-5$, the effect on the graph of the 2 before the brackets is:

A no effect
B translates the graph up 2
C makes the graph narrower
D makes the graph wider
E inverts the graph

Dilation factor $=2$ $a>1$
$\therefore$ steeper

6- Which graph has the smallest y-intercept?

A $y=3(x-2)^2+6$
B $y=5(x-1)^2-2$
C $y=\frac{1}{2}(x+2)^2-1$
D $y=-2(x+2)^2+5$
E $y=-5(x-2)^2+3$

A: let $x=0$ $\therefore y=3\times4+6=18$
B: let $x=0$ $y=5\times(-1)^2-2=3$
C: let $x=0$ $y=\frac{1}{2}\times(2)^2-1=1$
D: let $x=0$ $y=-2(2)^2+5=-3$
E: let $x=0$ $y=-5(0-2)^2+3$
$=-5\times4+3$
$=-17$

7- For $y=-2x^2+7x-3$ the x-intercepts and y-intercept, respectively, are:

A $\frac{1}{2}$, -3 and -3
B $-\frac{1}{2}$, 3 and -3
C $\frac{1}{2}$, 3 and 3
D $-\frac{1}{2}$, 3 and 3
E $\frac{1}{2}$, 3 and -3

y-int: let $x=0$
$\therefore y=-2\times0+7\times0-3=-3$

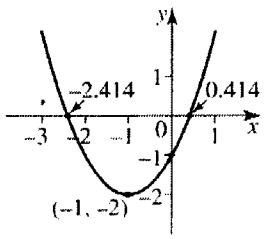
x-int: solve $(-2x^2+7x-3=0, x)$

$x=0.5, 3$

$\therefore x$-ints: $\frac{1}{2}$, 3

8- The graph that best represents $y = x^2 - 2x - 1$ is:

A

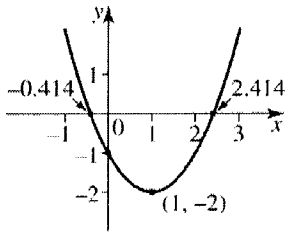


$y = x^2 - 2x + 1^2 - 1^2 - 1$

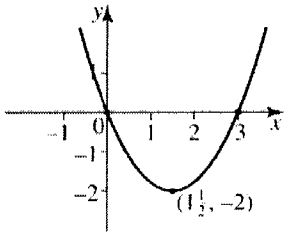
$= (x-1)^2 - 2$

$\therefore T/p = (1, -2)$

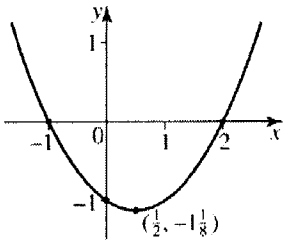
B



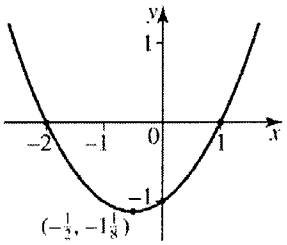
C



D



E



9- The point (0, 1) does not lie on the graph:

A $y = 3^x$

B $y = \left(\frac{1}{2}\right)^x$

C $y = 3 \times 2^x$

D $y = 4^{2x}$

Sub. $x = 0$ in

A: $y = 3^0 = 1$

B: $y = \left(\frac{1}{2}\right)^0 = 1$

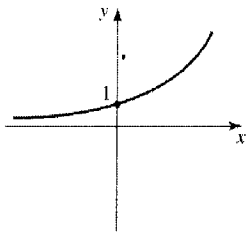
C: $y = 3 \times 2^0 = 3 \times 1 = 3$

E $y=-3^x$ Sub. $x=0$: $y=-3^0=-1$

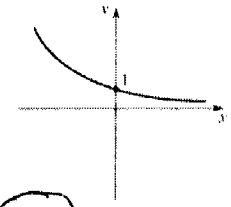
Does not lie On C or E !!

10- The graph of $y=-3^x$ is best represented by:

A

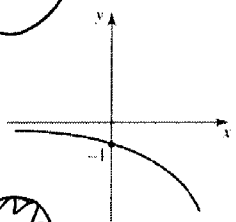


B

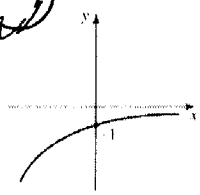


C

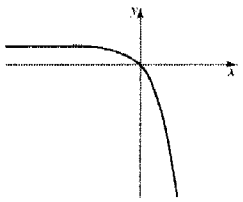
Graph on CAS.



D



E



Section C- Short answer (25 marks)
Working out must be shown to gain full marks.

Question 1

(a) Complete the square for $x^2 - 4x - 4$ (3 marks)

$x^2 - 4x - 4$

$= x^2 - 4x + 2^2 - 4 - 2^2$

$= (x-2)^2 - 8$

(b) What is the turning point of $y = x^2 - 4x - 4$ (1 mark)

$(2, -8)$

(c)Is the turning point a maximum or a minimum? (1 mark)

Minimum

(c) Find the value of the discriminant for $y = x^2 - 4x - 4$ and state how many solutions this gives (2 marks)

$\Delta = b^2 - 4ac$

$a = 1,\ b = -4,\ c = -4$

$\therefore \Delta = (-4)^2 - 4 \times 1 \times -4$

$= 16 - -16 = 32$

$\therefore$ Two solutions (since $\Delta > 0$)

(d) Find the value of any x-intercept(s) of the function $y = x^2 - 4x - 4$ (2 marks)
Write your answer in exact form.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \dfrac{+4 \pm \sqrt{32}}{2}$

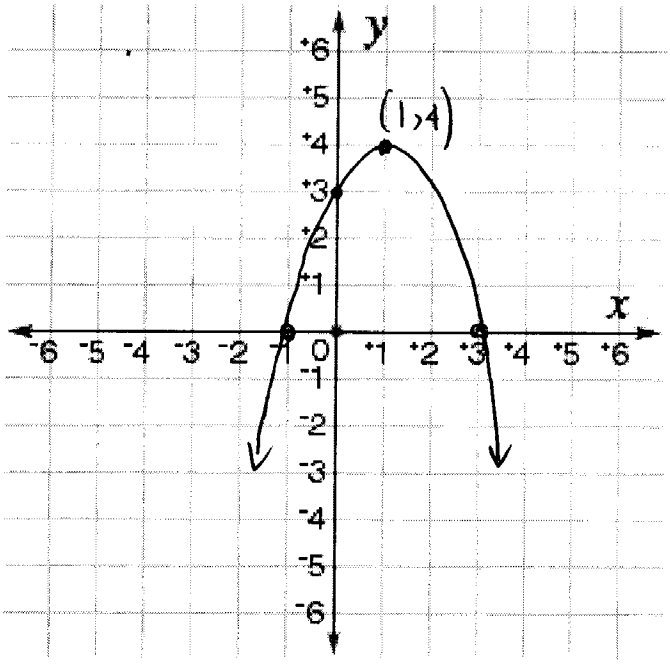
$= \dfrac{4 \pm \sqrt{16}\sqrt{2}}{2}$

$= \dfrac{4 \pm 4\sqrt{2}}{2}$

$= 2 \pm 2\sqrt{2}$

Question 2

(a) On the grid provided, sketch the function $y = -(x-1)^2 + 4$.
Make sure to include the coordinates of the turning point, y-intercept, and x-intercepts (if any).
(3 marks)



T/p: $(1, 4)$

x-int: let $y = 0$

$0 = -(x-1)^2 + 4$

$\therefore (x-1)^2 = 4$

$x - 1 = \pm\sqrt{4}$

$x - 1 = 2$ or $x - 1 = -2$

$x = 3$ or $x = -1$

y-int: let $x = 0$

$\therefore y = -(0-1)^2 + 4$

$= 3$

(b) What type of the turning point is it? (1 mark)

Maximum

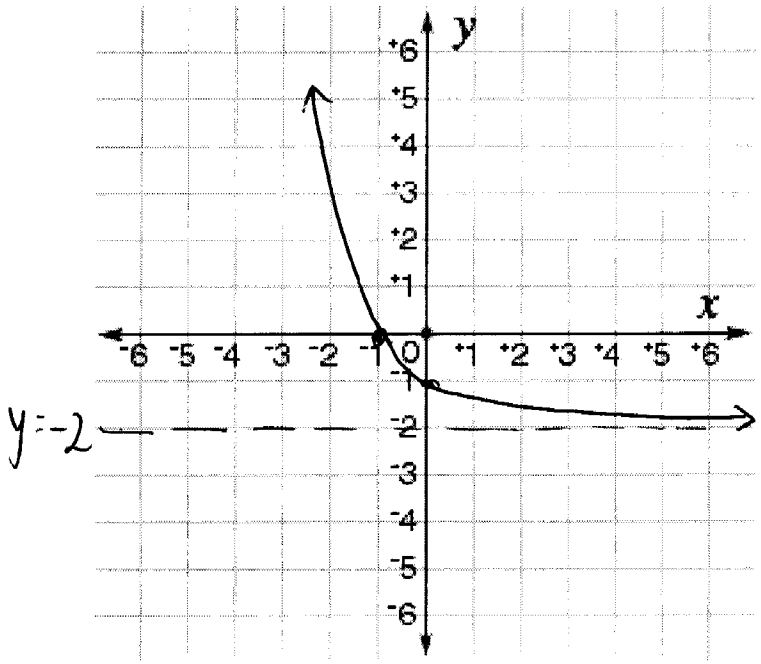
(c) What is the equation of the axis of symmetry? (1 mark)

$x = 1$

Question 3

(a) Sketch the graph defined by the equation $y = 2^{-x} - 2$

Include any intercepts and asymptote. (3 marks)



x-int: let $y = 0$

solve $(2^{-x} - 2 = 0, x)$

$\therefore x = -1$

y-int: let $x = 0$

$y = 2^0 - 2$

$= 1 - 2$

$= -1$

Asymptote: $y = -2$

***Section D: Analysis Questions*: (10 marks)**

Question 1

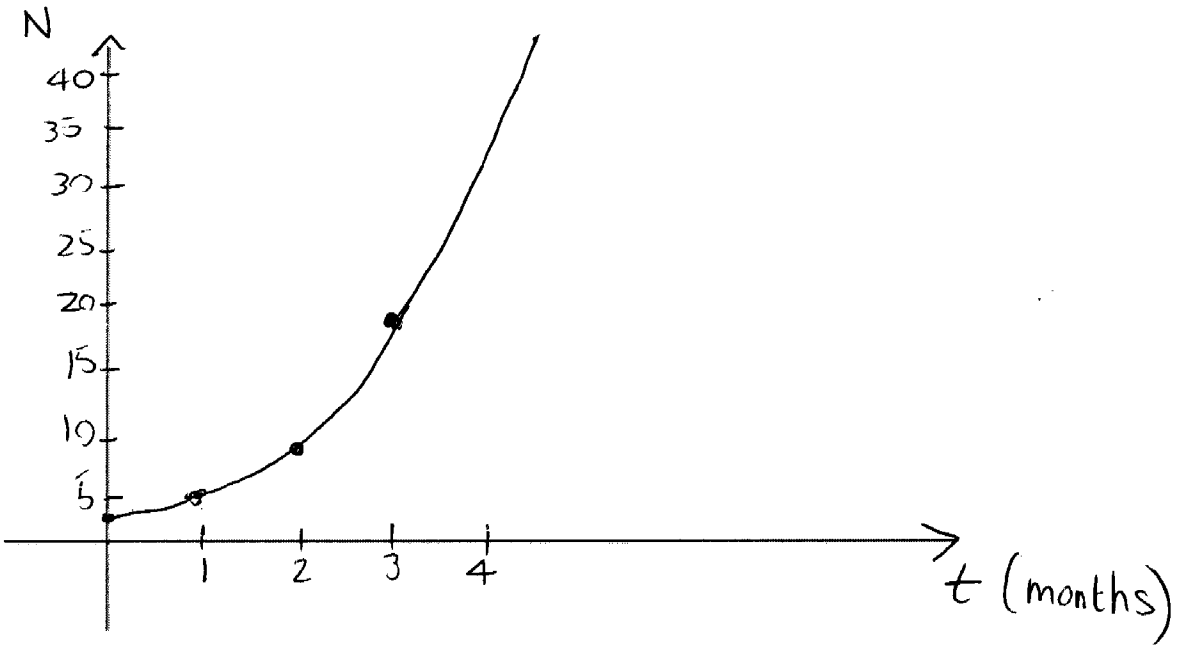
A scientist working at the CSIRO found that the number of mice kept in the laboratory for testing purposes could be found by the equation $N = (2.5)^t + 2$

Where N = the number of mice and t = the number of months after the breeding program began.

(a) Complete the table of values for $0 \leq t \leq 4$. Substitute integer values into the equation and use a calculator to find the corresponding values for N. Round your answer to the nearest whole number. (2 marks)

t	0	1	2	3	4
N	3	5	8	18	41

(b) Sketch a graph using the table of values.
Use an appropriate scale and label both axes. (2 marks)



(c) What is the significance of the N value at t = 0? (1 mark)

No. of mice initially

(d) How many mice will there be after 1 year? (1 mark)

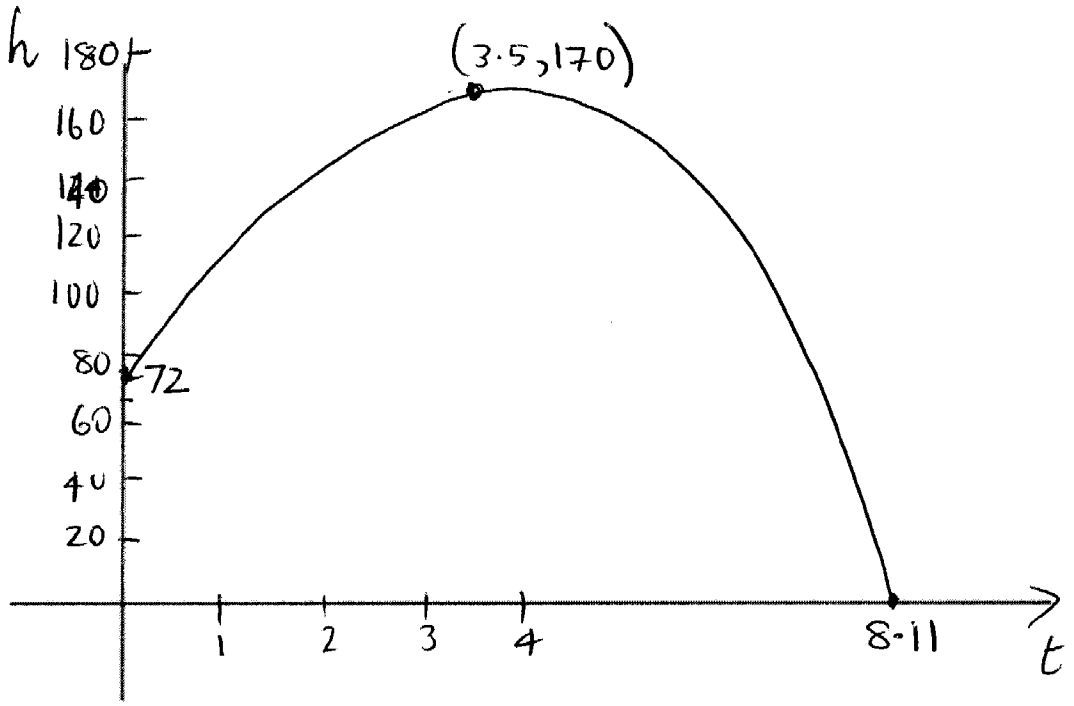
5

$f_1(x) = -8x^2 + 56x + 72$ on CAS.

Question 2

A flare is launched from the side of a cliff 72 m high. The flare follows the path given by the formula $h = -8t^2 + 56t + 72$ where h is the height in metres and t is the time in seconds.

(a) Sketch the graph using an appropriate scale for t > 0. (2 marks)



(b) Using your calculator, determine for how many seconds the flare is in the air. (1 mark)

Solving $0 = -8t^2 + 56t + 72$

$t = -1.11, 8.11$

∴ The flare is in the air for 8·11 seconds

(c) What is the maximum height that the flare achieves? (1 mark)

170 m